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Geant4 Tool for Simulation of Arbitrarily Defined Fresnel Lenses

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Abstract—We report on creating a module for the Geant4 toolkit for a simulation of point-by-point defined Fresnel lenses and mirrors. This module provides creation of cylindrical Geant4 shapes with one or two Fresnel surfaces, defined by the arrays of points ($z(\rho)$).

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INTRODUCTION

A Fresnel lens is a composite lens consisting of individual rings of definite thickness (Fig. 1). The lens profile can on average be both flat and spherical or of a more complex shape depending on the purpose. The surface of the rings is calculated starting from the requirements on the lens characteristics and in the general case has no particular analytical representation.

Fresnel lenses (as well as mirrors) are rather widely used in everyday life (lighting devices, photo flares, light magnifying glasses, overhead projectors, etc.) and in science, for construction of space telescopes and detectors. When Fresnel lenses are employed in pursuing scientific aims, one may need a simulation of not only the optical properties of a detector but also of the response to the passage of any particles in addition to the photons. To do this, it is necessary to describe an optical system as part of a complex detector in the format of one of the available simulation toolkits, for example, Geant4 [1].

Geant4 is a toolkit for simulating the passage of particles through matter. The major areas of application are high-energy physics, nuclear physics, and accelerator physics, as well as medicine and dosimetry. This module contains a code for simulating a large number of particles and their interactions for which various models and parametrizations are realized. Aside from the major set of physical interactions, Geant4 also describes optical processes, so Geant4 may be used to describe and simulate complex optical systems.

The complexity of simulated optical systems is, however, limited by the resources of Geant4 in describing the geometry. This limitation may be obviated by developing independently a code for simulating complex elements in the geometry of the optical system. Having run into problems when simulating an optical system in the JEM-EUSO experiment [2], we decided to describe the geometry of a Fresnel lens to be used in Geant4. By means of the module we developed, we could simulate one- or two-surface Fresnel lenses and Fresnel mirrors, as well as conventional thin and thick lenses with a surface of arbitrary cylindrically symmetric shape.

1. SIMULATION OF FRESNEL LENSES

1.1. Terms and Designations Used

Here, we present the designations used in this paper:

Fresnel surface—a cylindrically symmetric surface with piecewise smooth profile (Fig. 1), one of the surfaces of the Fresnel lens or mirror.

“Special surface”—a surface with profile described by the function $\rho^2 = az^2 + bz + c$. The form of the function has been selected because of the simple way of finding the intersection with a straight line.

CSG (Constructive Solid Geometry)—constructive block geometry. This is the basic method of geometry description used in Geant4.

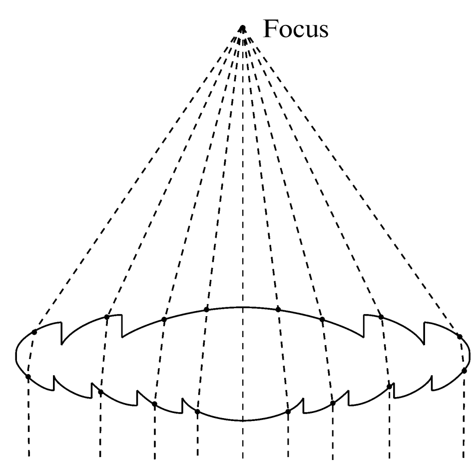

Fig. 1. Schematic image of a Fresnel lens.

CSG_U and CSG_S denote the method of creating geometric objects in Geant4: with logic unification of the component parts of the lens (CSG_U, "united") and without logic unification (CSG_U, "separate").

Own_S and Own_D denote our method of simulating the Fresnel lenses in Geant4. Own_D ("direct"): a direct exhaustion of possible solutions is used to seek an intersection. Own_S ("smart"): an optimized exhaustion (default behavior) is used.

1.2. Problems of Fresnel Lens Description in Geant4

Within the CSG, the detector geometry is described by means of elementary blocks. The blocks can be both simple (cube, sphere, cone) and more intricate (paraboloid, twisted parallelepiped, and so on). More complex elements can be created by operations of "intersection" of blocks, "unification," or "subtraction"; however, as prescribed in Geant4, individual blocks are so arranged in space that they must not intersect each other or have common surfaces—they should necessarily be separated by a minimum distance. Otherwise, the block must be integrated into a single logic object.

