Transport properties of the plasma at RHIC

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Physics Seminar

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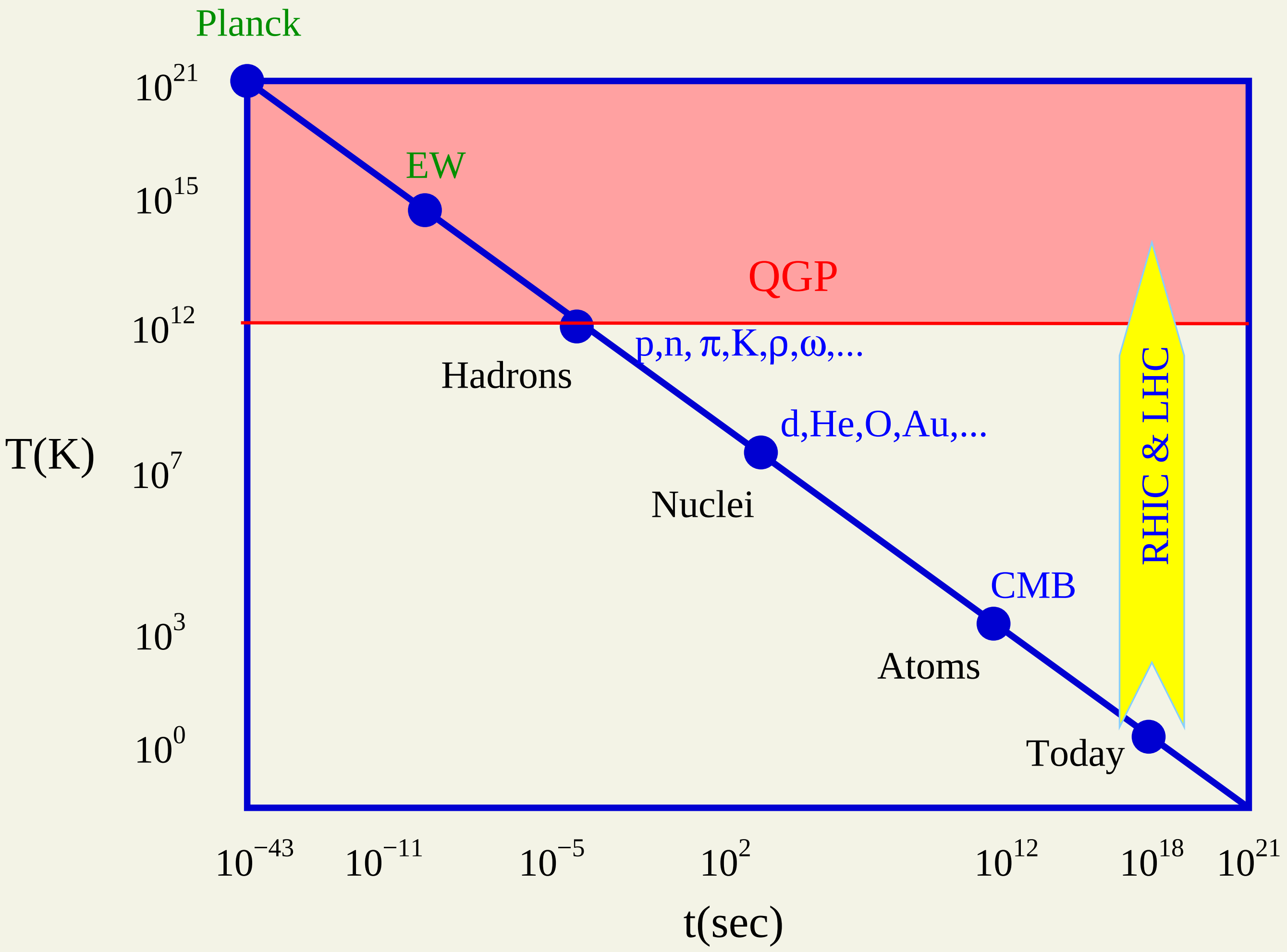
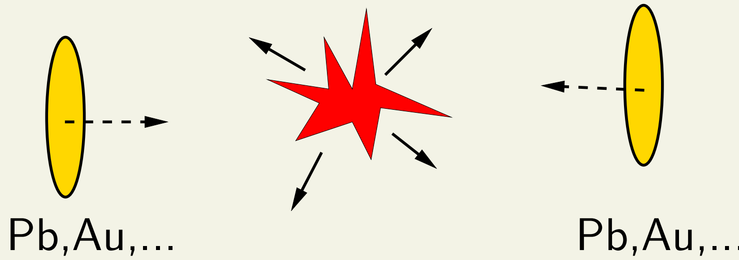
- covariant parton transport theory
- RHIC results - implications of a strongly-interacting parton system
- open questions - what makes it so opaque?

Heavy ion time machines



"Little-Bang" McLerran '82

2008	**LHC**	- 10^7 **A GeV**
2000	**RHIC**	- 10^4 **A GeV**
1996	**SPS**	- 10^2 **A GeV**
1986	**AGS**	- 10^1 **A GeV**
1984	**BeV**	- 10^0 **A GeV**



goal: recreate primordial parton plasma

mixes particle, nuclear, cond. matter, plasma physics, nonlinear dynamics

Double challenge

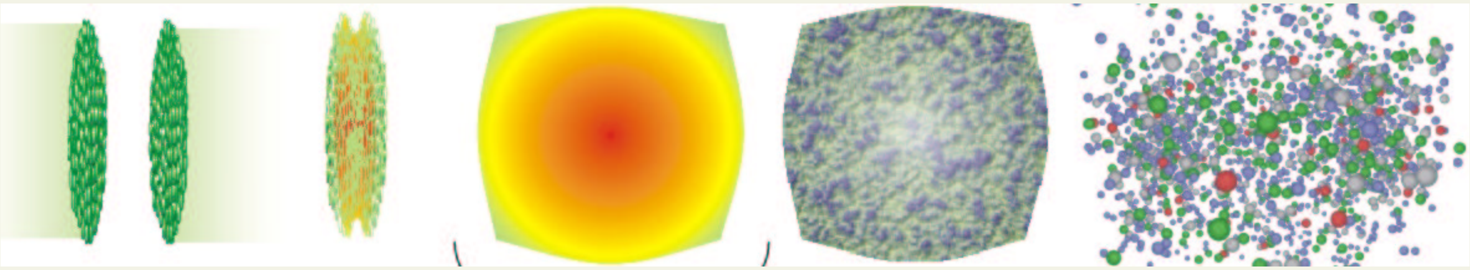
- **partonic condensed matter physics** Kajantie '96

many-body system $\gg \sum$ constituents

- **collision dynamics ($\sim$ plasma physics, nonlinear systems)**

evolving system $\gg$ system in a box

initial nuclei parton plasma hadronization hadron gas



$\uparrow$

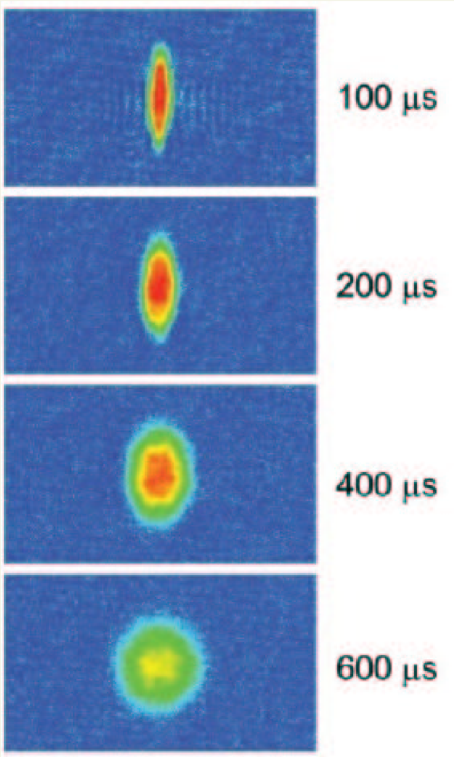
$\sim 10^{-14}$ **m**

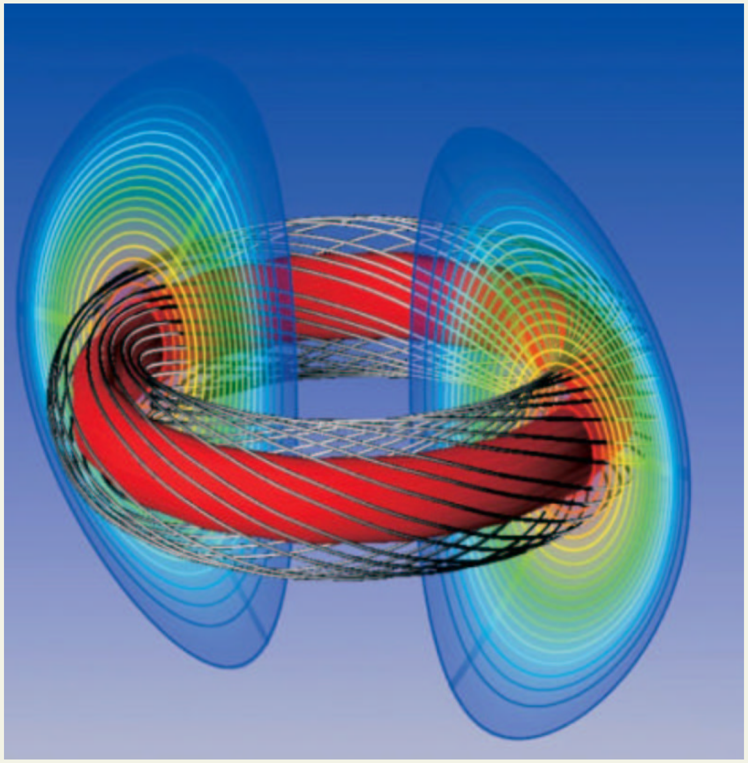
= 10 fermis

$\downarrow$

<—— $\sim 10^{-23}$ **sec = 10 yocto(!)-secs** ——>

Exciting commonalities with





The Crab Nebula in Taurus (VLT KUEYEN + FORS2)

ESO PR Photo 40f/99 (17 November 1999)

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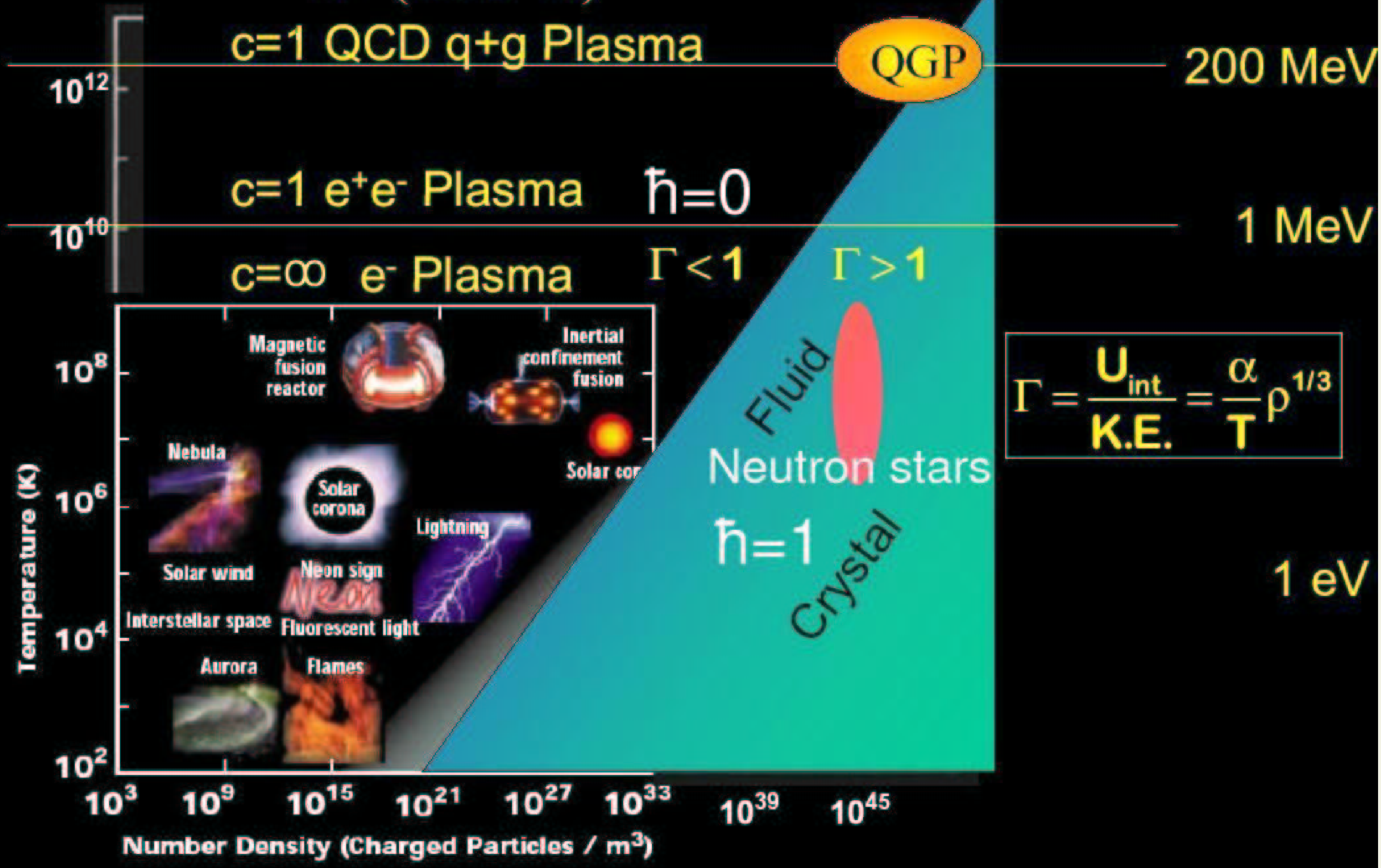
strongly-coupled cold atoms - **E&M plasmas** - **supernovas**

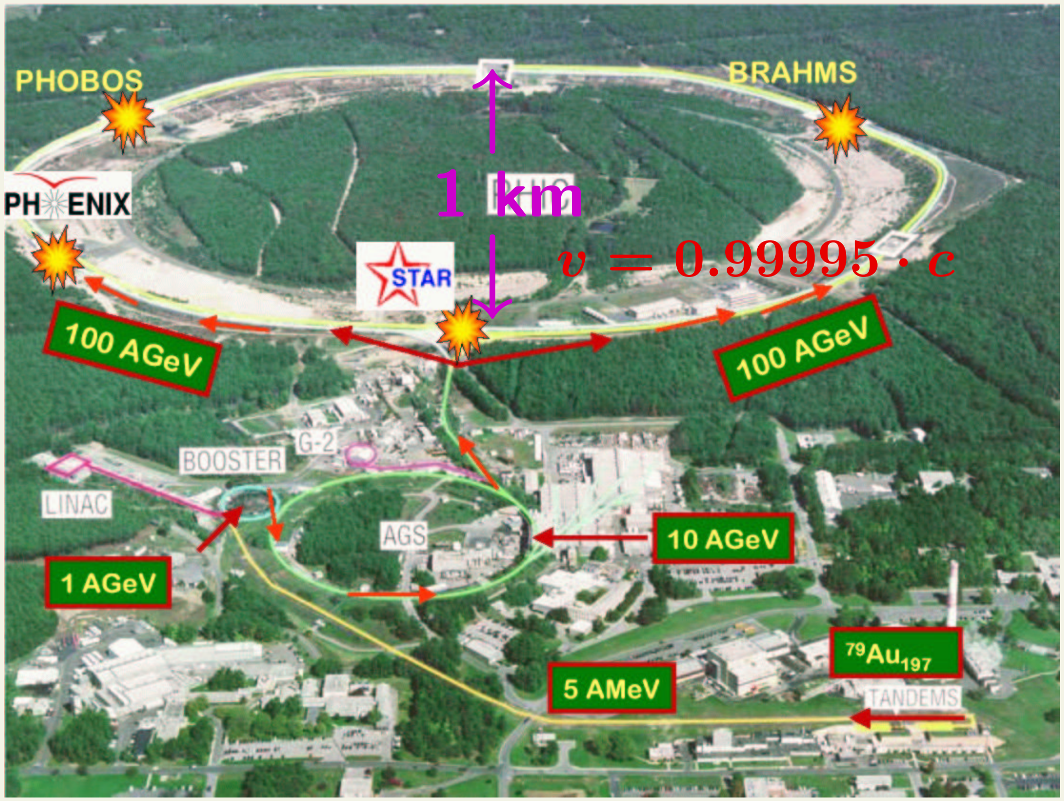
$$\rho_{QGP} \approx 4T^3 = 4\,\frac{10^{45}}{m^3}\left(\frac{T}{2\times10^{12}K}\right)^3$$



$$\Gamma = \frac{U_{int}}{K.E.} = \frac{\alpha}{T}\rho^{1/3}$$

http://fusedweb.pppl.gov/CPEP/Images/CPEP-Fusion-2000-EN-Front.PDF

Relativistic Heavy Ion Collider



present best machine, 2000-

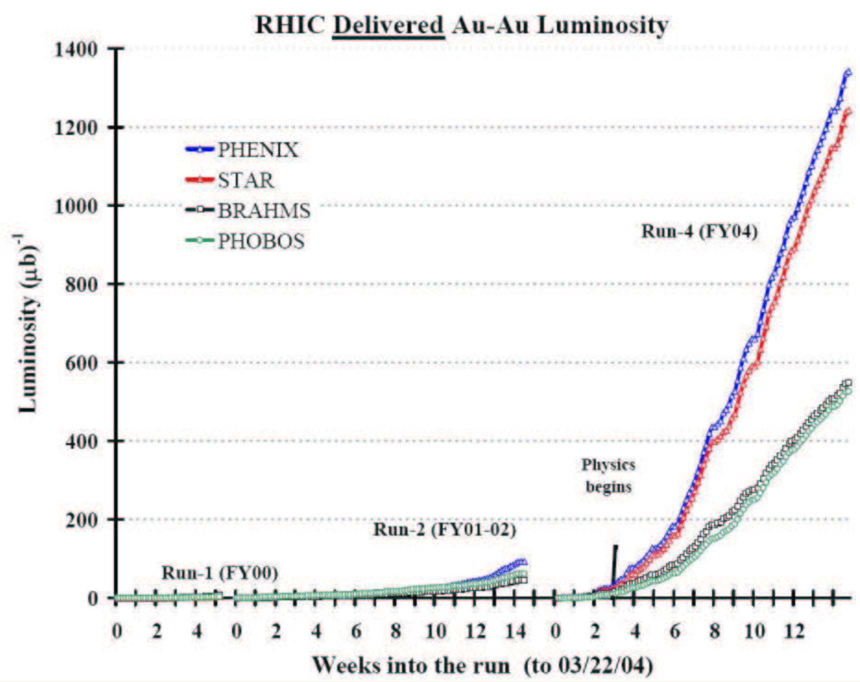
Au+Au: $\sqrt{s} = 200$ **GeV/nucleon**

Large Hadron Collider

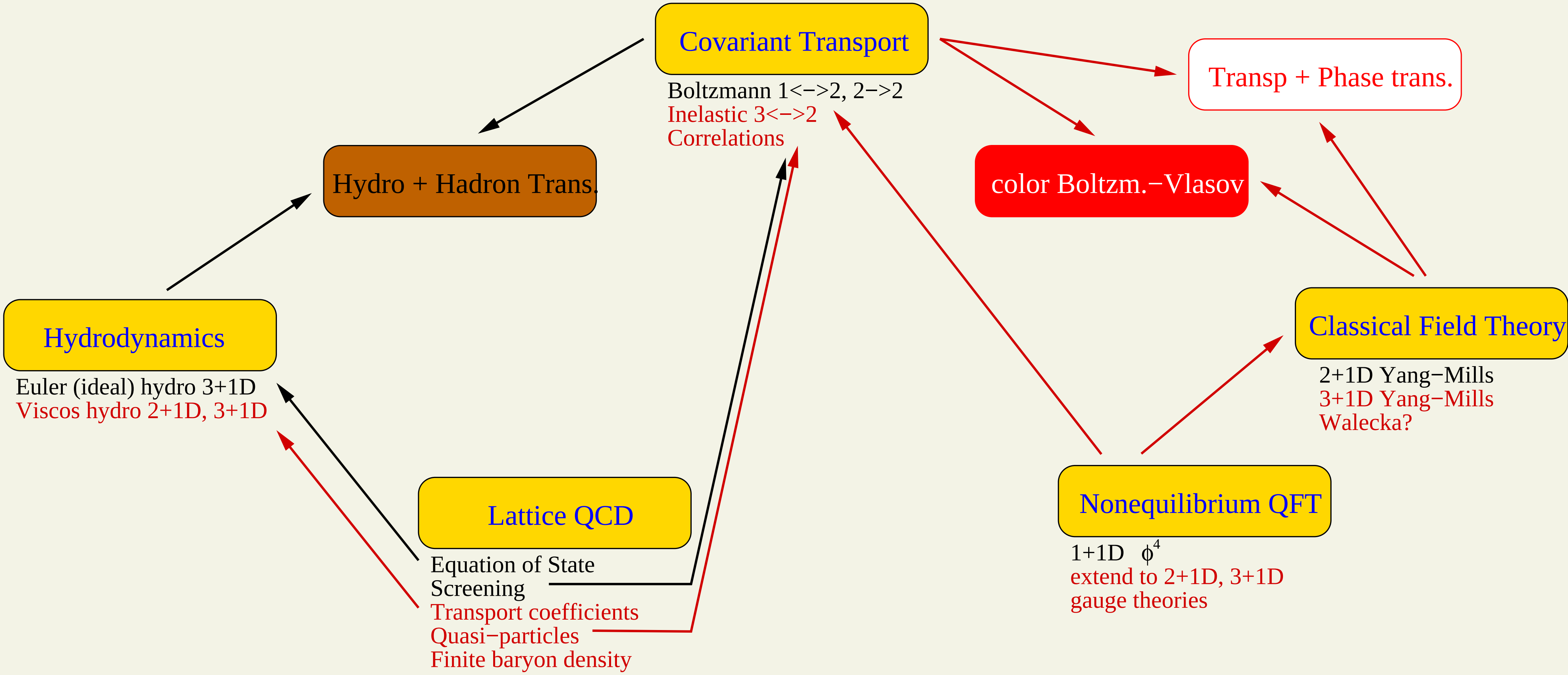
near future, 2008 -

Pb+Pb: $\sqrt{s} = 5500$ **GeV/nucleon**

… lots and lots of data already …



Dynamical frameworks



Covariant Transport

Boltzmann 1<−>2, 2−>2
Inelastic 3<−>2
Correlations

Transp + Phase trans.

