A Geant4 Based Engineering Tool for Fresnel Lenses

João Costa, Mário Pimenta, and Bernardo Tomé

***Abstract*—Geant4 is a Monte Carlo radiation transport toolkit that is becoming a tool of generalized application in areas such as high-energy physics, nuclear physics, astroparticle physics, or medical physics. Besides the electromagnetic and hadronic physics processes, Geant4 provides also an optical physics process category, allowing the simulation of the production and propagation of light. Such capabilities are well tailored for the simulation of optics systems namely in cosmic-rays experiments based in the detection of Cherenkov and fluorescence light. The use of Geant4 as an engineering tool for the optics design and simulation of Fresnel lens systems is discussed through a specific example. It is thus possible to implement a realistic end-to-end simulation of a physics experiment using Fresnel lenses in the framework of Geant4.**

***Index Terms*—Cherenkov telescope, cosmic-rays, Fresnel lens, Geant4, Monte Carlo simulations, optics analysis, optics design.**

I. Introduction

The use of Fresnel lenses has been discussed in the last few years in the context of cosmic-rays experiments based on the detection of the Cherenkov and fluorescence light produced by the extensive air showers [1], [2], [17], [3]. Nevertheless, the first attempt to detect this fluorescence light, was made in 1967 by the Cornell Experiment using a detector based on a Fresnel lens [4].

Most of Cherenkov telescopes have large diameter mirrors but their field of view is small due to the degradation of imaging quality for off-axis angles and to the need of using small cameras at the focal plane to limit the mirror obscuration. To cover a large area of the sky, a possible alternative is the use of refractive optics with the camera located behind the lens. However several problems are posed by normal lenses, namely: the amount of light absorbed in the lens due to its thickness, the lens weight and the mechanical positioning of the lens in the detector. Fresnel lenses are a viable alternative [5] due to their small thickness and lightness.

The design and optimization of the optical system of such detectors is usually performed with commercial programs. In a first step, qualitative studies are undertaken for the more relevant wavelengths allowing to determine the main characteristics of the lens. Thorough and quantitative studies taking into account the precise geometries and the properties of the optics materials are then performed using feature laden commercial programs. However, these engineering packages are usually not included in the simulation programs of the experiments, where simplified ray-tracing models or lookup tables have to be used.

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In this paper a simulation tool for Fresnel lenses using the Geant4 toolkit [6]–[8] is presented. It is shown how this tool can be explored to perform realistic simulations and optimizations of optics systems using Fresnel lenses. The implementation of the lens simulation using Geant4 is first presented. Then an example of a specific lens is used to illustrate some of the performance studies that can be undertaken with this simulation tool. The dependence of the point spread function with the lens groove-density is first shown. The chromatic performance is studied for monochromatic light and for polychromatic light following a spectrum characteristic of the Cherenkov-light emission. The lens light-transmission is also investigated. Finally the effects of mechanical misalignments of the lens components are illustrated.

II. The Geant4 Toolkit

Geant4 is a simulation toolkit widely used in particle physics, medical physics and astroparticle experiments. It is a Monte Carlo code to simulate the particle transport and interaction in matter, with tracking capabilities in 3D geometries of arbitrary complexity. This toolkit is based on an Object Oriented design which entails adaptability to various simulation applications and permits the upgrade of existing physics processes or the implementation of new ones.

The Geant4 toolkit includes an extensive set of electromagnetic, hadronic and optics physics processes [6]–[9]. Optical photons are generated through scintillation, Cherenkov and transition radiation or can be explicitly emitted by a light source. The tracking of optical photons includes refraction and reflection at medium boundaries, Rayleigh scattering and bulk absorption. The optical properties of a medium, such as refractive index, absorption length, or reflectivity, are expressed as a function of the wavelength. The optical characteristics of the interfaces between different media can be described through the UNIFIED optical model [10]. This model provides a realistic description of surface finish and reflector coatings. Surface roughness is described by considering a surface to be made of micro-facets with normal vectors following a given distribution around the nominal normal to the surface at the specified position. The UNIFIED model provides also several reflection mechanisms where the radiant intensity of a surface is controlled through parameters defining the relative probabilities for specular reflection about the normal of a micro-facet, specular reflection about the average normal of the surface, backward reflection and internal Lambertian reflection.