When used to describe Fresnel lenses, such an approach has the following drawbacks:

—Because of its intricate shape, the description of a Fresnel lens requires a combination of a large number of blocks. In the simplest case, for every lens ring there is a block. Upon unification, the computational complexity of the simulation increases in proportion to N^3 of the number of the combined parts. The simulation of 10^5 photons passing through a Fresnel lens with 800 rings may take over a day.[1]

—If the lens blocks are not combined, the simulation time grows with N and the preceding example will take a few minutes. In this case, the description of the geometry is, however, distinctly more intricate because the parts of the lens may neither intersect nor be apart. The minimum distance to be left between the blocks produces hollows in the lens on which a photon can be reflected. In this case, the share of improperly calculated trajectories may amount to 5% of their total number. Therefore, this solution cannot be recommended for precision computations.

—The basic geometry elements suitable for describing a Fresnel lens are a cone and a paraboloid. When the lens ring can be described by none of these elements within the required accuracy, it should be made up of a few blocks, thus increasing the number of individual parts of the lens.

1.3. New Module for Fresnel Lens Description

It is clear from the foregoing subsection that a Fresnel lens had to be described as a separate object for which the intersection search algorithm is realized in an explicit form. This will allow one to compensate for a slowing down caused by the unification of the lens parts and to avoid the errors associated with the hollows inside the lens and deviations of the approximated surface from the real one.

In the development of the module, the following goals were set:

—Simplicity of the lens description, which is unaffected by the complexity of the lens itself and the number of rings. The optical system of the JEM-EUSO experiment contains lenses with up to a few thousand rings.

—High accuracy of simulation. It is necessary to simulate events with the number of photons being $\sim 10^5 - 10^7$ (light from cosmic rays with $E \gtrsim 10^{20}$ eV) in a reasonably short time.

In the new module, the lens is "assembled" from individual parts (surfaces), each properly connected to the neighboring ones. At present, the following simple surfaces are available: plane, cylinder, sphere, paraboloid, "special surface," and a surface described by an array of points.

Two surfaces are required to describe a lens. This may be a thin spherical lens, a thick lens with one spherical and one parabolic surface, a lens with one or two Fresnel surfaces, and so on.

The Fresnel surface is given as a set of simple surfaces. For optimizing the calculation, it is also essential to specify two additional surfaces, each coinciding in radius with the lens and bounding it on one side. The closer the bounding surfaces adjoin the Fresnel surface, the faster the search for the ring that crosses the path of a particle. The cases with multiple intersections are processed properly.

The basic element in the description of a Fresnel lens is a surface given by points (a numerical surface). A special-purpose algorithm is developed to search for such a surface. The algorithm is optimized and converges in a minimum number of iterations; the convergence rate depends little on the position and direction of motion of a particle. In addition, if required, it is possible to search for intersection by a direct exhaustion of the array of points.

The Fresnel surface can also be described by using other available types of surfaces. Of particular note is the "special surface" which is realized for approximation of a numerical surface and is well suited for description of the lens rings, because, on the one hand, this surface has more degrees of freedom than, for instance, a paraboloid and, on the other hand, the equation of intersection of a straight line with such a surface is solved analytically.

[1] The simulation was performed using a computer with an Intel Core2 Quad CPU @ 2.66 MHz processor.

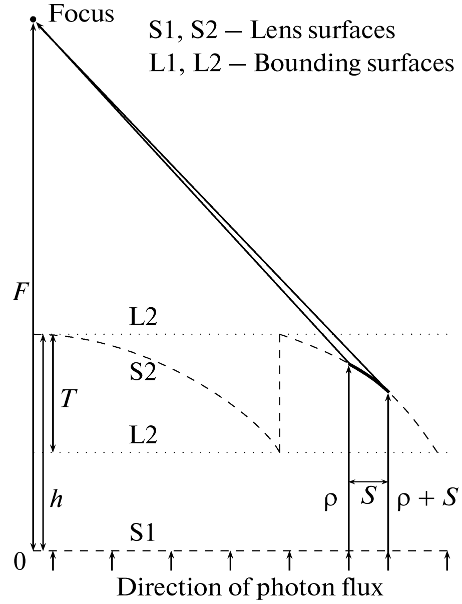

Fig. 2. Diagram of constructing a test lens.