Hydro + Hadron Trans.

color Boltzm.−Vlasov

Hydrodynamics

Euler (ideal) hydro 3+1D
Viscos hydro 2+1D, 3+1D

Classical Field Theory

2+1D Yang−Mills
3+1D Yang−Mills
Walecka?

Lattice QCD

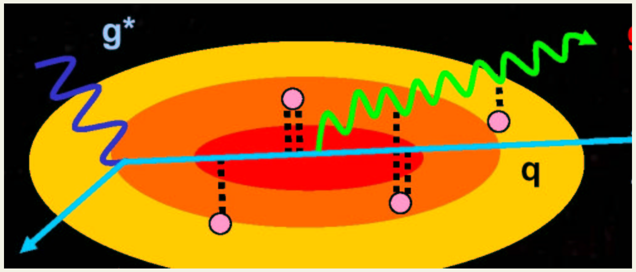
Equation of State
Screening
Transport coefficients
Quasi−particles
Finite baryon density

Nonequilibrium QFT

1+1D ϕ^4
extend to 2+1D, 3+1D
gauge theories

Dynamics matters even for high pT

parton energy loss → color charge density Wang, Gyulassy, Vitev, Wiedemann et al



static Yukawa scattering centers (GLV approach)

$$V(q) \propto e^{-i\vec{x}\vec{q}}\,\delta(q^0)\frac{\alpha_s}{\vec{q}^{\,2}+\mu_D^2}\;T_a^{jet} \times T_a^{center}$$

$$\sigma_{tot} \sim \alpha_s^2/\mu_D^2, \quad \langle \Delta q^2 \rangle \sim \mu_D^2$$

radiative parton energy loss can be computed perturbatively, if location of scattering centers known

⇒ needs input: space-time evolution of medium

e.g., GLV "pocket" formula:

$$\Delta E^{(1)} \approx const \times \alpha_s^3 \int \underbrace{dz \cdot z}_{\text{gluon self-coupling (non-Abelian theory)}} \cdot\, \rho(z, t = z/v) \qquad\qquad dE/dx \propto x(!)$$

Quark-gluon kinetic theory

Pang, Zhang, Gyulassy, DM, Vance, Csizmadia, Pratt, Cheng, ...

Incoherent, particle limit of underlying quantum theory (QCD). Nonequilibrium approach.

mean free path:

$$\lambda(x) \equiv \frac{1}{\textbf{cross section} \times \textbf{density(x)}} \qquad \begin{cases} \lambda = 0 & \text{– ideal hydrodynamics} \\ \lambda = \infty & \text{– free streaming} \end{cases}$$

Transport opacity: most relevant parameter [DM & Gyulassy NPA 697 ('02)]

$$\chi \equiv \langle n_{coll} \rangle \langle \sin^2 \theta_{CM} \rangle \quad \sim \# \text{ of collisions} \times \text{deflection weight}$$

Boltzmann transport eqn: f_i **- quark/gluon phase space distributions**

$$p^\mu \partial_\mu f_i(\vec{x}, \vec{p}, t) = \overbrace{S_i(\vec{x}, \vec{p}, t)}^{\text{source}} + \overbrace{C_i^{el.}[f](\vec{x}, \vec{p}, t)}^{2 \to 2\ (\text{ZPC}, \text{GCP}, \ldots)} + \overbrace{C_i^{inel.}[f](\vec{x}, \vec{p}, t)}^{2 \leftrightarrow 3\ (\text{MPC})} + \ldots$$

highly relativistic case → only a few covariant algorithms: ZPC, MPC, Bjorken-τ, ...

*** OSCAR code repository @ http://nt3.phys.columbia.edu/OSCAR ***

Molnar's Parton Cascade (MPC)

Elementary processes: elastic $2 \to 2$ **processes** + $gg \leftrightarrow q\bar{q}$, $q\bar{q} \to q'\bar{q}'$ + $ggg \leftrightarrow gg$

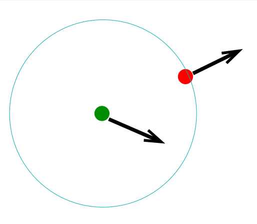
Equation for $f^i(x,\vec{p})$: $\qquad i = \{g, d, \bar{d}, u, \bar{u}, \ldots\}$

$$\begin{aligned}
p_1^\mu \partial_\mu \tilde{f}^i(x,\vec{p}_1) \quad = \quad & \frac{\pi^4}{2} \sum_{jkl} \int\limits_2 \int\limits_3 \int\limits_4 \left(\tilde{f}_3^k \tilde{f}_4^l - \tilde{f}_1^i \tilde{f}_2^j \right) \left| \bar{\mathcal{M}}_{12\to 34}^{i+j\to k+l} \right|^2 \delta^4(12-34) \quad \swarrow 2 \to 2 \\
& + \frac{\pi^4}{12} \int\limits_2 \int\limits_3 \int\limits_4 \int\limits_5 \left(\frac{\tilde{f}_3^i \tilde{f}_4^i \tilde{f}_5^i}{g_i} - \tilde{f}_1^i \tilde{f}_2^i \right) \left| \bar{\mathcal{M}}_{12\to 345}^{i+i\to i+i+i} \right|^2 \delta^4(12-345) \quad \swarrow 2 \leftrightarrow 3 \\
& + \frac{\pi^4}{8} \int\limits_2 \int\limits_3 \int\limits_4 \int\limits_5 \left(\tilde{f}_4^i \tilde{f}_5^i - \frac{\tilde{f}_1^i \tilde{f}_2^i \tilde{f}_3^i}{g_i} \right) \left| \bar{\mathcal{M}}_{45\to 123}^{i+i\to i+i+i} \right|^2 \delta^4(123-45) \quad \swarrow 3 \leftrightarrow 2 \\
& + \tilde{S}^i(x,\vec{p}_1) \quad \leftarrow \text{initial conditions}
\end{aligned}$$

with shorthands:

$$\tilde{f}_i^q \equiv (2\pi)^3 f_q(x,\vec{p}_i), \qquad \int\limits_i \equiv \int \frac{d^3 p_i}{(2\pi)^3 E_i}, \qquad \delta^4(p_1+p_2-p_3-p_4) \equiv \delta^4(12-34)$$

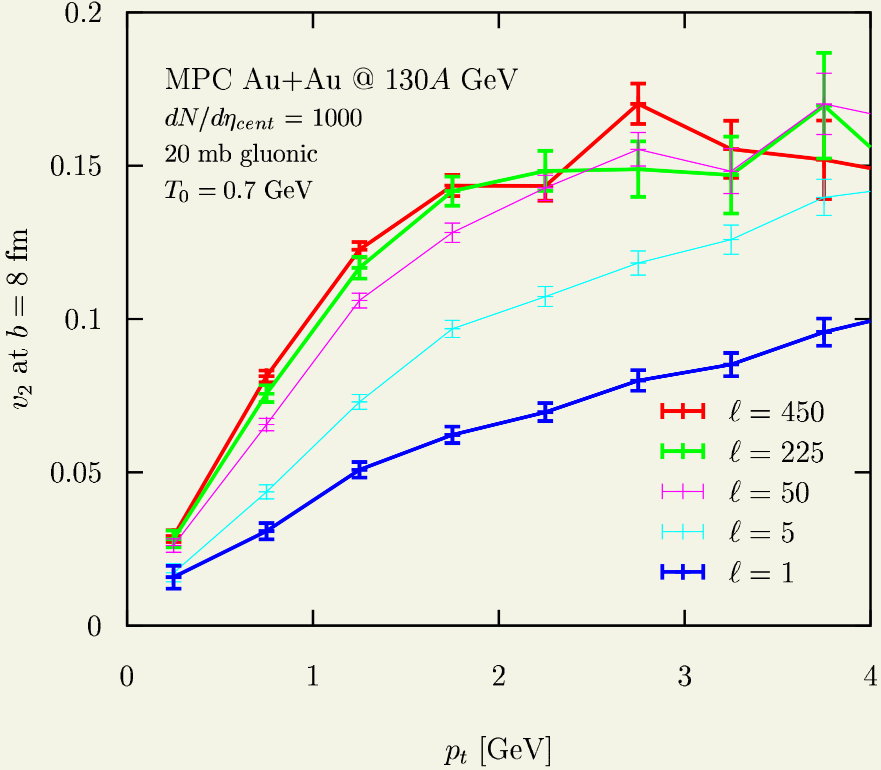
Nonlocal/acausal artifacts



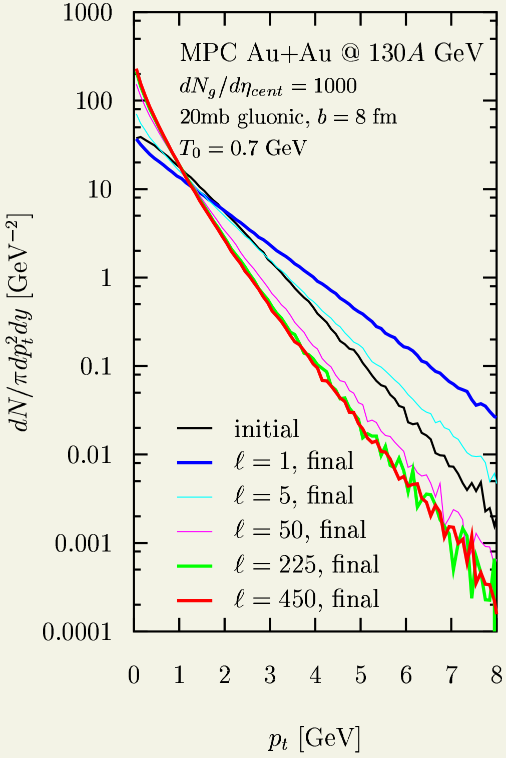
Naive $2 \to 2$ cascade nonlocal - **action at distance $d < \sqrt{\frac{\sigma}{\pi}}$**

subdivision: rescale $f \to f \cdot \ell$, $\sigma \to \sigma/\ell$ $\Rightarrow$ $d \propto \ell^{-1/2}$ local as $\ell \to \infty$

DM & Gyulassy ('02): $v_2(p_T)$



spectra



at RHIC: need subdivision $\ell \sim 200$ to eliminate large artifacts

$\to$ computational challenge - CPU time scales as $\ell^{\sim 3/2}$ per run

Covariant transport - RHIC results

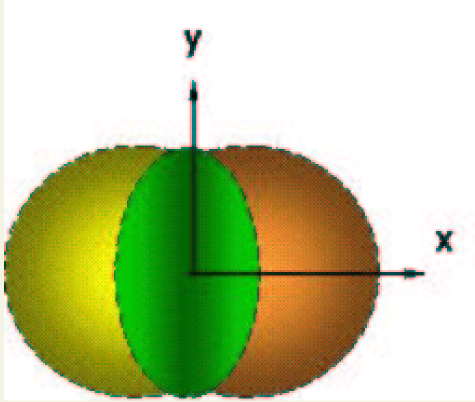
- **elliptic flow and thermalization**
- **heavy flavor (charm)**
- **soft tails at high pT (acceleration)**

[• pion HBT interferometry]

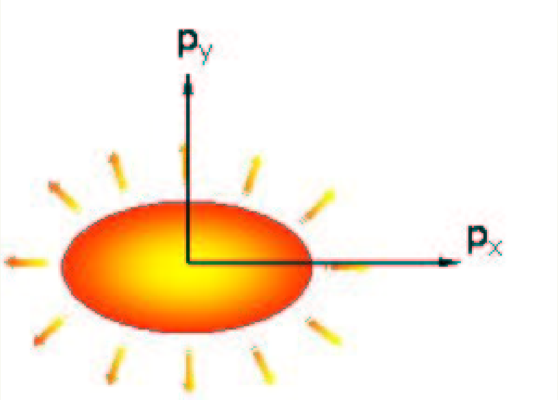
Elliptic flow (v_2)

spatial anisotropy → final azimuthal momentum anisotropy

$$\varepsilon \equiv \frac{\langle x^2 - y^2\rangle}{\langle x^2 + y^2\rangle}$$



$$\longrightarrow \quad v_2 \equiv \frac{\langle p_x^2 - p_y^2\rangle}{\langle p_x^2 + p_y^2\rangle}$$