III. Lens Geometry Implementation

A Fresnel lens is constructed from a given lens surface by decomposing it into small pieces, each corresponding to a groove

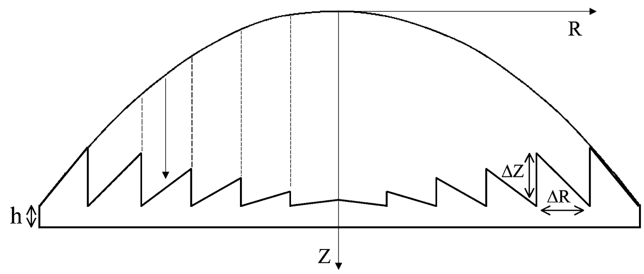

Fig. 1. Cross-section sketch illustrating the decomposition of a plano-convex lens into a Fresnel lens. Each groove of the Fresnel lens is build from a piece of the original lens surface translated to the plano side of the lens. h is the thickness of the Fresnel lens, while ΔR and ΔZ are the radial span and height of each groove, respectively.

of the Fresnel lens, as illustrated in Fig. 1. Although the surface of each groove may have the shape of the corresponding piece on the original lens, for a sufficiently large number of grooves a good approximation consists in implementing conical shaped grooves, each being a frustum of a cone with a cross-section represented by a straight line. The slopes of these lines are obtained from the sagita equation of the original surface, $z = f(r)$, where z is the distance of the surface from the plane tangent to the lens vertex $(z = 0)$ and r is the distance of the point on the lens from the lens axis.

The present implementation of the Fresnel lens simulation was developed for a Geant4-based simulation of a gamma-ray Cherenkov telescope under development. The class FresnelLens (Fig. 2) holds the geometrical and physical properties of the lens. A FresnelLens object is defined by specifying the minimum and maximum lens radius, the starting azimuthal angle and azimuthal angle span, the lens thickness, material and number of grooves. The lens can additionally be rotated around a specified axis. These features allow to describe either a monolithic lens or a generic lens petal, covering a limited azimuthal range. The lens parameters are data members of the FresnelLens class which can be retrieved through access operations, while the lens profile equation is defined by the GetSagitta() method. In the present simulation the lens grooves have a conical shape, with cross-sections represented by straight lines. The concentric grooves have the same radial span ΔR-constant width grooves. The geometry of a Fresnel lens is then defined using a parameterized replication of G4Cons volumes, through the FresnelLensParameterisation class which derives from the Geant4 class G4VPVParameterisation. Geant4 parameterized volumes are multiple copies of a volume differing in size, shape parameters, solid type or material, where these properties can be parameterized as a function of the copy number. The GetSagitta() method of the FresnelLens class is used by FresnelLensParameterisation to compute the slope of each groove and thus the parameters of each corresponding G4Cons volume.

IV. Simulation of a Specific Lens

As a working example a 1.7 m diameter plano-convex Fresnel lens was simulated. This example was inspired by the innovative approach exploited by the GAW [3] collaboration. The implemented Fresnel lens consists of a central monolithic lens and

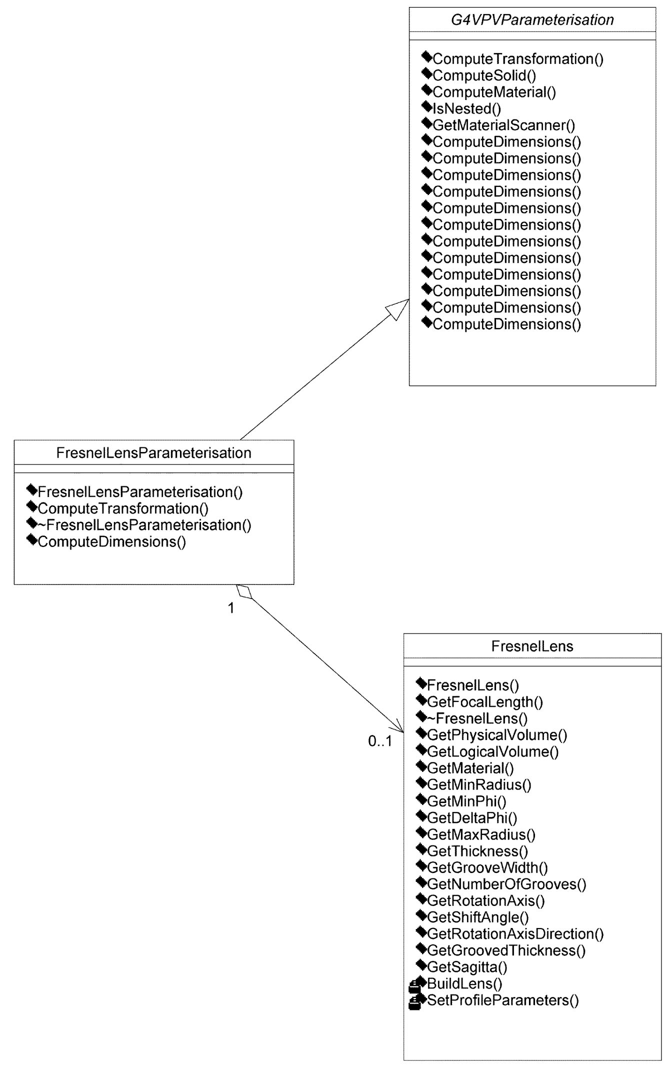

Fig. 2. The design of the Fresnel Lens class. The lens geometry is defined through the FresnelLensParameterisation class.

two concentric rings of lens petals, as it is shown in Fig. 3. The zoomed image shows the groove structure of the lens.