The module includes a set of auxiliary functions to simplify the creation of a Fresnel lens and automatic approximation of its rings.

2. SIMULATION

2.1. Description of a Test Lens

A flat Fresnel lens with one focusing side (Fig. 2) was parametrized to check the method. The parameters of the lens are as follows:

- Lens thickness (h)—6 mm.
- Thickness of the Fresnel surface (T)—3 mm.
- Focused wavelength (λ)—500 nm.
- Focal length (F)—10 m.
- Number of rings (N_r)—10.
- Lens transverse radius (R)—60 cm.
- Step of surface radius discretization (S)—0.5 μm.

The lens surface is described from the center to the edge with step S. Each step is a cone [ρ, $\rho + S$] (Fig. 2) that focuses photons with wavelength λ at distance F from the lens center. The coordinates of the surface points at each step are related by

$$z^{\text{true}}(\rho) = \begin{cases} \rho_0 = 0, \quad z_0 = h; \\ \rho_i = \rho_{i-1} + S; \\ z_i = z_{i-1} + S\dfrac{\sin\alpha}{n(\lambda) - \cos\alpha}. \end{cases} \quad (1)$$

$$\tan\alpha = \frac{\rho_{i-1}}{F - z_{i-1}}, \quad (2)$$

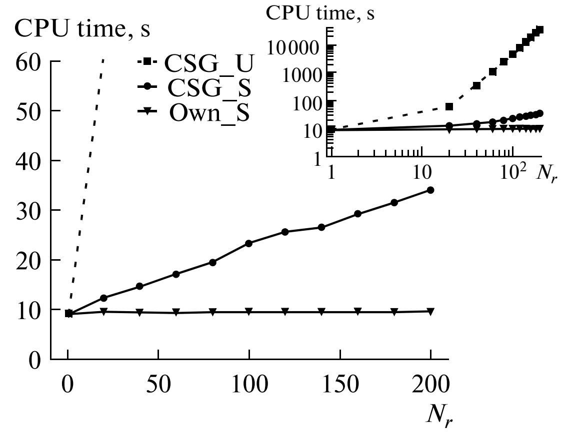

Fig. 3. Simulation time versus number of rings in the lens.

where $n(\lambda)$ is the refraction index of the lens material.

The construction of one ring is finished when its thickness reaches T. The lens is constructed until N_r rings are described. Thus, for the given lens, the transverse radius depends on the number of rings at constant T, F, and λ. The focal distance is chosen such that the ratio R/F is kept rather small. In this case, the lens is close to a flat lens and can be represented as a sphere or a paraboloid.

In testing the algorithms, primarily the variables that affect the output were tested: the number of lens rings, surface discretization step, and angle of photons incidence.

For the CSG_U and CSG_S methods, the lens rings are approximated ($z^{\text{approx}}(\rho)$ by paraboloids so that the approximation satisfies the constraint

$$\left|z^{\text{approx}}(\rho) - z^{\text{true}}(\rho)\right| \le 1\ \mu\text{m}. \quad (3)$$

In the case where the deviation of the paraboloid exceeds the permissible one, the lens ring is approximated by two paraboloids and so on until the deviations of all the paraboloids satisfy the accuracy requirements.

2.2. Simulation of a Test Lens

The simulation rate depends primarily on the number of elements according to which the exhaustion is performed: the number of lens rings and the number of ring segments. In our module, we tried to minimize the dependence on N_r through the use of bounding surfaces. Figure 3 shows the simulation time of the passage of 10^5 photons through the lens plotted versus the number of rings. Plots for the lens described by means of Geant4 are constructed for comparison. It is seen that, for Own_S, the simulation time depends little on the number of elements of the lens. For CSG_S and CSG_U, the relationship is proportional, respectively, to the first and third power of the number of lens elements, as was expected.