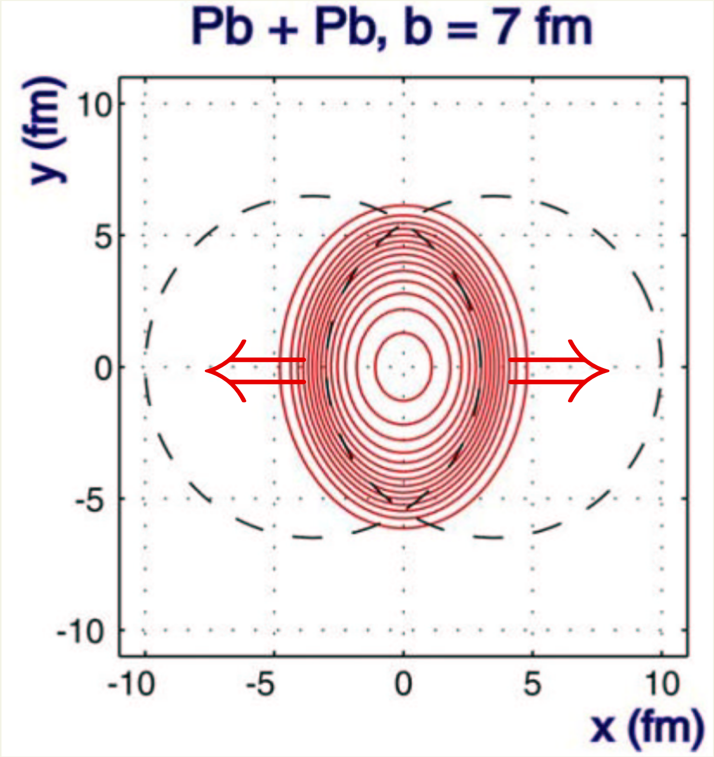


- measures strength of interactions
- self-quenching, develops at early times

What v_2 measures

macroscopically: **pressure gradients**

$$\Delta\vec{F}/\Delta V = -\vec{\nabla}p$$

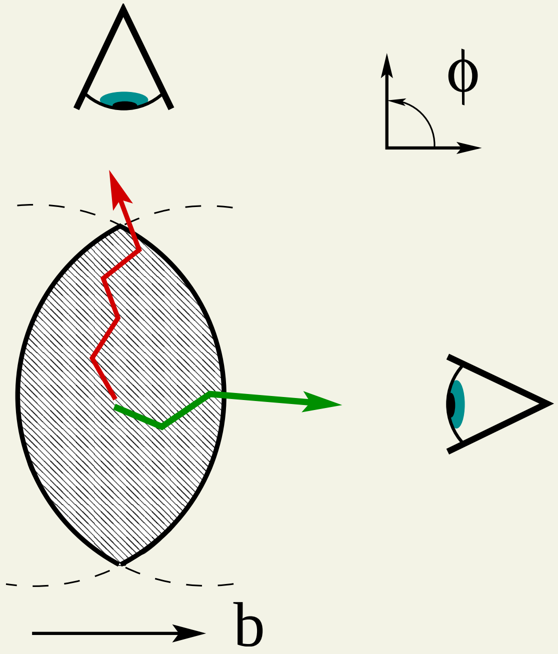


$\Rightarrow$ **larger acceleration in impact parameter direction**

microscopically: **transport opacity**

beam axis view

smaller momenta
more deflection

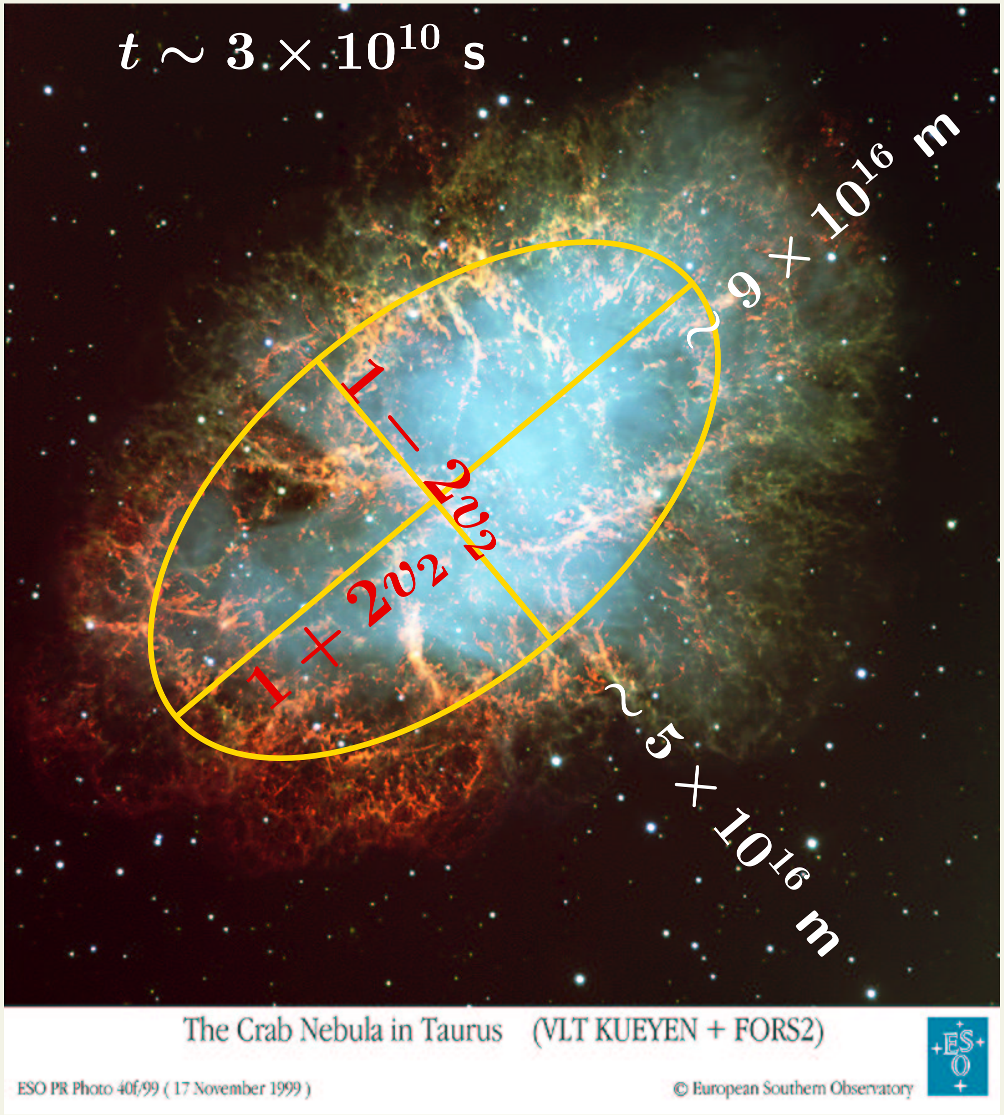


larger
momenta
less
deflection

variation in pathlength

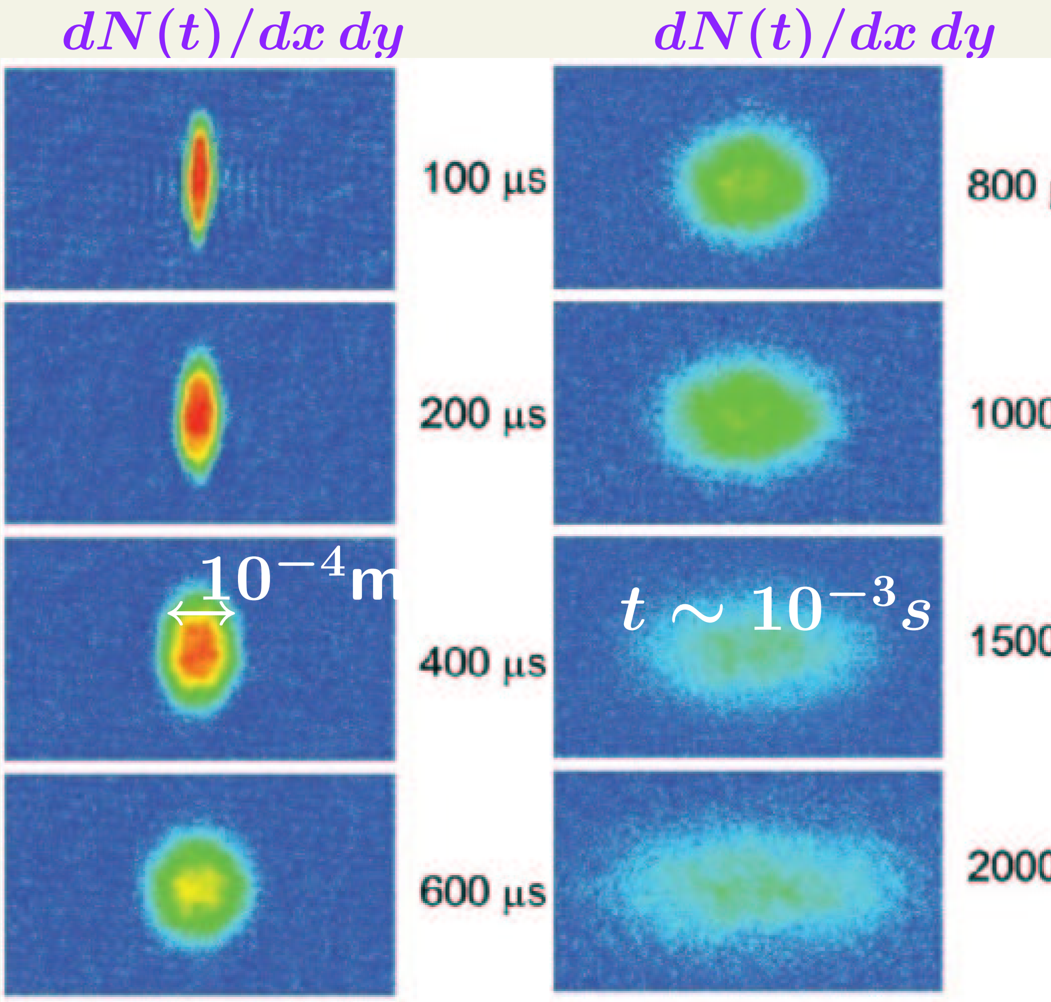
$\Rightarrow$ **momentum anisotropy** v_2

supernova remnant (CRAB)



droplet of ultra-cold ^6Li atoms

O'Hara et al, Science 298 ('03)



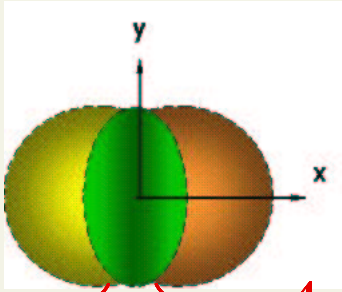
$$T \sim \text{micro-K},\ n \sim 10^{19}/\text{m}^3$$

Elliptic flow (v_2) at RHIC

beam axis view:

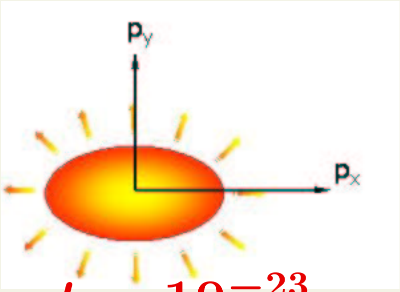
$\phi = 90$

$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$



$\longrightarrow$

$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

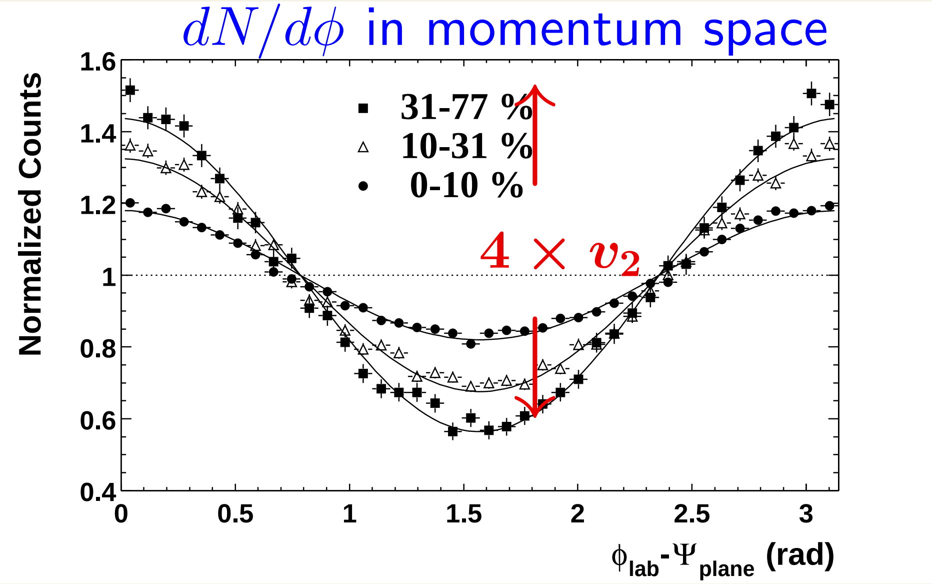


$\phi = 0$

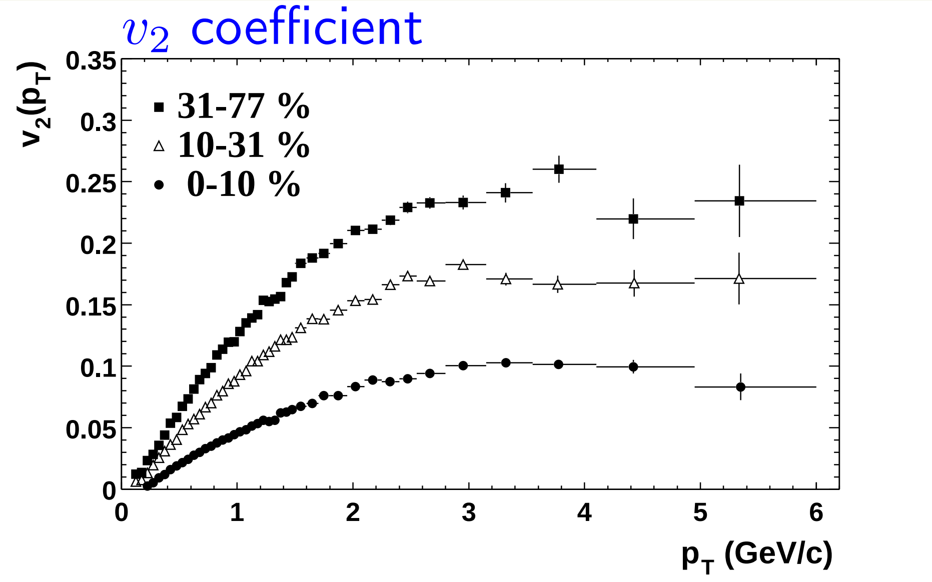
$\longleftrightarrow \ \sim 4 \times 10^{-15}$ **m**

$t \sim 10^{-23} s$

$$dN/d\phi \propto [1 + 2v_1 \cos\phi + 2v_2 \cos 2\phi + ...]$$


$dN/d\phi$ in momentum space

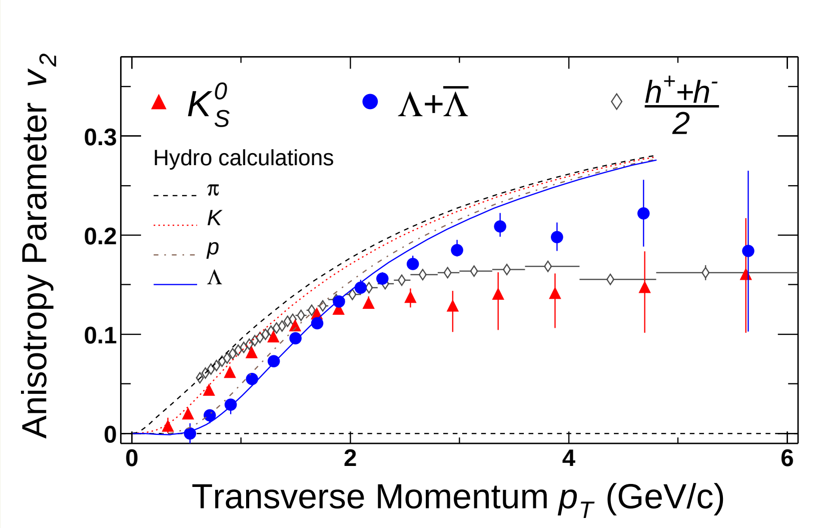
$\longrightarrow$


v_2 coefficient

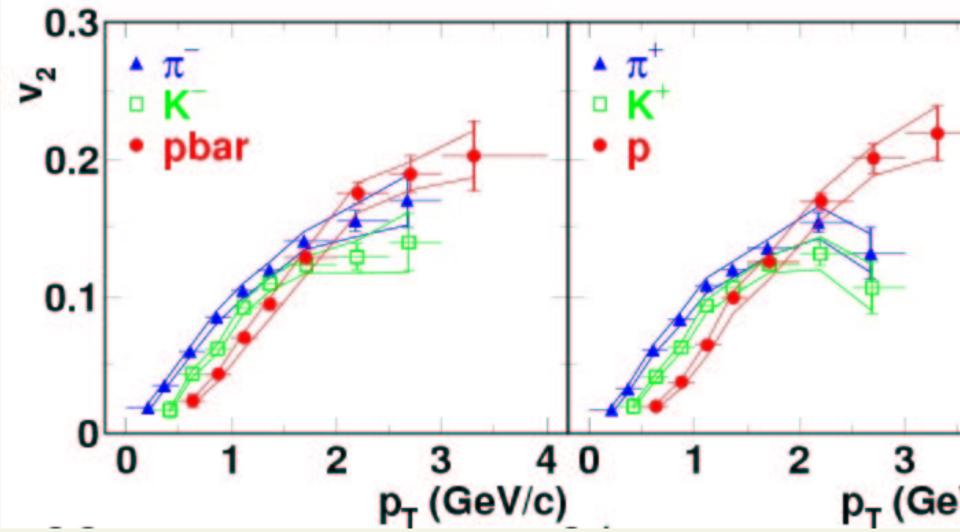
$v_2 \sim 0.15 \quad \Rightarrow$ **large 2 : 1 azimuthal anisotropy**

All particle species show large, up to $\sim 2:1$ anisotropy, which saturates at high p_T

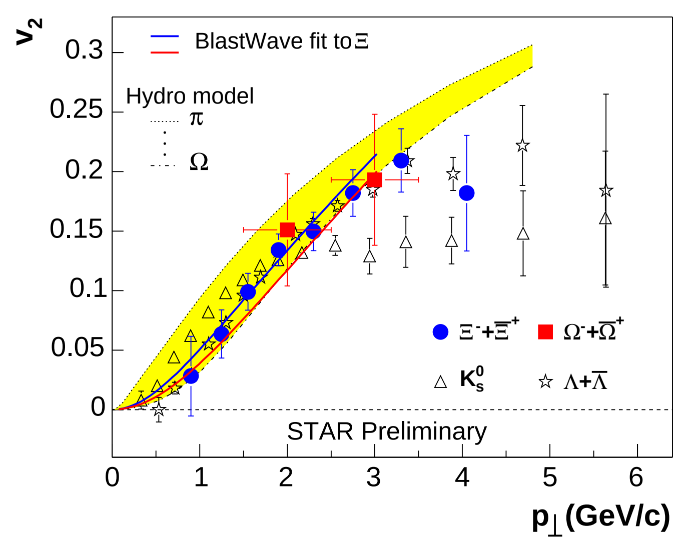
STAR, PRL92 ('04): $h^\pm$, K_S^0, Λ



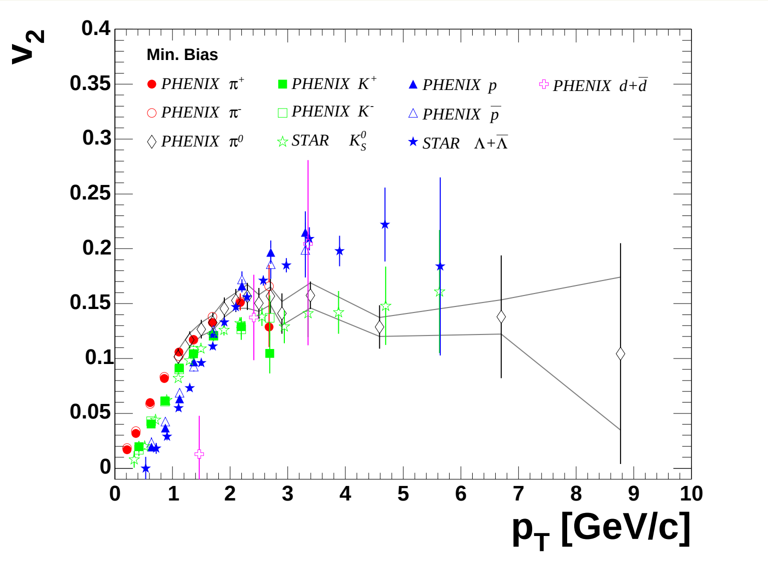
PHENIX, PRL91 ('03): $\pi^\pm$, $K^\pm$, p



STAR, JPG30 ('04): Ξ, Ω



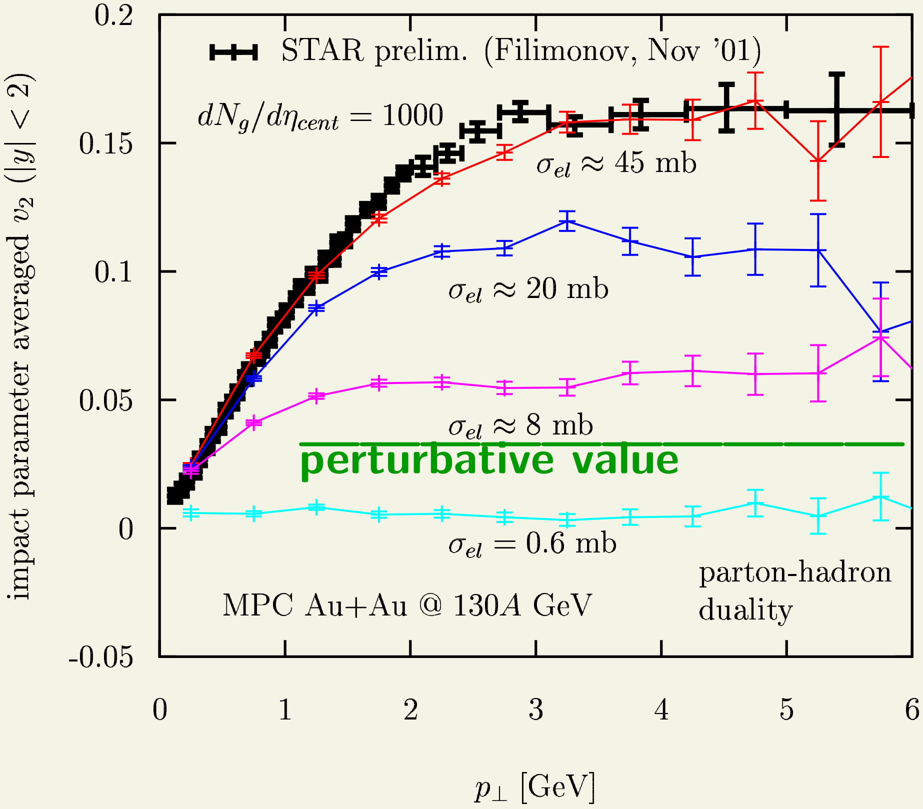
PHENIX, JPG30 ('04): π_0, d



Strong interactions at RHIC

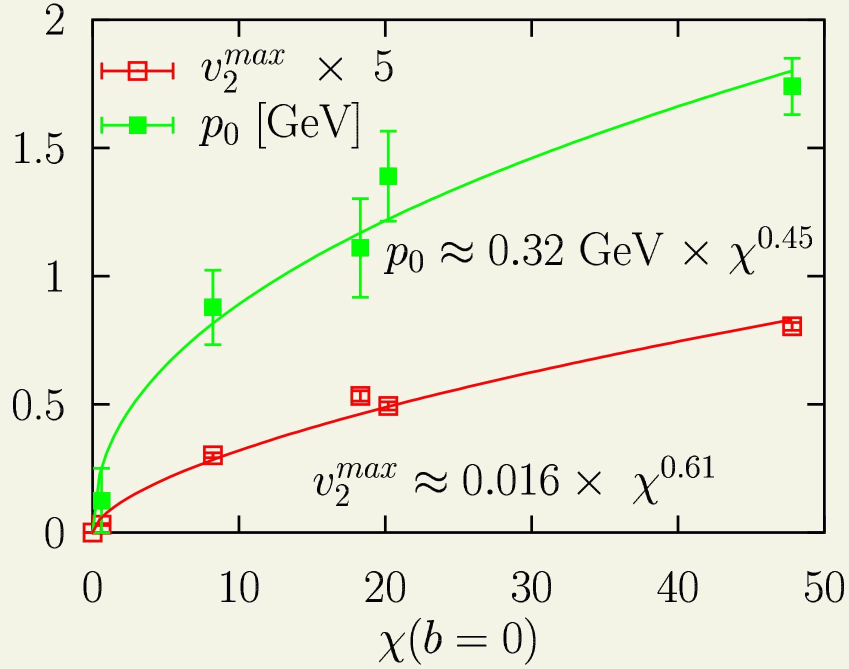
Au+Au @ 130 GeV, $b = 8$ fm

DM & Gyulassy, NPA 697 ('02): $\boldsymbol{v_2(p_T, \chi)}$



nonlinear opacity dependence

$$v_2(p_T, \chi) \approx v_2^{max}(\chi) \tanh(p_T / p_0(\chi))$$



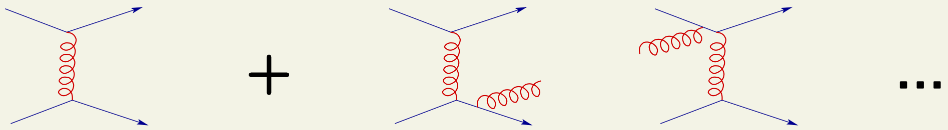
need $15\times$ perturbative opacities - $\sigma_{el} \times dN_g/d\eta \approx 45$ mb $\times 1000$

(saturated gluon $\frac{dN^{cent}}{d\eta} = 1000$, $T_{eff} \approx 0.7$ GeV, $\tau_0 = 0.1$ fm, 1 parton $\to$ 1 π hadronization)