A. Specifications of the Simulation

The lens profile was based on an optimization performed using the OSLO [11] optics software. The profile equation, $z = f(r)$, describes an aspheric even surface:

$$f(r) = \frac{C \times r^2}{1 + \sqrt{1 - (1+k) \times C^2 \times r^2}} + A_1 \times r^2 + A_2 \times r^4 + A_3 \times r^6 \quad (1)$$

where r is the distance of the point on the lens surface from the lens axis, 1/C is the curvature radius and k is the conic constant. The A_i's are the coefficients of the even polynomial describing the deviation of the surface from a conic. For the chosen lens profile a parabolic surface was used $(k = -1)$.

As a working example, the lens optimization was performed for $\lambda = 320$ nm. The main parameters defining the lens are the lens curvature, the aperture, the index of refraction of the lens

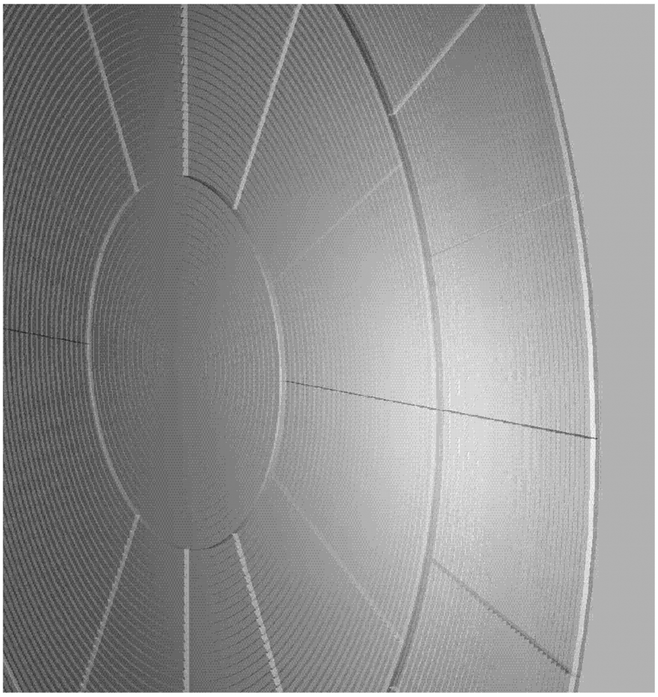

Fig. 3. Visualization of the Geant4 implementation of the Fresnel lens, showing a detail of the petal and groove structure.

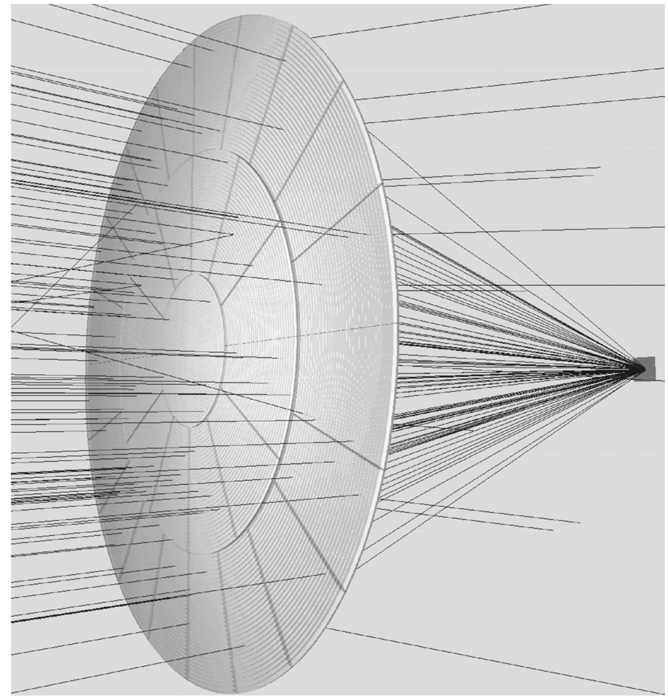

Fig. 4. Visualization of the simulated Fresnel lens, illustrating the focusing of on-axis light incident from the left.