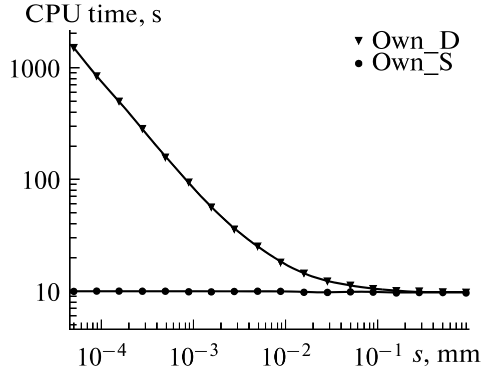

Fig. 4. Simulation time versus surface discretization step.

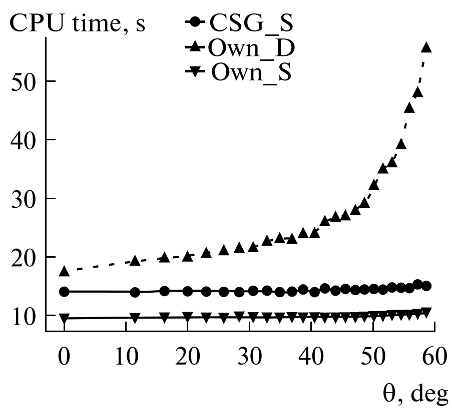

Fig. 5. Simulation time versus photon incidence angle.

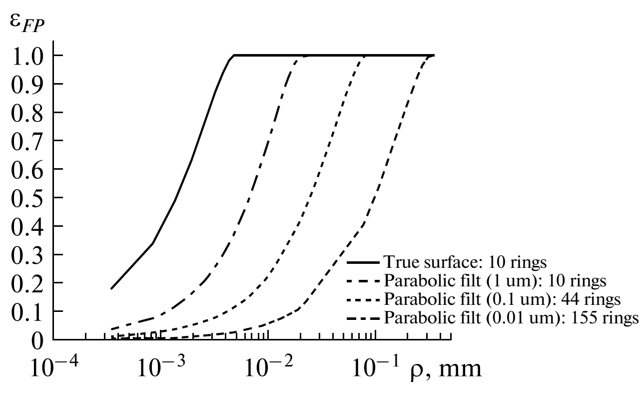

Fig. 6. Focusing efficiency as a function of the radius of a circle at the focal surface.

The number of the exhausted elements of the ring is in inverse proportion to the step with which the lens is constructed. To minimize the dependence on these variables, special-purpose algorithms were developed to search for the intersection of the ring with the lens. Figure 4 indicates that the simulation is almost unaffected by the surface discretization step. This is also why there is scarcely any dependence on the angle of photon incidence on the lens (Fig. 5). For comparison, both figures present the dependence for a search for intersection by using a direct exhaustion (Own_S), whose operating rate is directly related to the magnitude of the step.

Figure 6 shows the focusing efficiency of the photons incident at an angle of 0° for the given lens. The focusing efficiency is determined as the ratio of the number of photons situated inside a circle of radius r ($N(\rho < r)$) to the total number of photons at the focal surface ($N(FP)$):

$$\varepsilon_{FP}(r) = \frac{N(\rho < r)}{N(FP)}. \quad (4)$$

Normalized to the number of photons at the focal surface, the efficiency does not take into account losses associated with reflection, absorption, and peculiarities of describing the geometry in Geant4 (see subsection 1.2), which are not discussed in the present paper. The results of simulation of a true surface, as well as of its approximations done with various accuracies, are presented.

As is seen from the plots, with better accuracy, the CSG focusing efficiency approaches the efficiency of our method; however, the number of component blocks of the lens also increases. So the lens described in the CSG geometry with an accuracy of 0.01 μm consists of 155 paraboloids and an additional some 140 coupling cylinders, resulting in about 100 and 4 times longer simulation as compared to our method for CSG_U and CSG_S, respectively.

2.3. Application

The proposed module is used in the simulation program of the JEM-EUSO ESAF experiment [3] as an additional module for simulating the optical system of the detector with allowance for secondary elements and supporting structure. The module is used to simulate not only optical photons but also background from passing cosmic rays.

CONCLUSIONS

In the present paper, a new module for simulation of lenses and mirrors in the Geant4 environment has been proposed. Plots of the simulation rate and of the focusing efficiency have been given. The proposed software is distributed freely and available on request.

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