$\Rightarrow$ **strongly-interacting quark-gluon plasma (sQGP)**

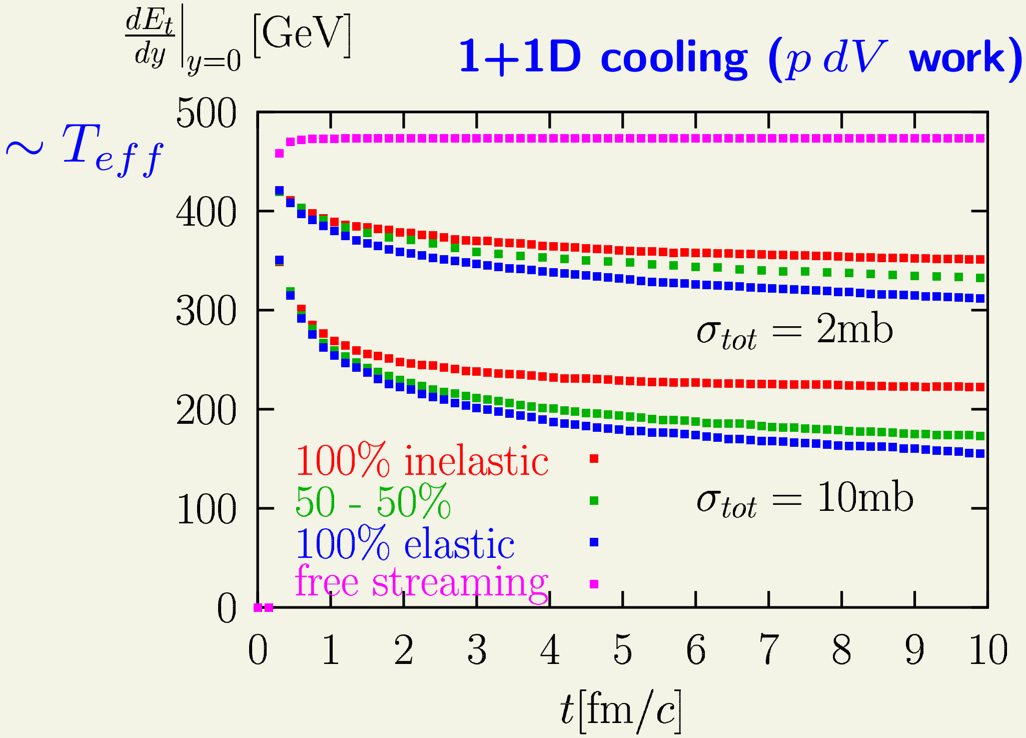
Radiative transport, $3 \leftrightarrow 2$

higher-order processes also contribute to thermalization



but enhance effective opacity only $2-3$ times → still s-QGP

1+1D cooling ($p\,dV$ work)



DM & Gyulassy, NPA 661, 236 ('99)

fixed transport cross section
but vary degree of inelasticity

100% elastic, 100% inelastic, 50-50%
$2 \to 2$ $3 \leftrightarrow 2$ mixed

⇒ inelastic $3 \leftrightarrow 2$ is roughly same as elastic with same transport cross section

Classical field limit misses v_2

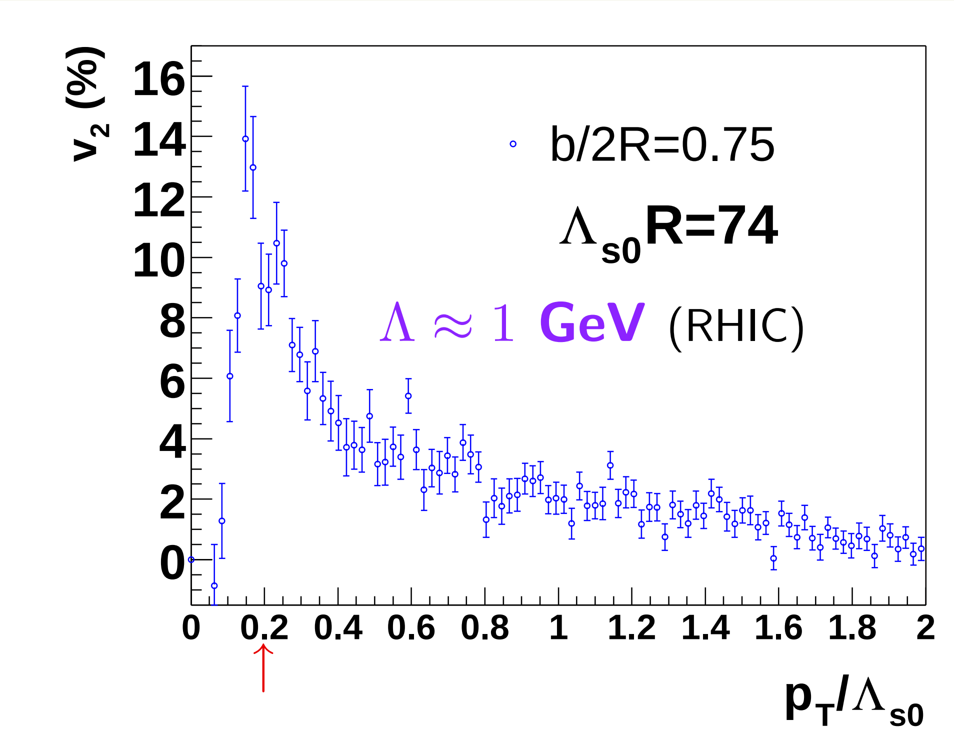
Boltzmann - particle limit $\leftrightarrow$ **long-wavelength limit - classical Yang-Mills**

applicable for early evolution at asymptotic energies & very large nuclei McLerran, Venugopalan, Kharzeev, Nardi, Kovchegov... "Color Glass Condensate"

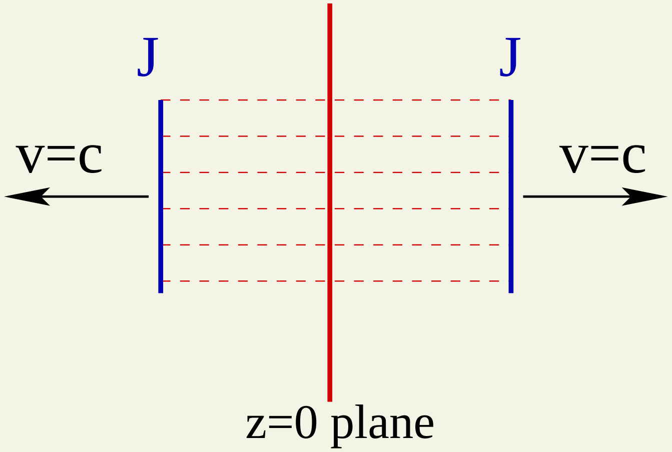
quark-gluon Maxwell eq: $D_\mu F^{\mu\nu}(x) = J^\nu(x)$

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu]$$
$$D_\mu = \partial_\mu - igA_\mu, \quad A_\mu = A_\mu^a t_a$$

v_2 **from 2+1D solutions** McLerran, Venugopalan, Nara, Krasnitz



$\Lambda \approx 1$ **GeV** (RHIC)



anisotropy peaks at very low momenta, disagrees completely with data

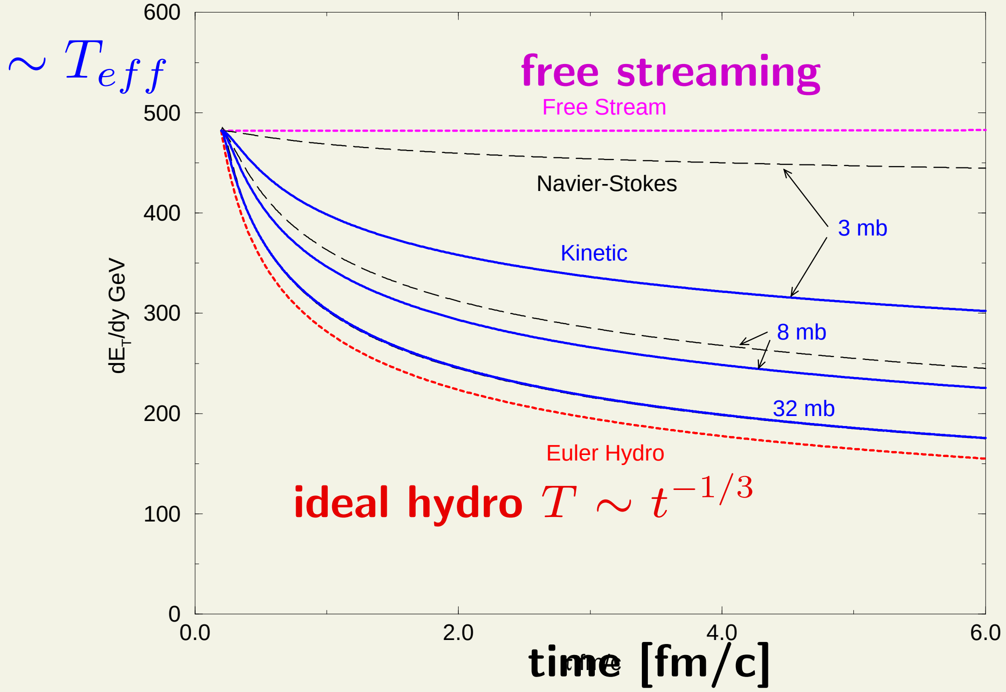
peak reflects charge-correlation length in random source current (~ nucleon size)

No, still not ideal fluid(!)

Even $\sigma_{gg\to gg} \sim 50$ mb is insufficient for ideal hydro (**perturbative QGP: ~ 3 mb**)

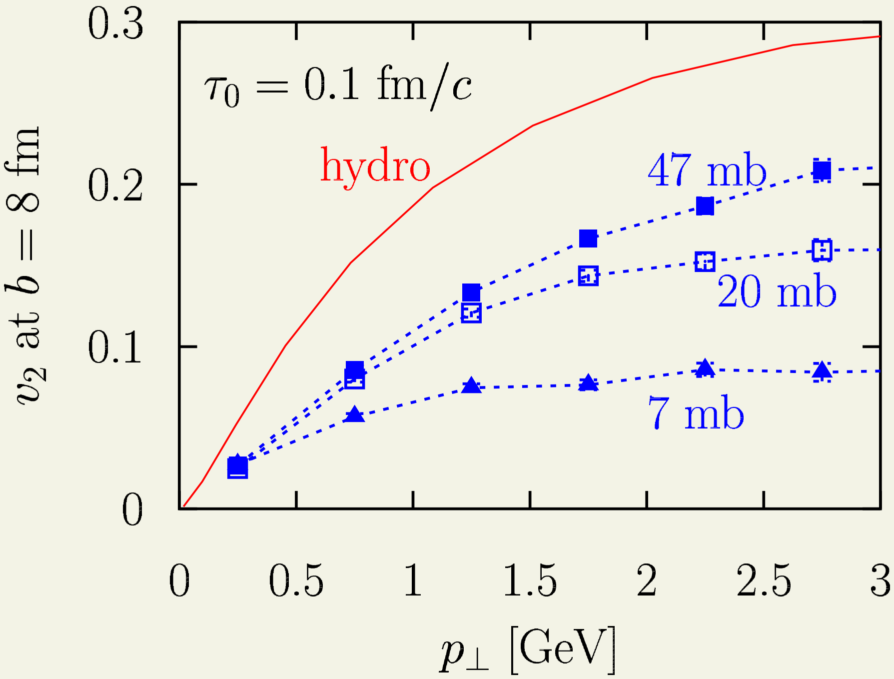
- **less pdV work, slower cooling**

Gyulassy, Pang & Zhang ('97): 1+1D



- **dissipation reduces v_2 by $30-50\%$**

DM & Huovinen, PRL94 ('05): 3+1D



$\longrightarrow$ **dense, strongly-interacting system, but dissipative**

But almost perfect fluid

$\sigma_{gg} \sim 50$ **mb** $\leftrightarrow \lambda_{MFP} \sim 0.08$ **fm** - **is likely the best one can get**

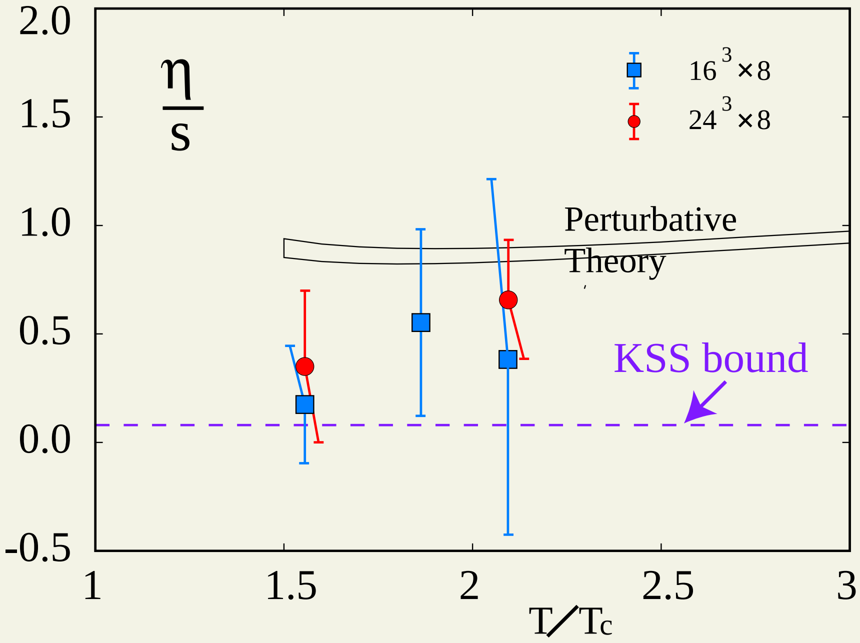
quantum mechanics: $\Delta E \cdot \Delta t \geq \hbar/2$

$\Rightarrow$ **kinetic theory:** $T \cdot \lambda_{MFP} \geq \hbar/3$ Gyulassy & Danielewicz '85

viscosity: $\eta = s\frac{\lambda T}{5}$ $\Rightarrow$ **minimal viscosity** $\eta/s \geq 1/15$

parton transport + large RHIC v_2 indicate QGP is most ideal fluid possible

Nakamura & Sakai ('04): **viscosity from lattice QCD**



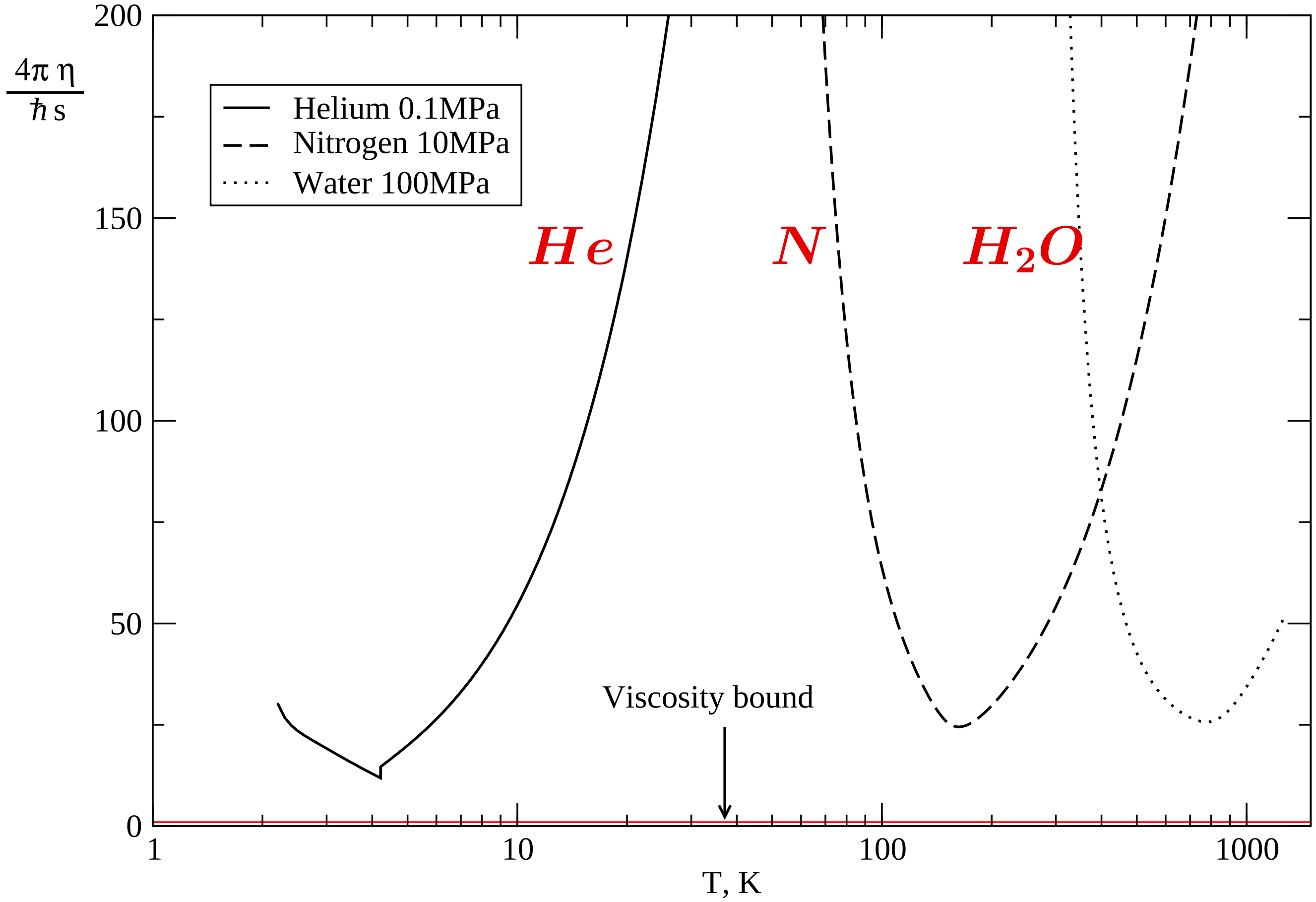
VERY difficult calculation - will take very long to converge with numerics

$\leftarrow$ **string theory conjecture:** $\eta/s \geq 1/4\pi$

Son et al ('02), ('04) - proof for $\mathcal{N} = 4$ SYM

- RHIC can test this theory bound

no other substance comes within a factor 10



perhaps strongly-interacting cold atoms?