TABLE I
PARAMETERS OF THE SIMULATED FRESNEL LENS

Curvature radius (1/C)	1065 mm
A_1 A_2 A_3	4×10^{-7} mm^{-1} 7×10^{-12} mm^{-3} -9×10^{-19} mm^{-5}
Focal length (λ = 320 nm)	2025.56 mm
Aperture radius	844 mm
Groove density	2/mm and 0.333/mm
Material	UV transmitting acrylic
Refraction index	Wavelength dependent
Transmittance of 3 mm thick slab (for λ >350 nm to the near infrared)	90%

material and its thickness. The lens material is a typical ultraviolet (UV) transmitting acrylic, with optical properties similar to those previewed for the ULTRA experience [2], [17]. The light transmittance of 3 mm of this material is of the order of 90% for $\lambda > 350$ nm, dropping to about 50% at $\lambda \sim 270$ nm and falling to zero for $\lambda < 250$ nm.

The main specifications for the Geant4 simulation are shown in Table I. The results presented here were obtained using version 7.1 of Geant4, with the following optical photon processes: absorption inside optical media (G4OpAbsorption), Rayleigh scattering (G4OpRayleigh) and the processes at boundaries between media (G4OpBoundaryProcess).

The approach followed in this example allowed to define in a simple way a base lens profile with negligible spherical aberration. However the OSLO description of Fresnel lenses presents several limitations. In OSLO no account is taken for the actual height of the Fresnel lens grooves [12], introducing some error in the trajectory of the refracted rays and not accounting for light losses due to multiple refractions at the grooves, as is shown below. In addition, aberrations due to flat surface grooves are also not covered by the Fresnel lens description in OSLO. With Geant4 the lens geometry can be implemented in greater detail, allowing the development of realistic simulations. The polychromatic analysis in OSLO is limited to a finite number of discrete wavelengths. Although each wavelength can be assigned a weight, the implementation of a generic light spectrum is not straightforward. User-defined emission spectra can be readily introduced in Geant4 using the General Source Particle module [13]. Since these features affect the lens focusing performance, a possible alternative would be to explore Geant4 also in the optimization of the Fresnel lens profile.

Two lenses were simulated, with groove densities of 2 grooves/mm and 0.333 grooves/mm, made of a central monolithic lens with several lens petals arranged in two layers (Fig. 3). The light detection was achieved through a Geant4 sensitive detector placed at the desired focal distance. Its size, about 30×30 cm^2, reproduces the size of an array of photomultipliers. Finally, in order to describe realistic observation conditions the dependence of atmospheric parameters such as pressure, temperature, density and index of refraction, with altitude was implemented following the U.S. Standard Atmosphere [14] parameterization. The wavelength dependence of the air index of refraction was also taken into account. The optimization studies described herein were performed at sea-level conditions, but simulations and optimizations of a specific setup should take into account the atmosphere properties at the specified location.