Should use viscous hydro (Navier-Stokes)

no guarantee for success - η small but gradients large - but may worth a try

zero baryon density limit: $\partial_\mu T^{\mu\nu} = 0$ **with**

$$T^{\mu\nu} = (\epsilon+p)u^\mu u^\nu - p g^{\mu\nu} + \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu - \tfrac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha) + \zeta\Delta^{\mu\nu}\nabla_\alpha u^\alpha, \qquad \begin{matrix}\Delta^{\mu\nu}=g^{\mu\nu}-u^\mu u^\nu \\ \nabla^\mu=\Delta^{\mu\nu}\partial_\nu\end{matrix}$$

η, ζ - shear and bulk viscosities

sound attenuation: $\omega(k) = c_s k - \frac{i}{2}\frac{1}{\epsilon+p}\left(\zeta + \frac{4}{3}\eta\right)k^2 = c_s k - \frac{i}{2}k^2\Gamma_s \qquad \left(c_s^2 \equiv \frac{\partial\epsilon}{\partial p}\right)$

slower cooling in 1D Hubble expansion: $\frac{d\epsilon}{d\tau} = -\frac{\epsilon+p}{\tau}\left(1 - \frac{\Gamma_s}{\tau}\right)$ Gyulassy & Danielewicz

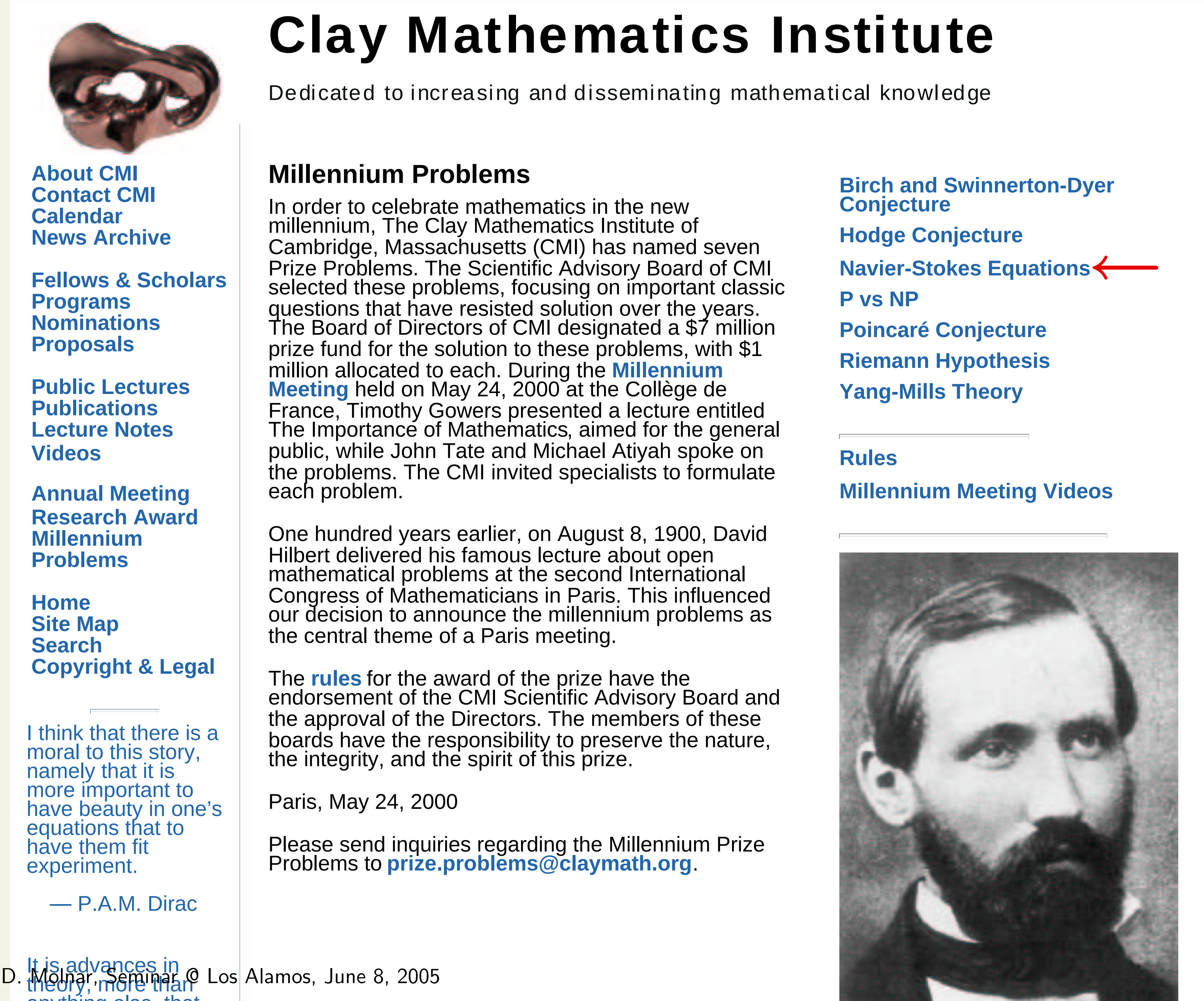
(!)Causal formulation: microscopic relaxation times as additional matter properties

Israel & Stewart

$\Rightarrow$ these could be inferred from heavy-ion data at RHIC & LHC

2+1D solution algorithms are just being developed ...

Millennium Prize Problems http://www.claymath.org/millennium/



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I think that there is a moral to this story, namely that it is more important to have beauty in one's equations that to have them fit experiment.

— P.A.M. Dirac

It is advances in theory, more than

Millennium Problems

In order to celebrate mathematics in the new millennium, The Clay Mathematics Institute of Cambridge, Massachusetts (CMI) has named seven Prize Problems. The Scientific Advisory Board of CMI selected these problems, focusing on important classic questions that have resisted solution over the years. The Board of Directors of CMI designated a $7 million prize fund for the solution to these problems, with $1 million allocated to each. During the **Millennium Meeting** held on May 24, 2000 at the Collège de France, Timothy Gowers presented a lecture entitled The Importance of Mathematics, aimed for the general public, while John Tate and Michael Atiyah spoke on the problems. The CMI invited specialists to formulate each problem.

One hundred years earlier, on August 8, 1900, David Hilbert delivered his famous lecture about open mathematical problems at the second International Congress of Mathematicians in Paris. This influenced our decision to announce the millennium problems as the central theme of a Paris meeting.

The **rules** for the award of the prize have the endorsement of the CMI Scientific Advisory Board and the approval of the Directors. The members of these boards have the responsibility to preserve the nature, the integrity, and the spirit of this prize.

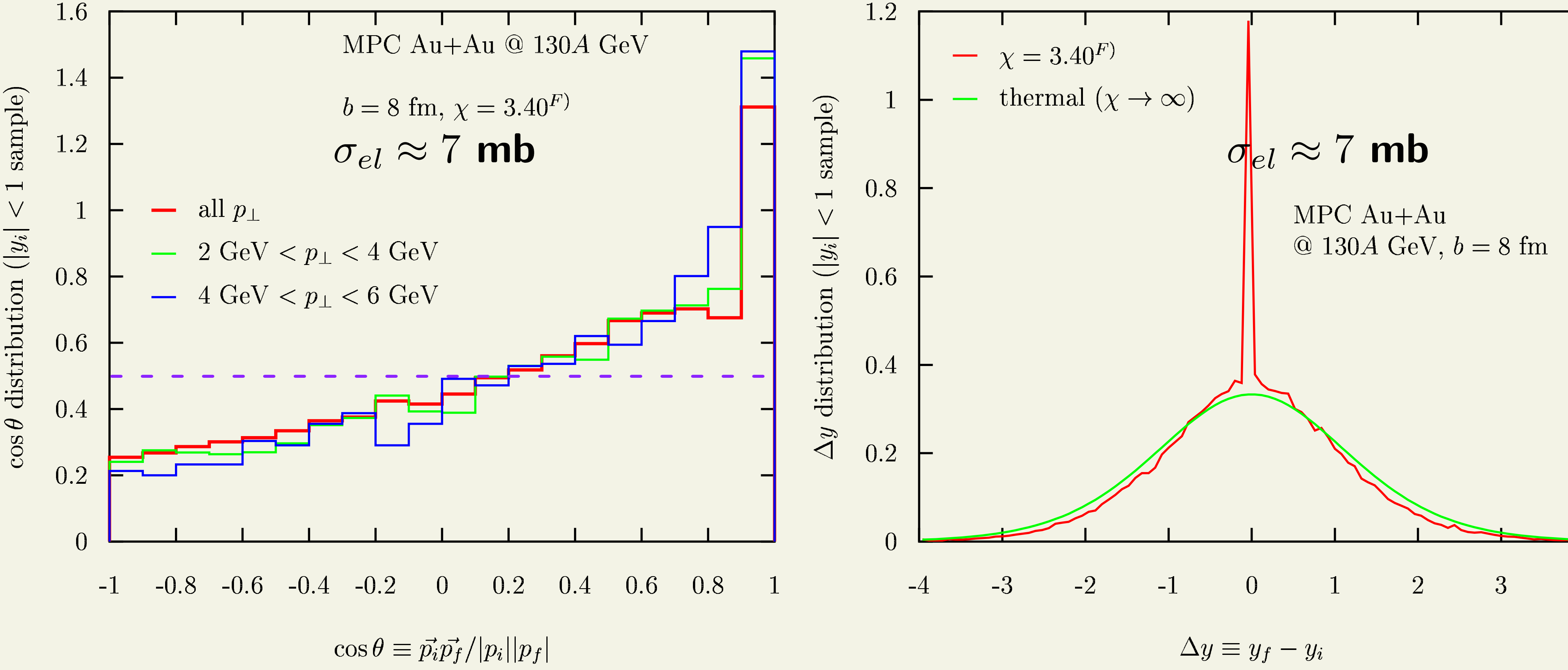
Paris, May 24, 2000

Please send inquiries regarding the Millennium Prize Problems to **prize.problems@claymath.org**.

- **Birch and Swinnerton-Dyer Conjecture**
- **Hodge Conjecture**
- **Navier-Stokes Equations** ←
- **P vs NP**
- **Poincaré Conjecture**
- **Riemann Hypothesis**
- **Yang-Mills Theory**

- **Rules**
- **Millennium Meeting Videos**

Significant randomization



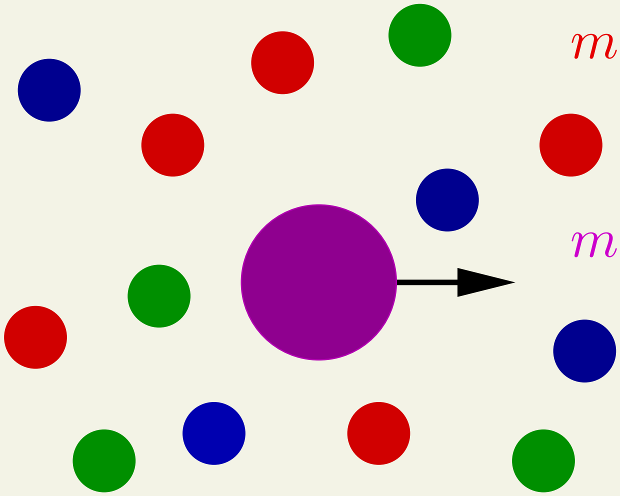
a) cosine of deflection angle $\vec{p}_i \angle \vec{p}_f$

b) rapidity shift $y_f - y_i$

light parton momenta randomize to large degree, already for $\sigma \sim 7$ mb ($\chi \sim 7$)

Cross-check: heavy quarks

$\sim$ **"Brownian motion" in plasma**



$m_g, m_{u,d}, m_s \sim T$

$m_c \sim 1.2$ **GeV** $\gg T$

$$v \sim \sqrt{T/m}$$

$$p \sim \sqrt{m \cdot T}$$

$$N_{coll} \sim p/\Delta q \sim \sqrt{m/T}$$

charm quarks very heavy $\Rightarrow$ need more collisions to randomize

$\Rightarrow$ **at low momenta: expect reduced anisotropy** v_2

$\Rightarrow$ **at high momenta: mass difference should not matter as** $m/p \to 0$

expect: all $2 \to 2$ processes get enhanced by same factor in opaque plasma

based on Combridge NPB 151 ('79) 429:

$$\begin{aligned}
\sigma_{gg\to q\bar q} &= \frac{2\,r}{27}\frac{1+r}{1+2r}\ln(1+\frac{1}{r})\,\sigma_{gg\to gg} \qquad , \qquad \sigma_{q_i\bar q_i\to q_j\bar q_j} = \frac{16\,r}{243}\,\sigma_{gg\to gg}\\
\sigma_{gg\to c\bar c} &= \frac{2\,r}{27}\Theta(1-4R)\left[(1+4R+R^2)\ln\frac{1+\sqrt{1-4R}}{1-\sqrt{1-4R}} - (7+3R)\frac{\sqrt{1-4R}}{4}\right]\sigma_{gg\to gg}\\
\sigma_{q\bar q\to c\bar c} &= \frac{16\,r}{243}\Theta(1-4R)(1+2R)\sqrt{1-4R}\;\sigma_{gg\to gg}
\end{aligned}$$

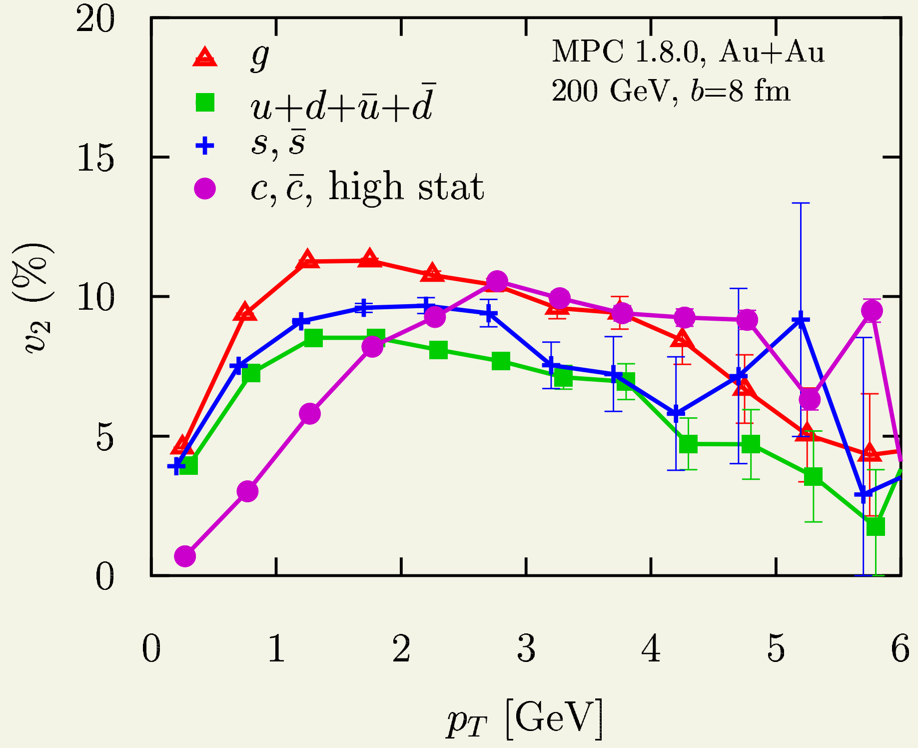
where $r \equiv \mu_D^2/s$**,** $R \equiv M_c^2/s$

take $\mu_D = 0.7$ **GeV,** $M_c = 1.2$ **GeV**

Predicted charm flow(!)

parton transport MPC 1.8.0 vs **indirect $D(qc)$ meson measurement:**

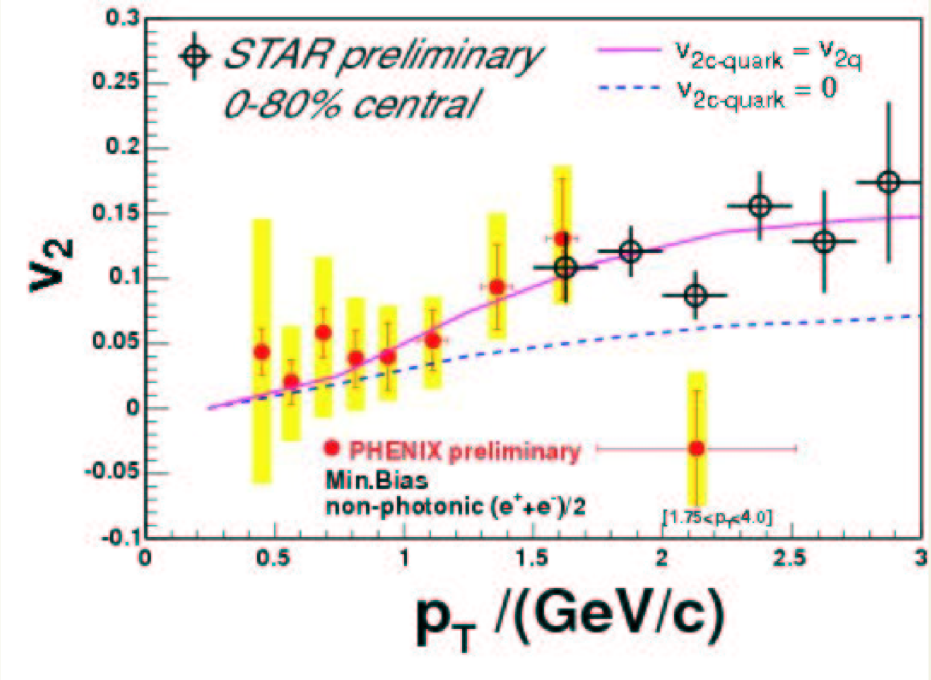
DM, JPG ('04): **parton v_2**



elastic & inel. $2 \rightarrow 2$

$6\times$ perturbative opacities

PHENIX, STAR ('04): **decay electron v_2** $\approx v_2^D$



uses decay electrons: $D \rightarrow K^{(*)}\,\nu\, e$

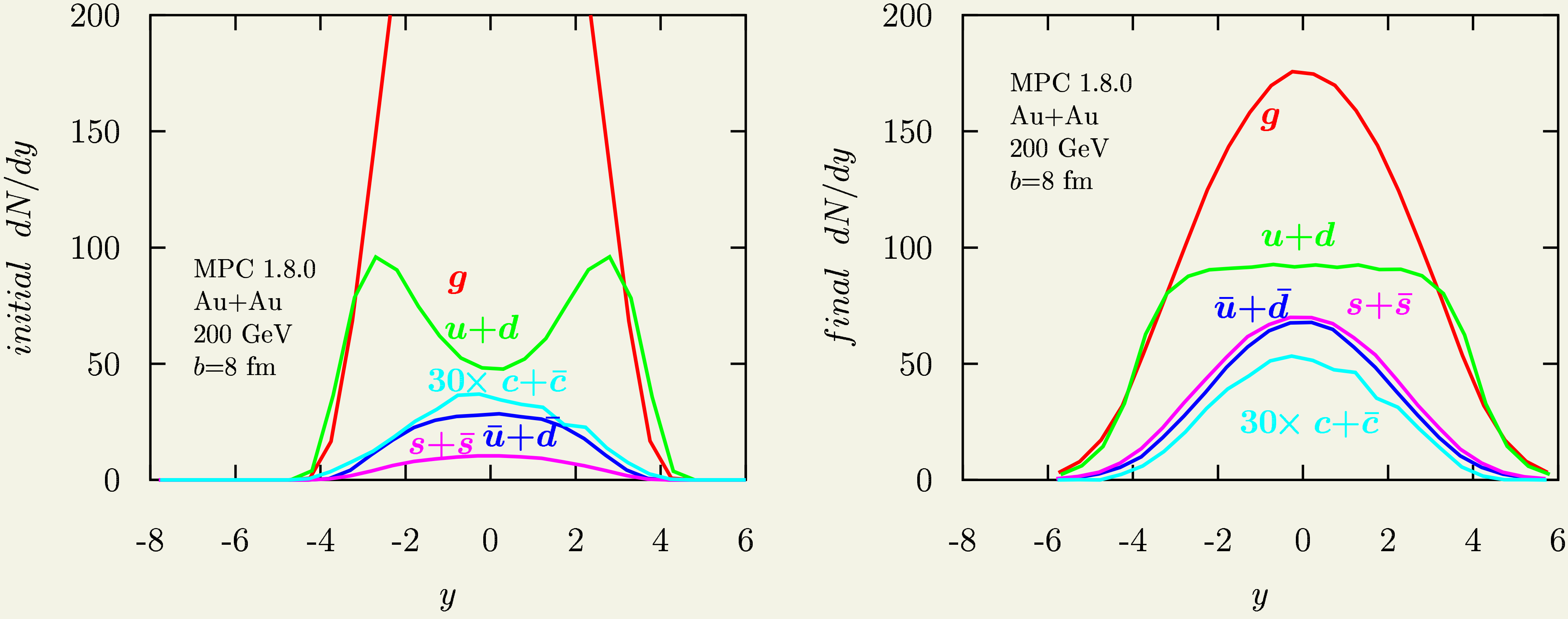
e's from hadron decays and γ-conversion subtracted
$\equiv$ **"non-photonic"**

qualitative agreement with data, detailed studies needed - $v_2(b, \chi, \sqrt{s})$

also expect secondary charm production from opaque plasma

DM, JPG ('04):