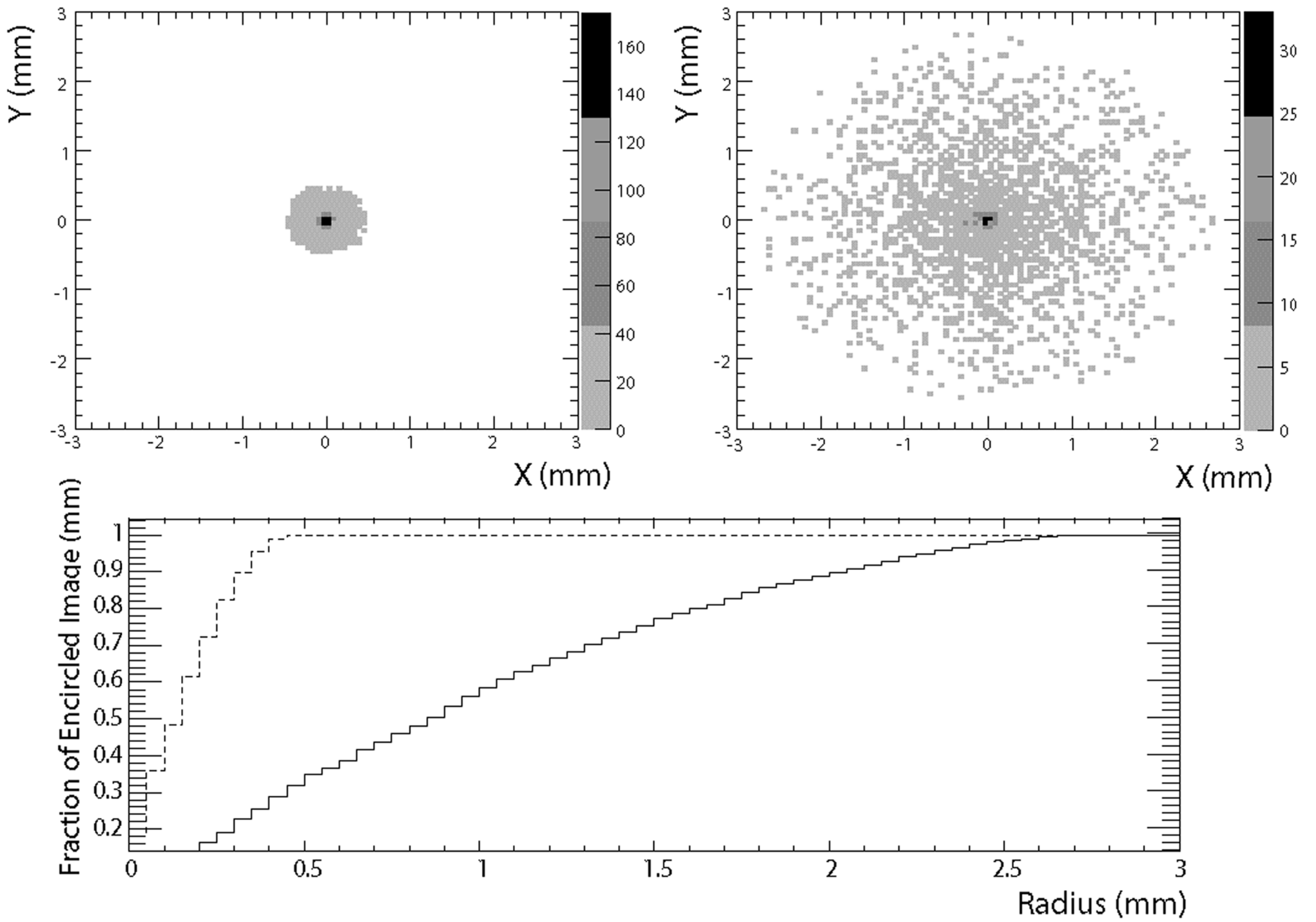

Fig. 5. Point spread function for on-axis incidence of monochromatic light ($\lambda = 320$ nm). Upper left: lens with groove density of 2/mm. Upper right: lens with groove density of 0.333/mm. Bottom: Encircled image versus radius for lenses with groove density of 2/mm (- - -) and 0.333/mm (—).

B. Analysis of the Optics

In the lens design and optimization several parameters must be taken into consideration, namely the magnitude of the aberration, characterized by the point spread function (PSF), the lens transmittance and the detector field of view (FOV). For the purpose of characterizing the present lens, some of its optimization parameters were varied: the density of the grooves, the thickness of the lens, and the distance from the lens to the detection plane. Some of the results obtained in this context are shown hereafter with the main purpose of illustrating the capabilities of the Geant4-based optics simulation tool. A complete optimization of a detector and the systematic characterization of its performance is out of the scope of this paper.

As a first example, the dependence of the PSF with the groove density was studied for UV light ($\lambda = 320$ nm) incident parallel to the lens optical axis ($\theta = 0^{\circ}$), as shown in Fig. 4. The spot images at the nominal focal distance ($\mathrm{d} = 2025.56$ mm) and the corresponding encircled image, the fraction of photons collected inside a circle of variable radius, are shown in Fig. 5 for the two simulated lenses.

The focusing was quantified by the parameter R_{90}, defined as the radius of the circle at the focal plane containing 90% of the detected light. In the following studies a light beam incident on-axis was considered, yielding a PSF which is symmetric about the lens axis. However, for large incidence angles coma aberration distorts the PSF and the definition of the spot size should be modified. The dependence of R_{90} with the focal distance is shown in Fig. 6, for the two simulated lenses. While for the lens with the smaller grooves the optimum distance $d_{\min}$