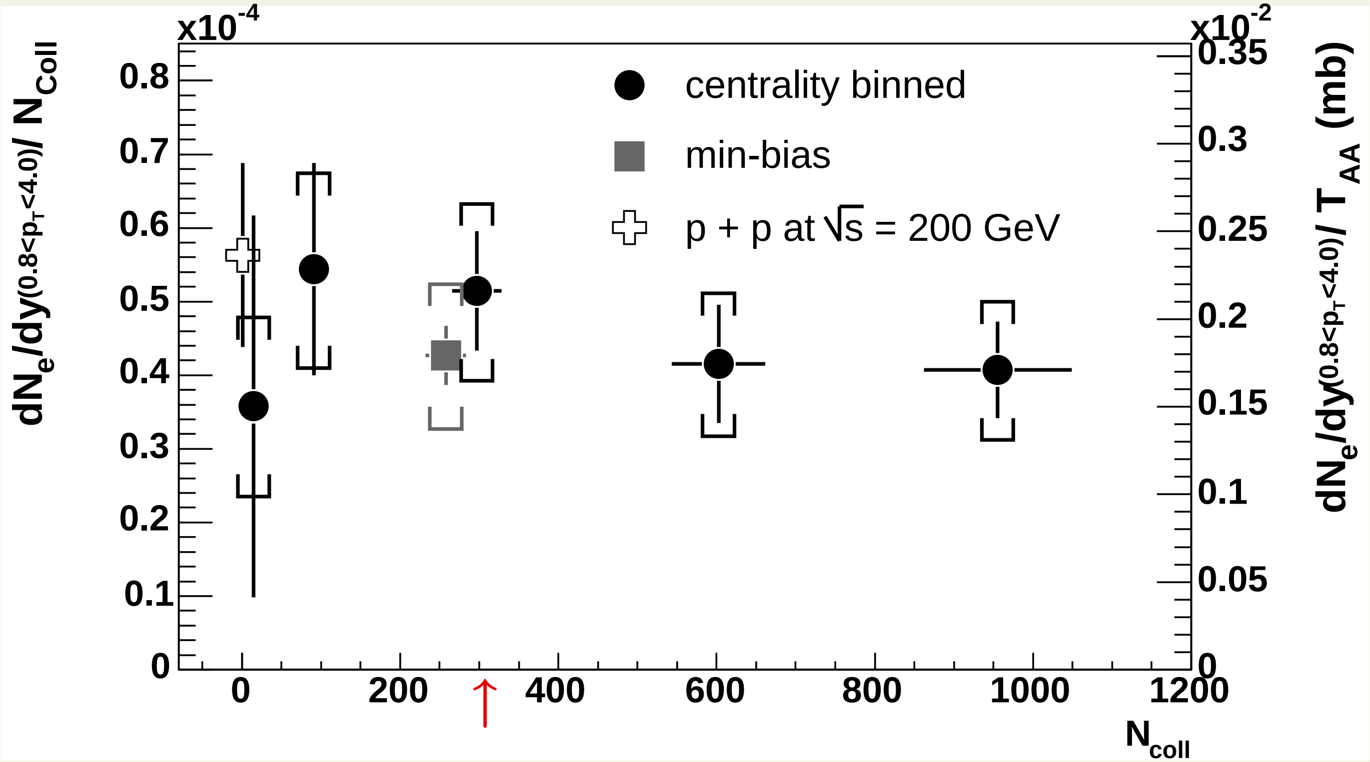
initial vs. final rapidity distribution



- roughly half the glue fuse to $q\bar{q}$
- extra $40-50$% charm yield due to secondary production
- strangeness is up by much more, factor 5 or so

PUZZLE - data show no secondary charm (no more than N_{coll} scaled p+p)

PHENIX, PRL 94 (2005) 082301



already trouble in p+p: heavy quark rates are above theory (NLO pQCD)

R. Vogt et al

Soft physics tails at high pT(!)

partons can end up with some final parton momentum (p_T, y) in three ways:

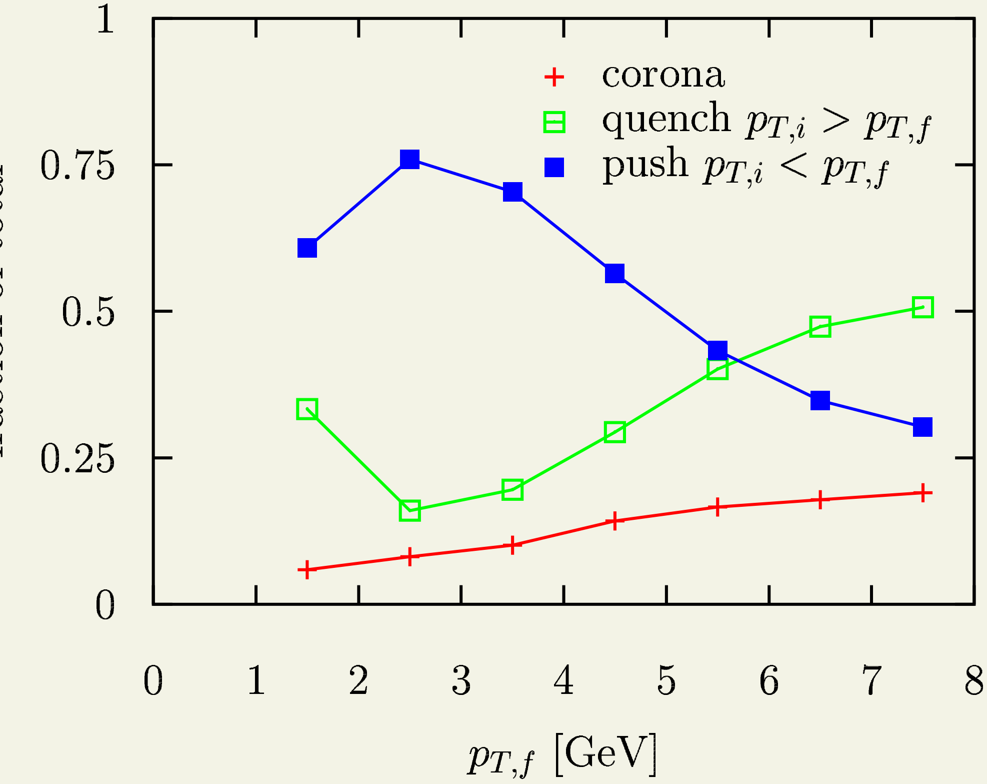
- **escape with no interaction - corona**
- **interact and lose energy - quench**
- **3rd possibility: interact and gain energy - “push”**

in opaque plasma, gain component can be relevant at surprisingly high p_T, pushing “pure” hard physics out to $p_T \gtrsim 10$ GeV

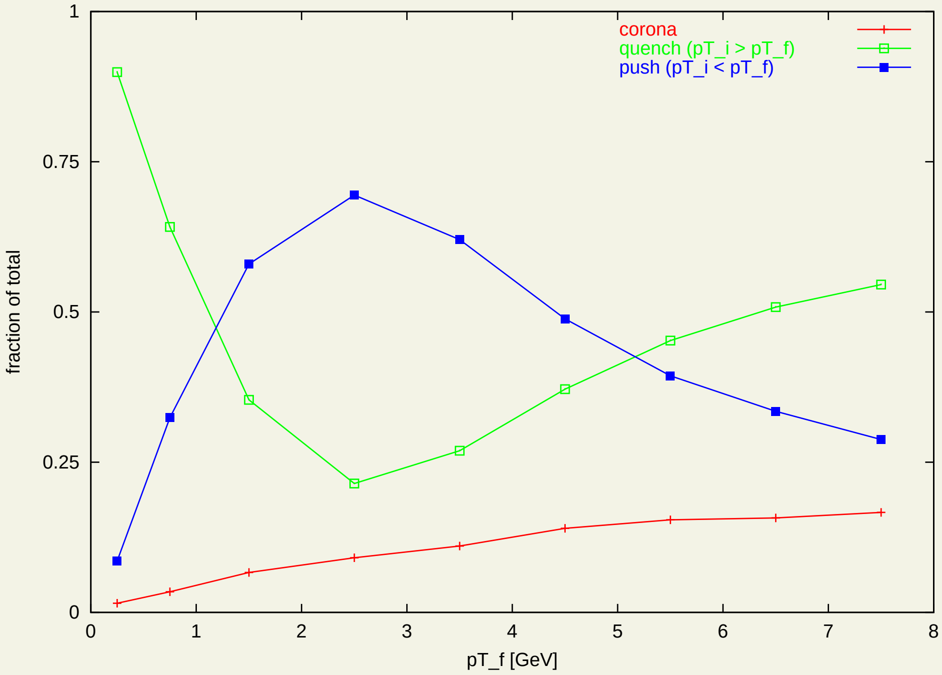
study using MPC 1.8.0 w/ elastic and inelastic $2 \to 2$, $dN^{cent}/d\eta = 2000$

fractions from corona, quench, push vs pT, $(|y_f| < 1)$

DM, nucl-th/0503051: $\sigma_{gg} = 10$ **mb**



$\sigma_{gg} = 5$ **mb**



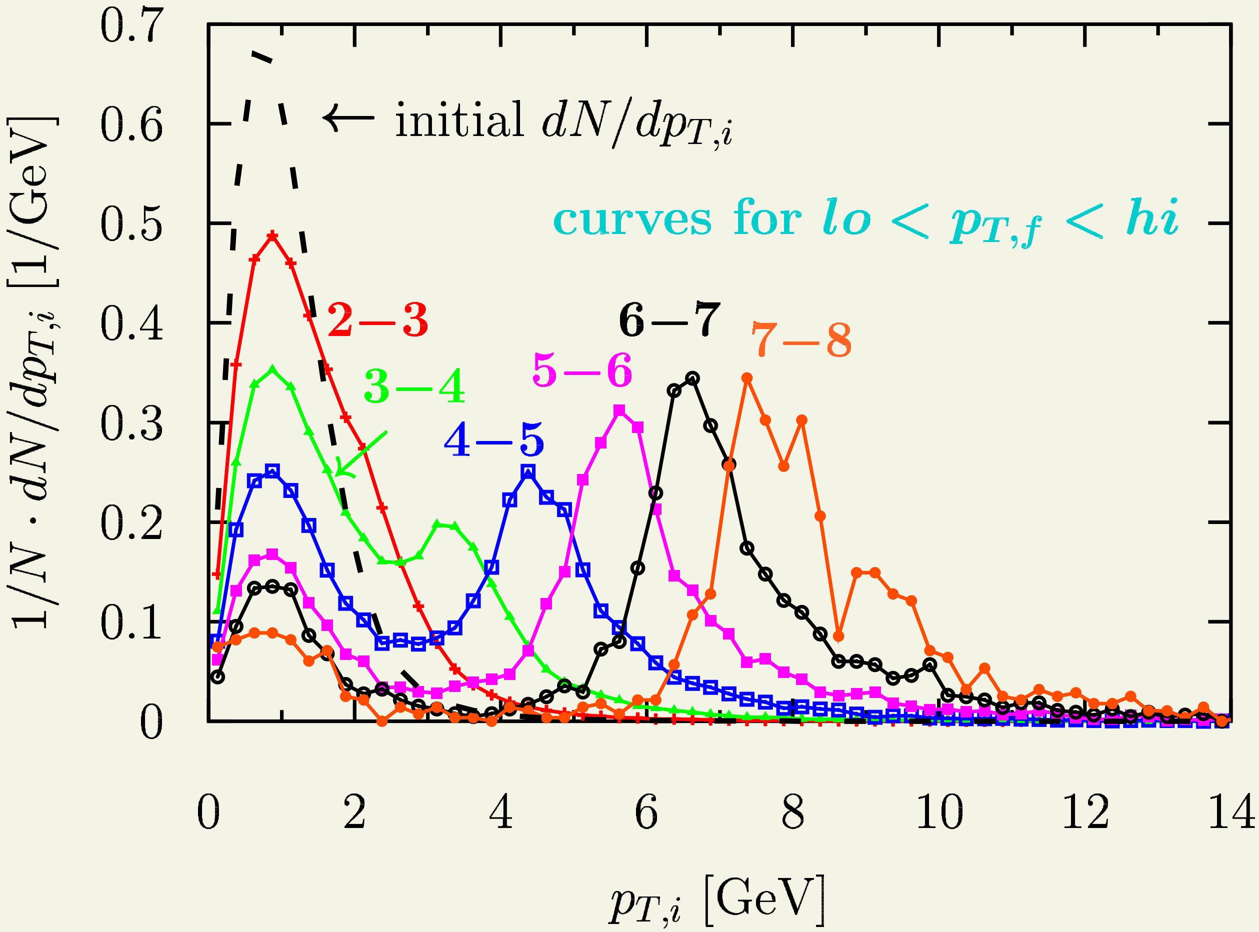
corona and "push" are significant even at $p_{T,parton} \sim 8$ **GeV**

fractions show surprisingly weak opacity dependence

distribution of initial momenta for fixed final momentum bins, $|y_{fin}| < 1$

(only quench + "push" plotted, normalized)

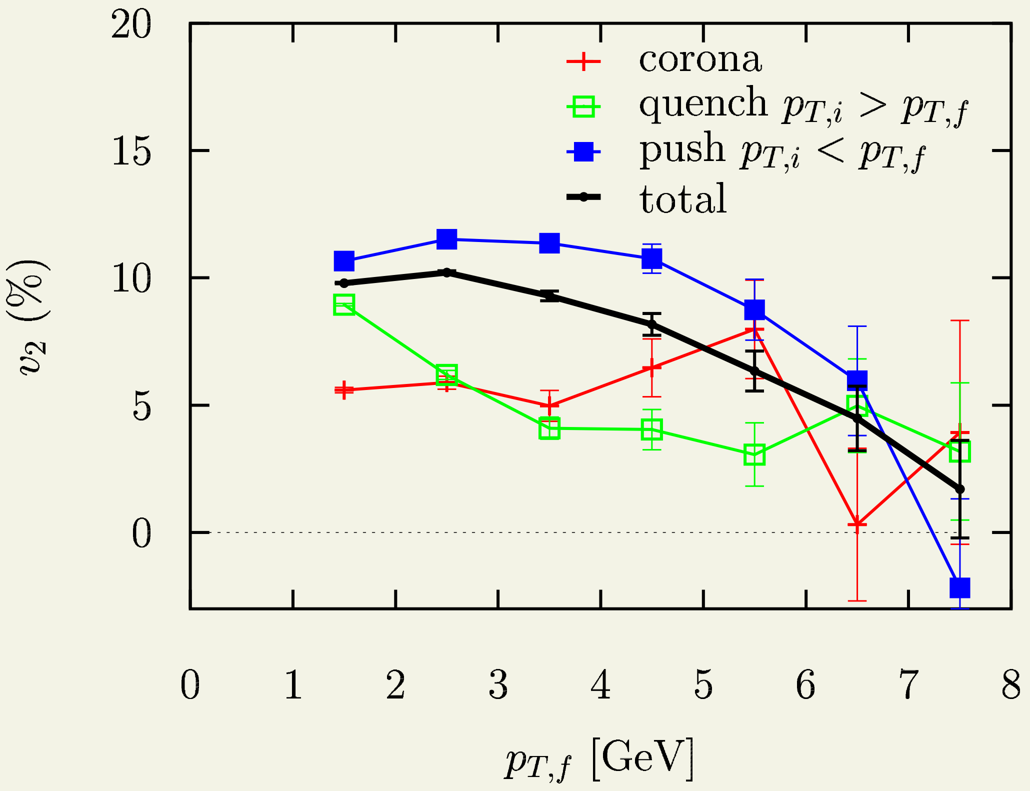
DM, nucl-th/0503051: $\sigma_{gg} = 10$ **mb**



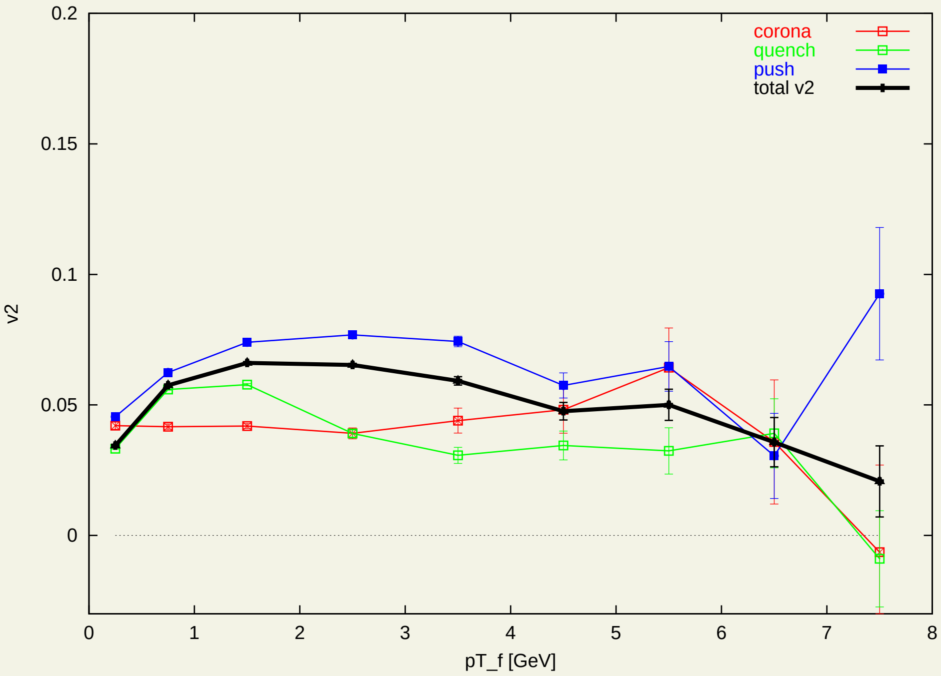
"lucky" $p_{T,i} \sim 1$ GeV soft partons can end up at $p_T \sim 5 - 6 - 7$ GeV

elliptic flow contributions vs pT

DM, nucl-th/0503051: $\sigma_{gg} = 10$ **mb**



$\sigma_{gg} = 5$ **mb**



rapid v_2 drop from quench at high p_T is compensated by large v_2 of "pushed-up" partons

combined $v_2(p_T)$ decreases more slowly at high p_T and can exceed "geometric" (extreme absorption) bounds Shuryak ('04), Voloshin ('04)

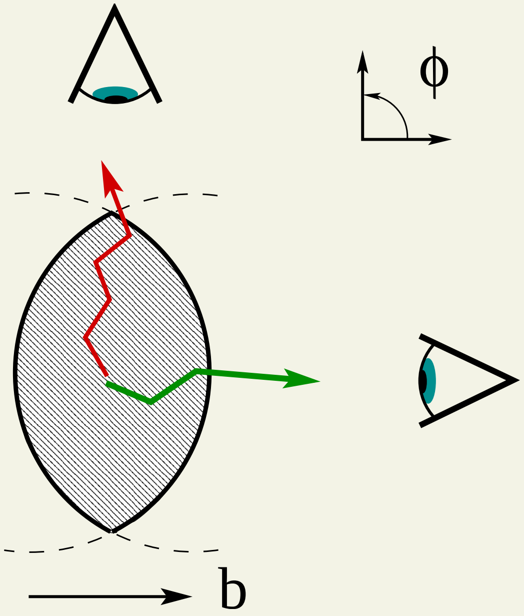
Quench v_2 - resembles inel. E-loss

radiative E-loss: $$\Delta E \approx const \times \alpha_s^3 \int \underbrace{dz \cdot z} \cdot \rho(z, t = z/v)$$

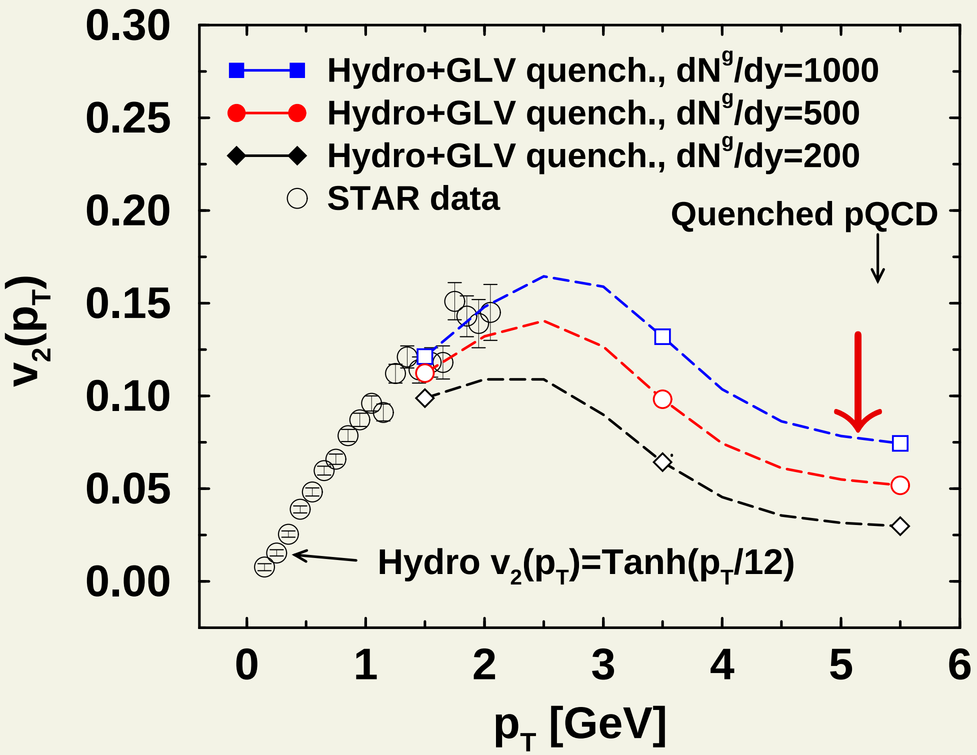
Wang, Gyulassy, Vitev, Wiedemann et al

non-Abelian gluon self-coupling: $\frac{dE}{dx} \propto x(!)$

variation in path-length, $\Delta E(\phi)$ → momentum anisotropy



Gyulassy & Vitev PRL86 '01:



v_2 drops fast at large $p_T \gtrsim 3$ GeV - $v_2(p_T)$ data look flatter

Where do the high opacities come from?