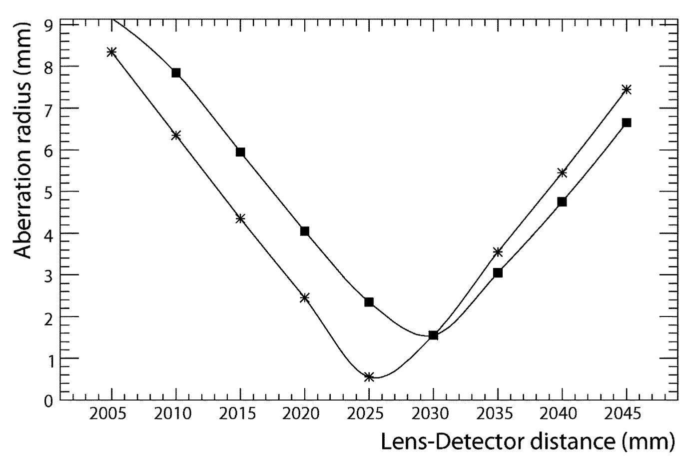

Fig. 6. Aberration radius (R_{90}) as a function of the distance from the lens to the detector for on-axis incidence of monochromatic light ($\lambda = 320$ nm). Results are shown for lenses with 2 grooves/mm ($\star$) and 0.333 grooves/mm (■); the lines join the points.

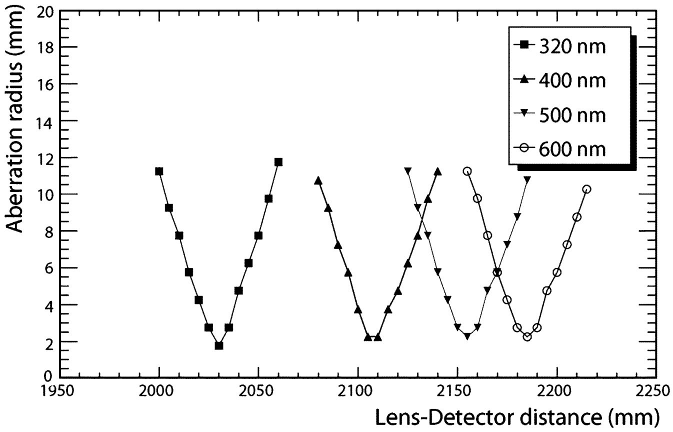

Fig. 7. Aberration radius (R_{90}) as a function of the distance from the lens to the detector for different wavelengths.

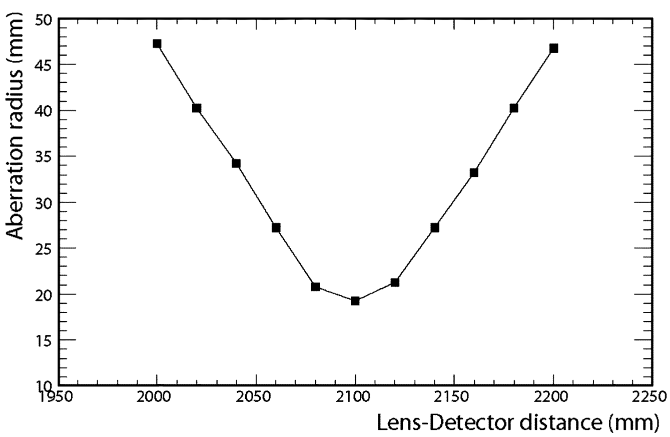

Fig. 8. Aberration radius (R_{90}) as a function of the distance from the lens to the detector. The incident light follows a Cherenkov-like spectrum as defined in the text.

(i.e., the distance that minimizes R_{90}) is found to be close to the nominal focal length (2025 mm), for the other lens the aberration can be reduced by about 60% through the displacement of the focal plane by 5 mm.

In both cases the spot size at $d_{\min}$ follows approximately the width of the grooves of the corresponding lens. This aberration is due to the geometry of the groove facets which have a flat surface, as illustrated in Fig. 1. A beam of parallel photons arriving at a given groove are all refracted in the same direction, exiting the lens in a parallel beam without undergoing any focusing. This intrinsic aberration, proportional to the size of the groove, is thus a characteristic of Fresnel lenses featuring flat surface grooves.

Fig. 7 shows R_{90} as a function of the focal distance for monochromatic light at several wavelengths. The size of the smallest PSF is approximately the same for all wavelengths, although obtained at a different $d_{\min}$. Since the refractive index of the lens decreases with the wavelength, $d_{\min}$ increases with increasing wavelength, ranging from about 2030 mm for $\lambda = 320$ nm to 2185 mm for $\lambda = 600$ nm.