BIGGEST PUZZLE

- **strong correlations?** - **critical scattering, (quasi)bound states** Shuryak, Zahed et al ('04)
- **plasma instabilities? - quark-gluon E & M** Mrowczynski '93, Arnold et al ('04)
- **only apparent (hadronization effects)? - quark coalescence**

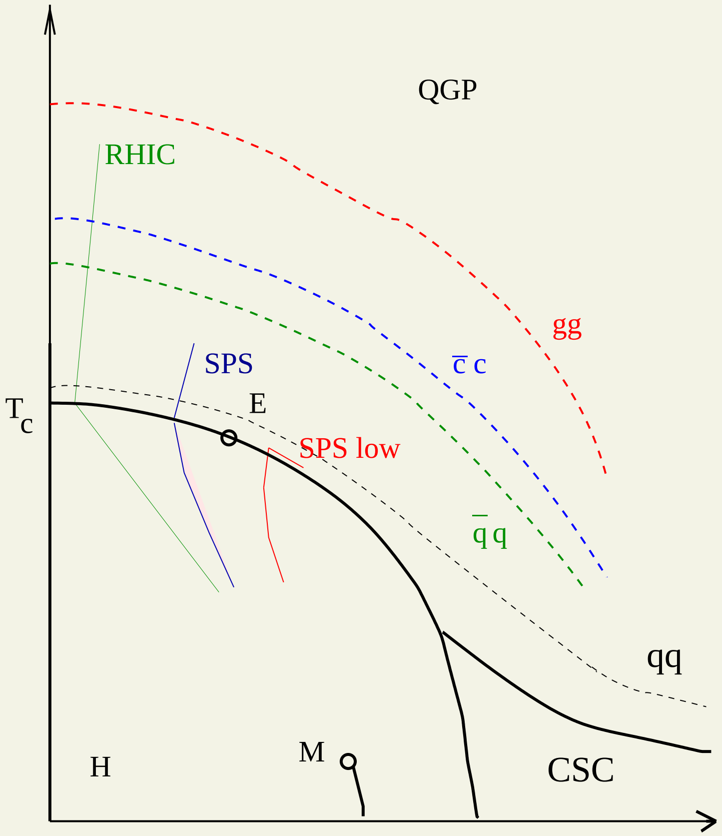
  Ko, Lin, Voloshin, DM, Greco, Levai, Mueller, Fries, Bass, Nonaka, Asakawa ...

Critical scattering

bound/quasi-bound states can enhance (low-energy) cross sections
by orders of magnitude

e.g., neutron, proton $r \sim 1$ **fm** $\sigma > 10^3 r^2 \gg 4\pi r^2$ **weakly-bound deuteron**

cold ^6Li atoms $r \sim 100$**Å** $\sigma > 10^6(!) r^2$ **weakly bound Li_2 molecules**



Shuryak, Zahed '04:
may be hundreds of weakly bound states in QGP

but does the rapidly expanding system stay long enough near "critical lines"?

Strongly correlated QGP challenge

BBGKY hierarchy: multiparticle distributions $f_1(x,p)$, $f_2(x_1,p_1,x_2,p_2)$, $f_3(\ldots)$, ...

$$\begin{aligned} D_1 f_1 &= C_1[f_2] \\ D_2 f_2 &= C_2[f_2, f_1] \\ D_3 f_3 &= C_3[f_3, f_2, f_1] \\ &\ldots \end{aligned}$$

D_n: derivatives - $\partial_t + \sum_j^n v_j \partial_{x_j} - F_{j,n}\partial_{p_j}$, C: collision terms (integrals)

0th-order: truncate $f_2 = f_3 = \ldots = 0$ → **Vlasov eqn.**

1st-order: truncate $f_3 = f_4 = \ldots = 0$

- for weak coupling: → hierarchy of relaxations $\tau_1 \gg \tau_2$ → **Boltzmann**
- if stronger coupling: coupled f_1 and f_2

$\Rightarrow$ transport eqn. for 2-particle distribution function

1st try: treat correlations as quasi-particles ($\sim$ bound states)

→ need particle-correlation and correlation-correlation scattering cross sections

Color plasma physics

couple particles and classical Yang-Mills fields - **analog of E&M plasma physics**

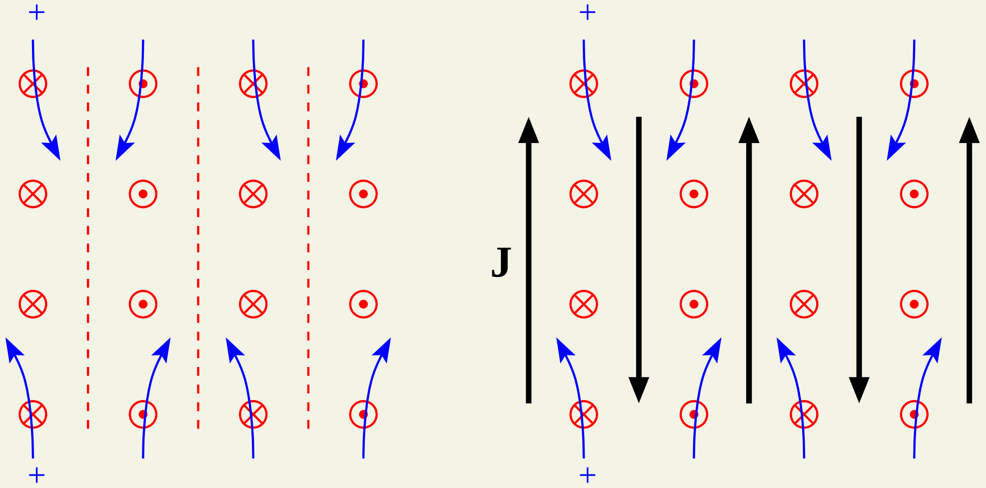
Wong equation: color Vlasov-Boltzmann Wong, Heinz

$$p^\mu \left(\partial_\mu + g t_a F^a_{\mu\nu} \frac{\partial}{\partial p_\nu} + g f_{abc} A^b_\mu t^c \frac{\partial}{\partial t_a}\right) f = C(f), \quad D_\mu F_a^{\mu\nu} = g \int p^\nu t_a \left(f_q - \bar{f}_q + f_g\right) dP dQ$$

essentially unexplored, even the magnetohydrodynamic limit unknown

fields may be key to thermalization: plasma instabilities Mrowczynski '93, Arnold et al '04

anisotropic momentum distributions → Weibel-like instabilities → isotropization



(a) (b)

Quark coalescence

Ko, Lin, Voloshin, DM, Greco, Levai, Mueller, Fries, Bass, Nonaka, Asakawa ...

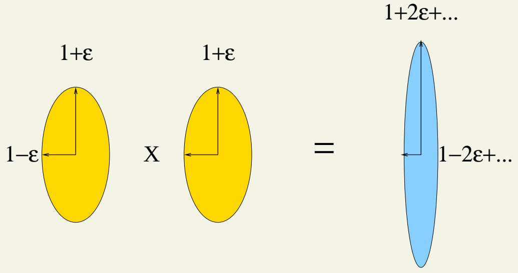
coalescence of comoving quarks: $q\bar{q} \Longrightarrow M \qquad 3q \Rrightarrow B$

DM & Voloshin, PRL91 ('03)

analog of $n + p \to d$

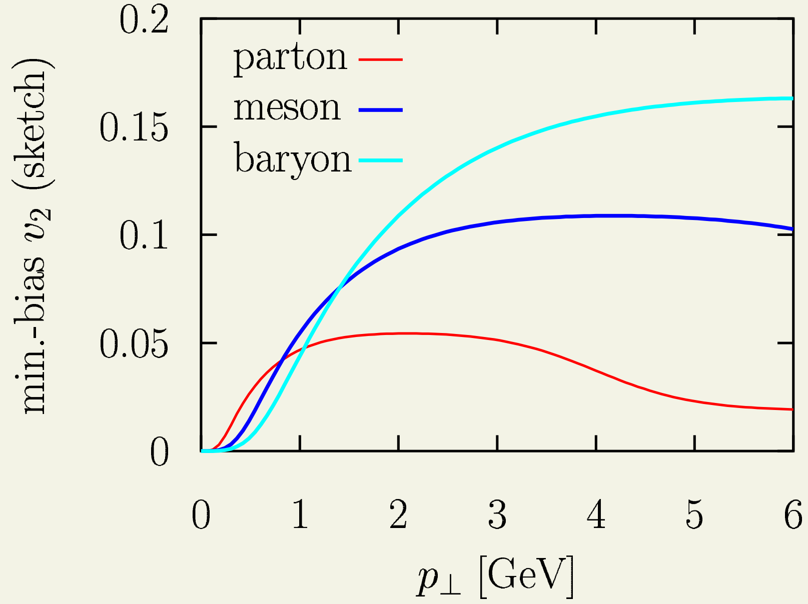
$$\frac{dN_M(p_T)}{d\phi} \propto \left[\frac{dN_q(p_T/2)}{d\phi}\right]^2$$

$$\frac{dN_B(p_T)}{d\phi} \propto \left[\frac{dN_q(p_T/3)}{d\phi}\right]^3$$



squared/cubed probability → amplified v_2

$$v_2^{hadron}(p_\perp) \approx n \times v_2^{quark}(p_\perp/n)$$



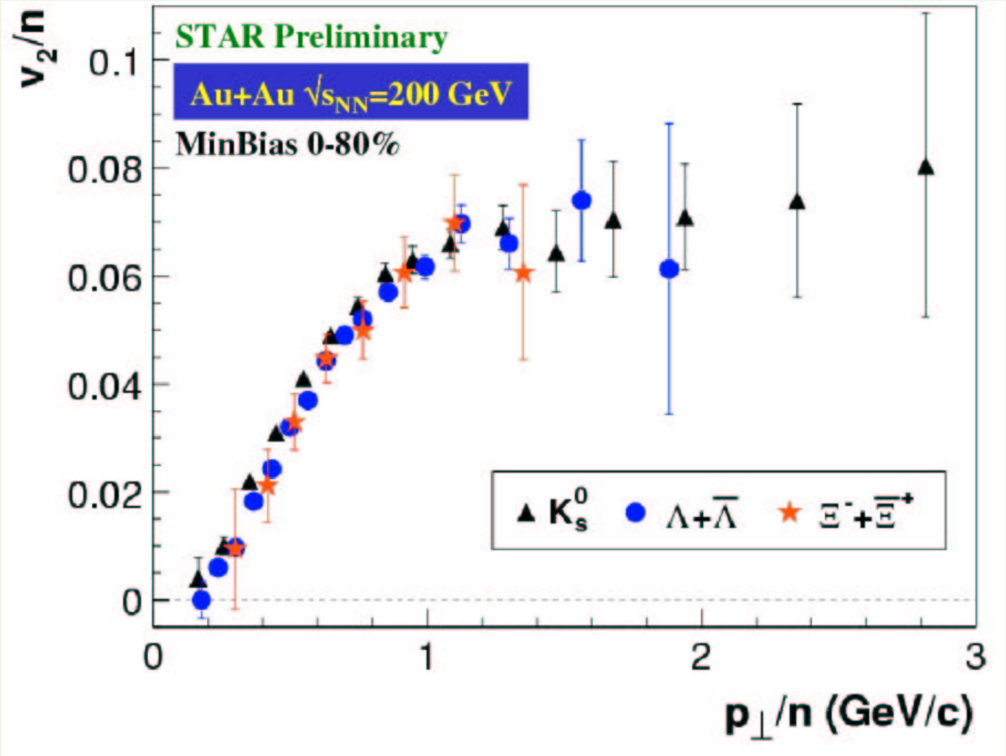
$3\times$ **for baryons**
$2\times$ **for mesons**
} **50% larger** v_2 **for baryons**

→ $5\times$ **for pentaquark,** $6\times$ **for deuteron**

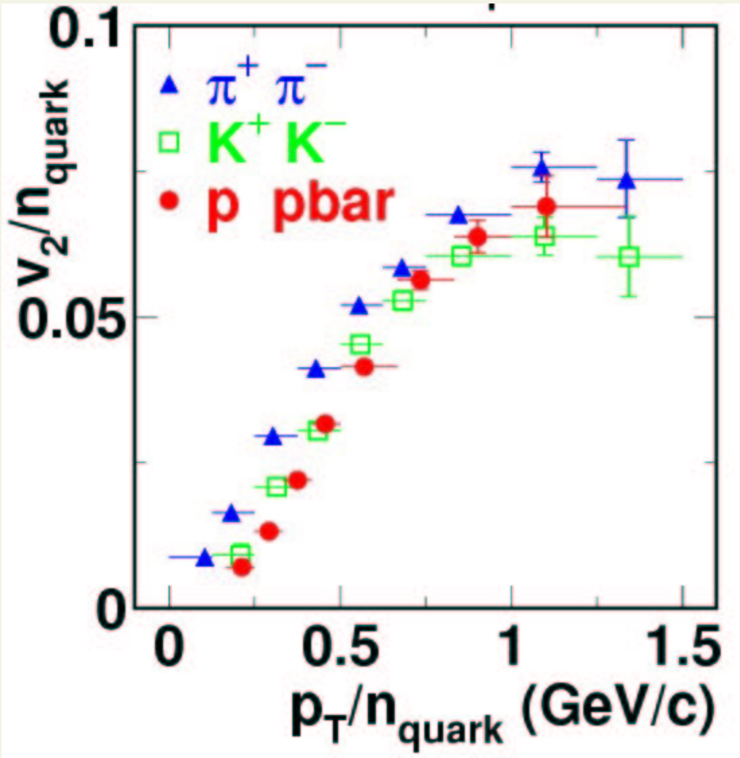
amplification greatly reduces opacities needed to reproduce v_2 **data**

EXP. SUPPORT: quark number scaling of elliptic flow (v_2/n_q vs p_T/n_q)

STAR '03



PHENIX '03



picture hangs together - coalescence also describes enhanced baryon/meson ratios Fries et al, Ko et al

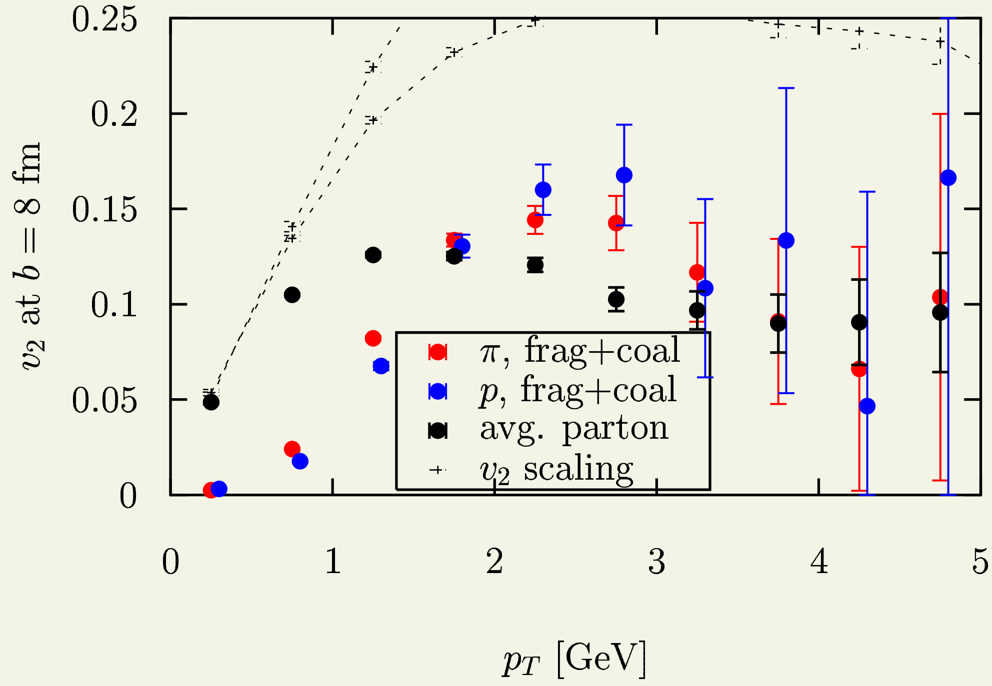
(!) provided quark phasespace distribution is a fit parameter

From dynamics, scaling is truly remarkable → PUZZLE

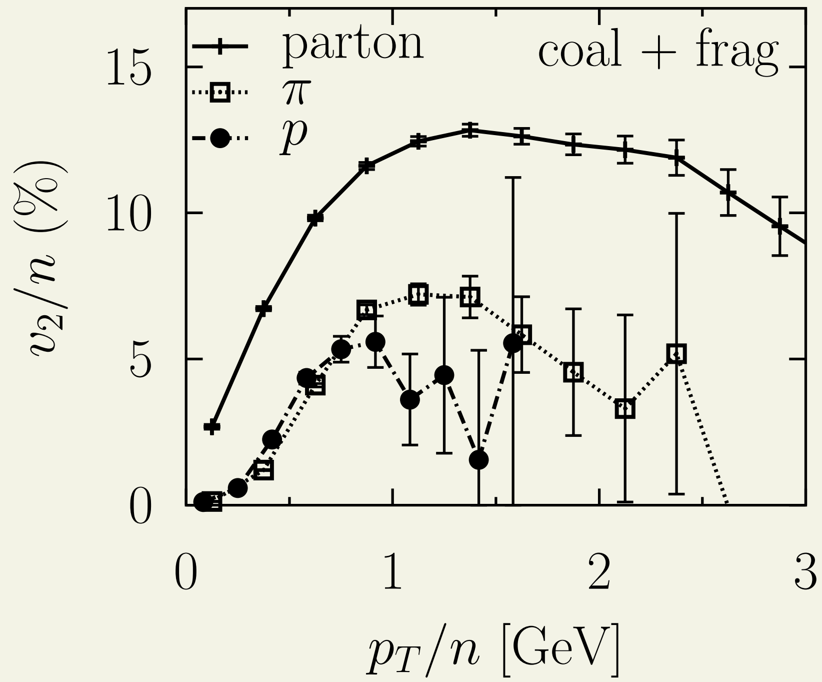
- significant fragmentation contributions
- strong space-momentum correlations (spatial anisotropies)
- surface emission

parton transport + dynamical 4D coalescence - Gyulassy, Frankel, Remler '83
and indep fragmentation -JETSET for partons without coal partner

DM ('04): $v_2(p_T)$ - π, p, q



scaled v_2



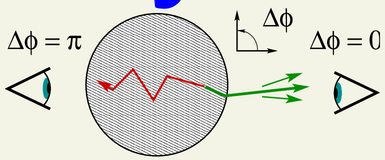
flow amplification greatly reduced, baryon-meson splitting mostly gone

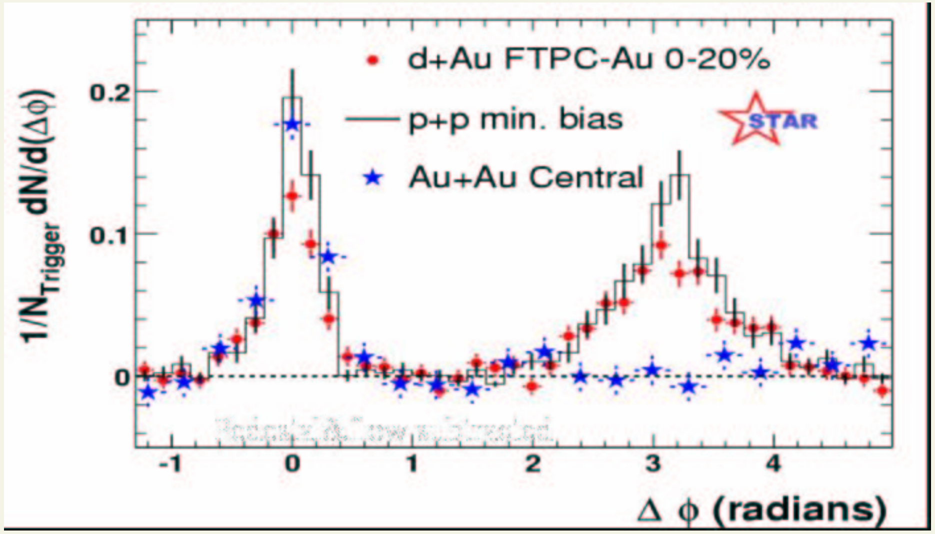
may still scale approximately ~ 15% err but scaled v_2 is NOT the quark v_2