In a real cosmic ray experiment, a Cherenkov telescope will operate in an extended wavelength range and the lens optimization should take into account possible chromatic aberration effects. The various properties of the incident light such as its

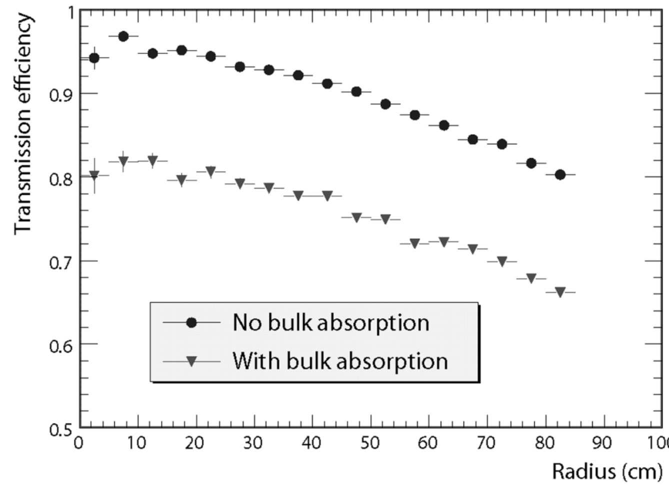

Fig. 9. Transmission of light with distance from center of the lens. A polychromatic light beam with a Cherenkov-like spectrum, as defined in the text, was used. The black circles were obtained setting the acrylic absorption-length to infinite (no bulk absorption), while the inverted triangles correspond to the typical absorption spectrum of the lens acrylic.

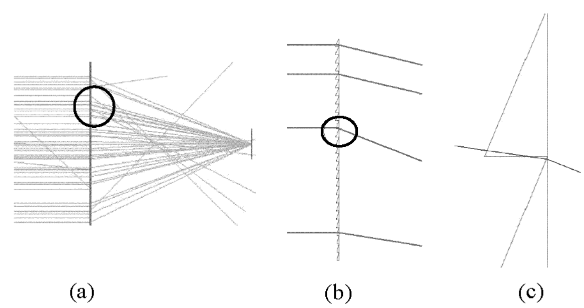

Fig. 10. Example of multiple refraction of photons in the lens grooves. (a) Some light rays exit the lens out of focus. The encircled region in (a) is shown in greater detail in (b) and (c), illustrating how photons impinging close to the edge of one groove can be refracted to the next groove.

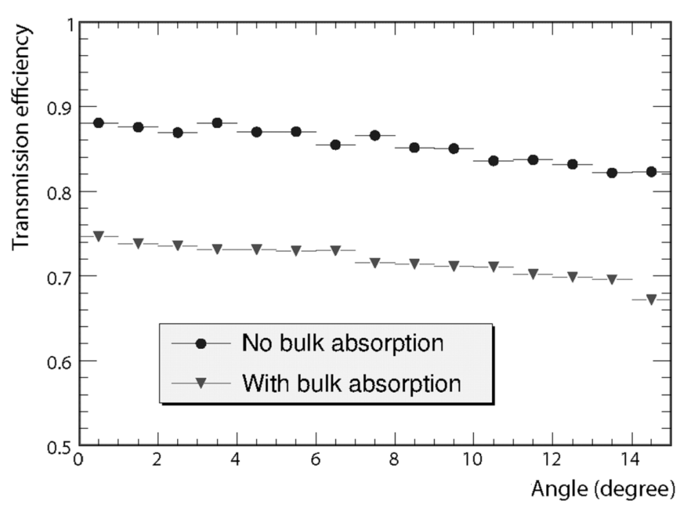

Fig. 11. Transmission of light with the incidence angle to the lens normal. A polychromatic light beam with a Cherenkov-like spectrum as defined in the text was used. The black circles were obtained setting the acrylic absorption-length to infinite (no bulk absorption), while the inverted triangles correspond to the typical absorption spectrum of the lens acrylic.

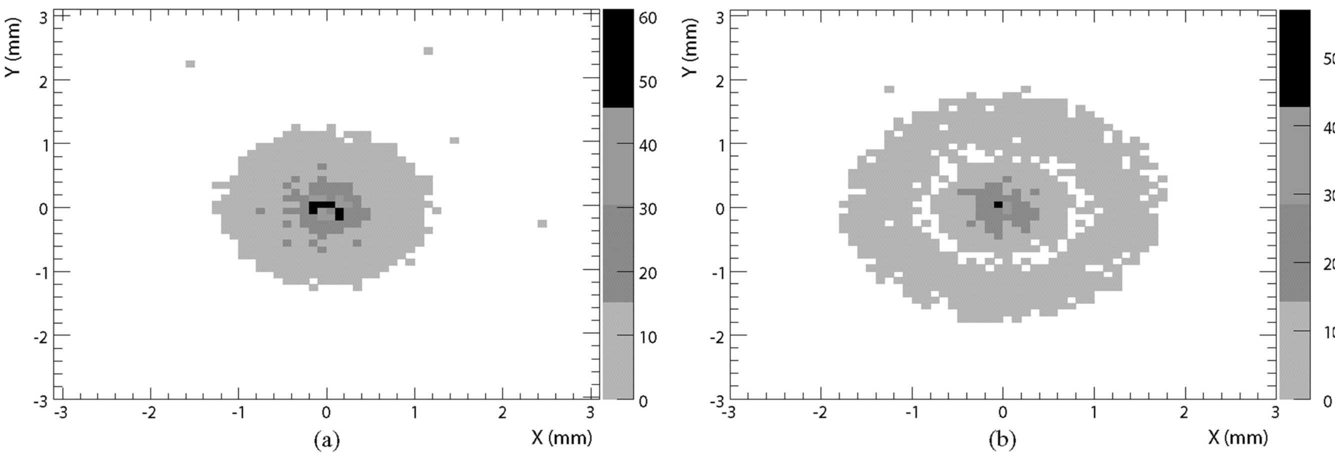

Fig. 12. Point spread function versus tilting of the lens petals. Left-rotation around the radial axis; Right rotation around the azimuth axis.

spectrum can be incorporated in the optimization studies thanks to the capabilities of Geant4. Effects such as the wavelength dependence of the light scattering in the atmosphere (Rayleigh and Mie scattering), of the light absorption in the lens material, or of the spectral sensitivity of the photo-detectors can also be implemented.

In the following optimization studies a polychromatic photon beam with a Cherenkov-like spectrum, $(dN_\gamma/d\lambda) \sim \lambda^{-2}$, was used in the wavelength range $320 < \lambda < 600$ nm. The photon spectrum was assumed to be zero for $\lambda < 320$ nm, to take into account the typical drop of the photodetector's sensitivity for small wavelengths.

The optimum focusing distance for the polychromatic beam is found to be $d_{\min} \sim 2100$ mm, as it is shown in Fig. 8, corresponding to $\mathrm{R}_{90} \sim 20$ mm. The optimum configuration for the polychromatic case yields a PSF which is about a factor 10 to 20 larger than the optimum PSF for any of the individual wavelengths in the range considered. As suggested by Fig. 7 this increase is due to the large concavity of the individual curves and to the large interval spanned by the $d_{\min}$ values, both effects arising from the lens chromatic aberration.

The lens transmittance was studied as a function of the angle of incidence and of the distance from the center of the lens. Fig. 9 shows the fraction of detected light as a function of the distance of the incident rays from the lens axis, for $\theta = 0°$. Two cases are shown: a lens with infinite absorption length and a lens with the typical absorption spectrum of the UV transmitting acrylic.

As observed, the light loss increases by about 20% towards the edge of the lens, while the loss due to bulk absorption is approximately constant with the radius. In fact, besides the loss due to absorption, which is less than 10%, there is an additional effect reducing the light transmission. As illustrated in Fig. 10 this is due to the multiple refraction of photons which exit the groove by which they entered the lens and cross an adjacent groove, thereby changing divergently their direction. As observed, this effect increases towards the lens periphery since, for constant width grooves, the groove height, ΔZ, increases with the radius (see Fig. 1).

Finally, as shown in Fig. 11 the light transmission averaged over the full lens area exhibits a slight decrease as a function of the angle of incidence, in the range covered by a large field of view Cherenkov telescope. This effect is due to the increase of the light path inside the lens with the angle.

Studies on the impact on the optics performance due to the mechanical alignment of the lens components can also be performed with the present simulation tool. As an example the effect of the precision of the placement of the lens petals is discussed hereafter. If the lens petals are misaligned or tilted from the nominal positions, part of the optical image will be defocused. The plots in Fig. 12 show the effect of rotating the lens petals around their primary axes of rotation. The PSF for on-axis incident monochromatic light ($\lambda = 320$ nm) is shown for a rotation around the radial axis (the line going from the center of the lens, passing through the center of each petal) and a rotation around the azimuthal axis (the line passing through the center of each petal and perpendicular to its radial axis). The tilting angle was chosen so that the border of each petal is shifted by about 1 mm in height.

V. Summary and Prospects

An engineering tool for the optics design and simulation of Fresnel lenses was developed using the Geant4 toolkit. Performance and optimization studies of a specific lens design were described for on-axis incidence as an example case. Effects of aberrations arising from off-axis angles should also be addressed in more detailed investigations. Besides this type of analysis, which can be carried out in standalone mode, it is possible to integrate the lens simulation in a full Geant4-based detector simulation, including detector components such as light guides and photomultipliers, as well as primary event generators and the signal digitization module. In the specific case of cosmic-ray telescopes, external air-shower generators like CORSIKA [15] or the DigitSim [16] simulation for the readout electronics could be interfaced with the detector simulation. These possibilities are particularly useful for the simulation of fluorescence and Cherenkov telescopes for present and future cosmic-ray experiments.

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