Other ways to probe the QGP

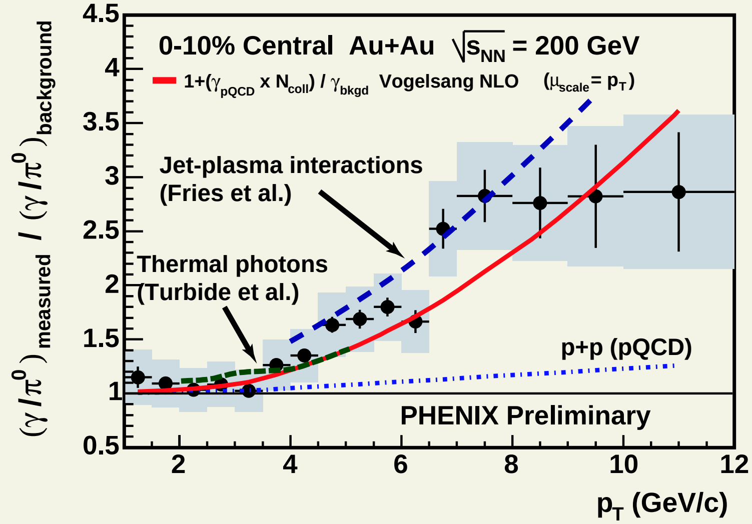
azimuthal correlations

STAR '04

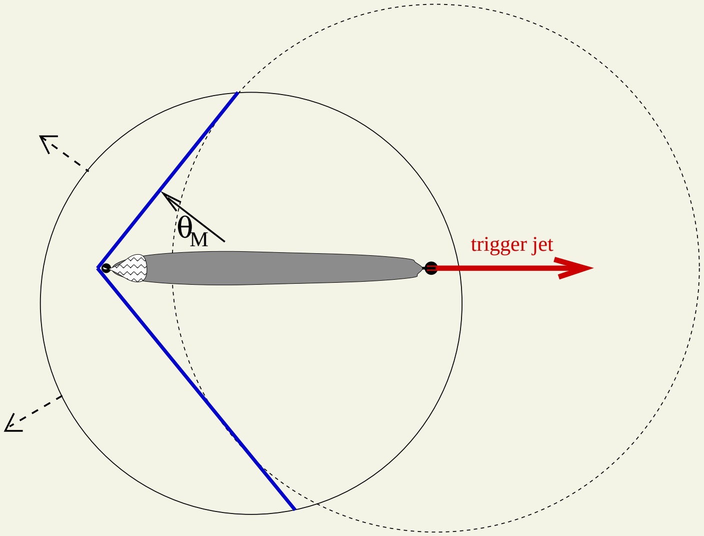




direct photons - PHENIX '05

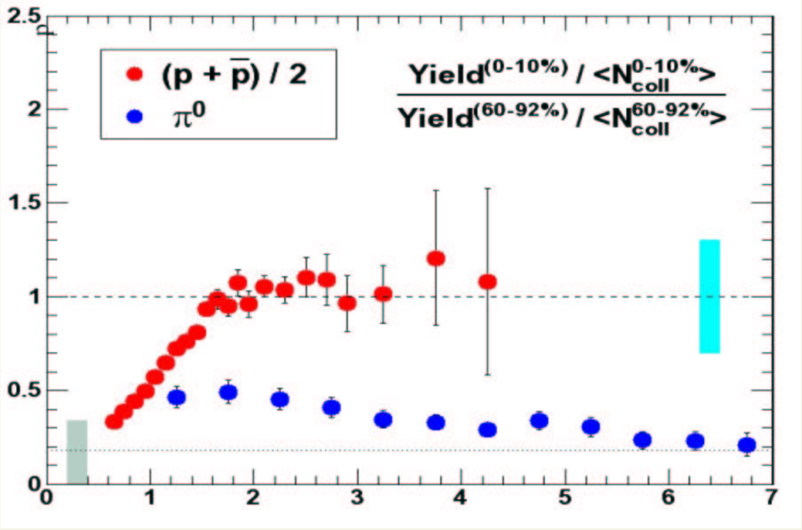


collective excitations - shocks



baryon production puzzle - PHENIX '03

$\sim R_{AB}$



other angles: nucleus (A) and collision energy ($\sqrt{s}$) dependence → **LHC**

Summary

- **Many indications of an opaque, largely randomized (but still dissipative) parton system at RHIC (at 10-100 times the densities of nuclei):**
  - **large elliptic flow, even for D mesons (prelim.)**
  - strong high-pT suppression of energetic particles, disappearance of back-to-back correlations
  - large baryon/meson ratios, **quark number scaling of** v_2
  - large "out" and "long" pion interferometry radii (HBT)

- **this matter seems to be the most ideal fluid ever observed**
  → experimental test of minimal viscosity derived from string theory.

- **at such high opacities, soft physics tails can reach up to $p_T \sim 10$ GeV**

- **many puzzles and open questions:**
  - **thermalization mechanism, origin of large opacities**
  - **consistent <u>quantitative</u> description of all observables**
  - **large charm v_2 but no secondary charm**
  - **no quark scaling of v_2** & B/M enhancement **from dynamical coalescence approach**
  - small HBT R_{side} independent of dynamics
  - **what will the plasma be like at the LHC (2007)?**

  ...

Backup slides

- quark coalescence
- pion HBT inteferometry
- “sonic boom”

coalescence formula

$$\frac{dN_M(\vec{p})}{d^3p} = g_M\int\Big(\prod_{i=1,2} d^3x_i\, d^3p_i\Big) W_M(x_1-x_2,\vec{p}_1-\vec{p}_2)\, f_\alpha(\vec{p}_1,x_1)\, f_\beta(\vec{p}_2,x_2)\,\delta^3(\vec{p}-\vec{p}_1-\vec{p}_2)$$

$$\frac{dN_B(\vec{p})}{d^3p} = g_B\int\Big(\prod_{i=1,2,3} d^3x_i\, d^3p_i\Big) W_B(x_{12},x_{13},\vec{p}_{12},\vec{p}_{13})\, f_\alpha(\vec{p}_1,x_1)\, f_\beta(\vec{p}_2,x_2)\, f_\gamma(\vec{p}_3,x_3)\,\delta^3\Big(\vec{p}-\sum\vec{p}_i\Big)$$

hadron yield   space-time   hadron wave-fn.   quark distributions

gives v_2 scaling trivially if:

1. **no other hadronization channels play a role**
2. **narrow wave functions** $W \sim \delta^3(\Delta x)\delta^3(\Delta p)$
3. **only small local harmonic modulations** $|v_2(x)| \ll 1$, $|v_n(x)| \ll 1$

$$\begin{aligned} v_2^{Meson}(p_T) &= \frac{2\,\langle f_q^2(x,p_T/2)\, v_{2,q}(x,p_T)\rangle_x}{\langle f_q^2(x,p_T/2)\rangle_x} \\ v_2^{Baryon}(p_T) &= \frac{3\,\langle f_q^3(x,p_T/3)\, v_{2,q}(x,p_T)\rangle_x}{\langle f_q^3(x,p_T/3)\rangle_x} \end{aligned}$$

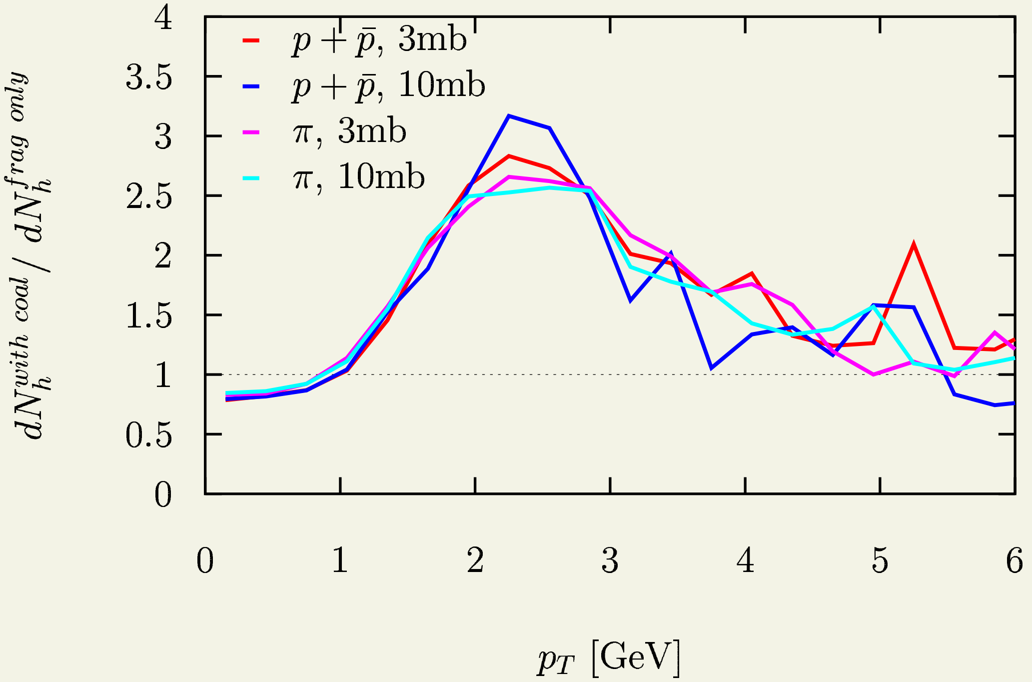
4. **spatial dependence can be ignored (factorizes out)** $\Rightarrow v_2^{hadron}(p_T) = n v_2^{quark}(p_T/n)$
   - **for example, global** $v_2(x,p_T) \equiv v_2(p_T)$**, or constant FO density**

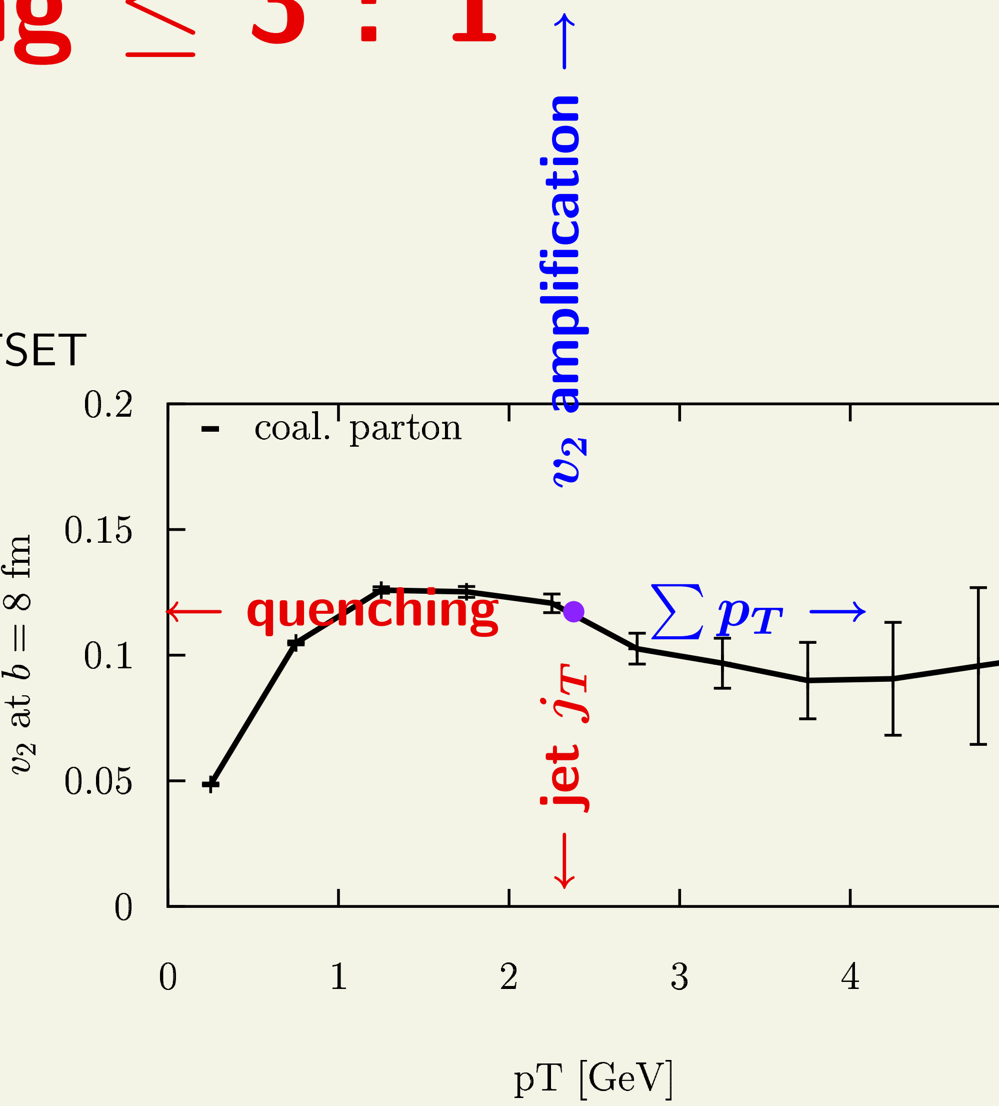
none of these satisfied in transport or hydro, contrary to parameterizations

1. Coal : Frag $\leq$ 3 : 1

coal+frag yield / frag only yield

DM ('04): dynamical calculation MPC + coal/JETSET



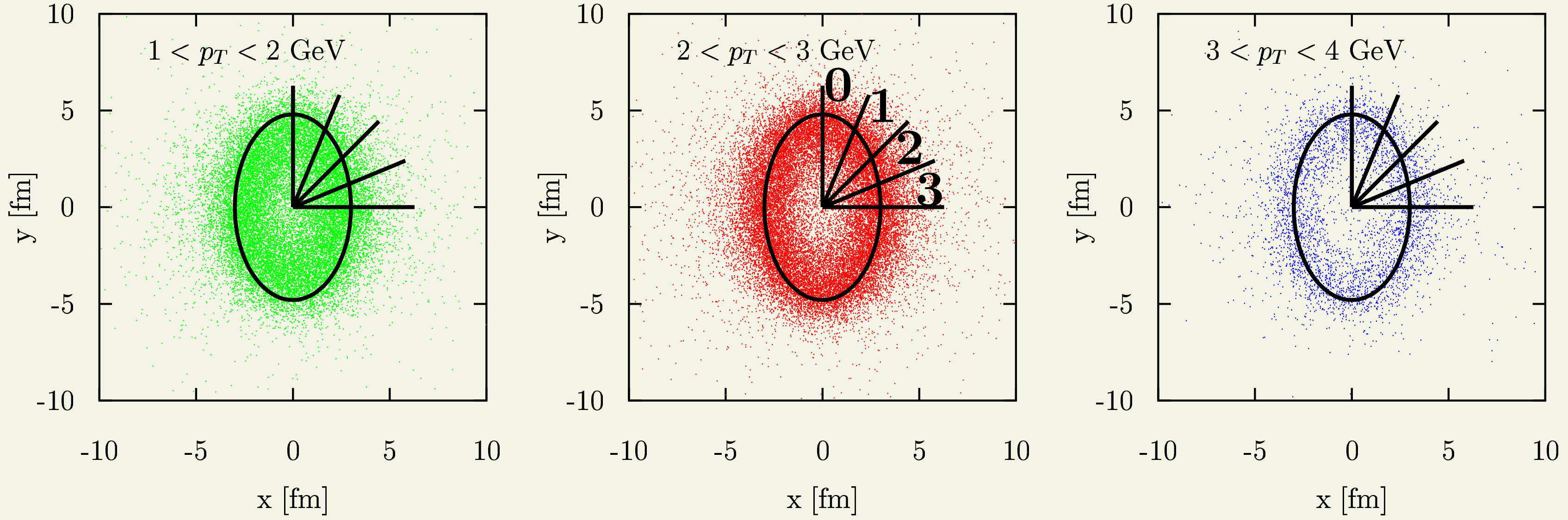


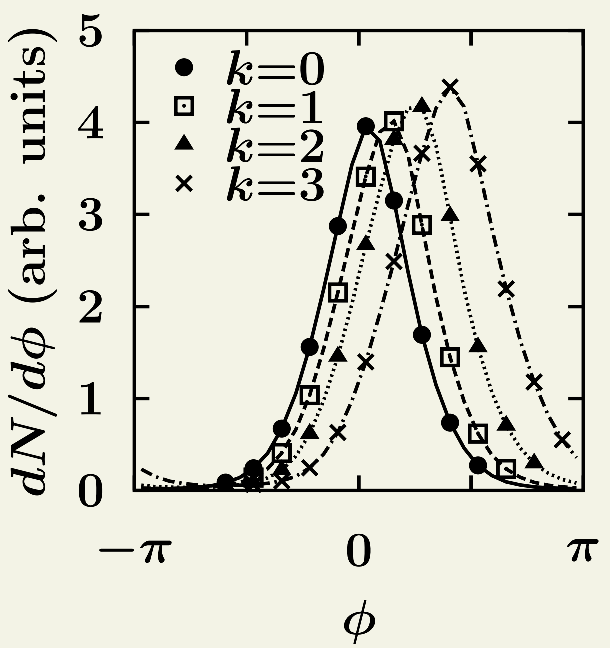
v_2 from $\sim 30\%$ fragmentation contribution does not amplify → scaling spoiled

also, about same enhancement for protons and pions → p/π not enhanced

2. Strong spatial variations

final **transverse position** **distributions** $(|y_{rap}| < 2)$





← **momentum $dN/d\phi$ in each spatial wedge**

show surface emission at high p_T $\Rightarrow v_2(x, p_T)$

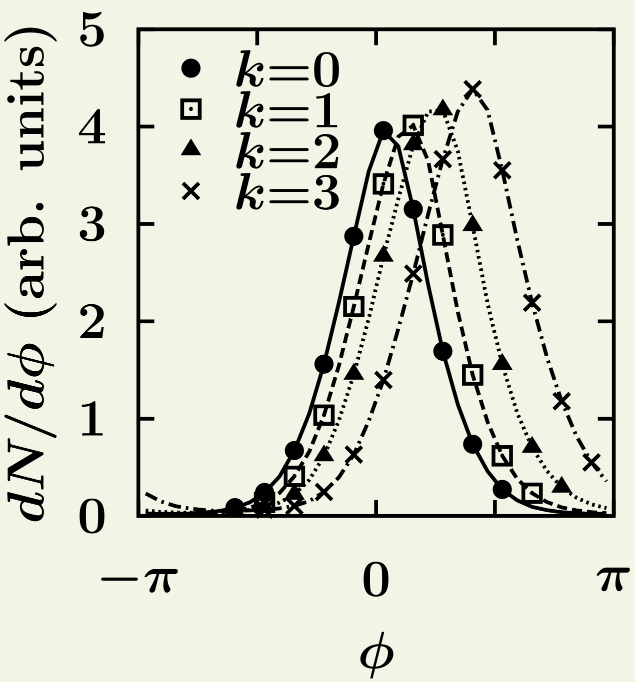
$k = 0$ **region:** $v_2 < 0$; $k = 3$ **region:** $v_2 > 0$

expect similar result from hydro

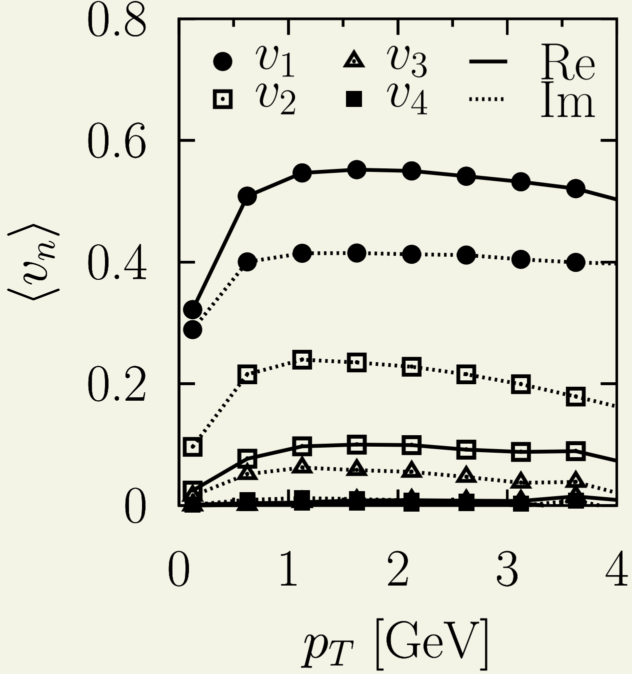
3. Large $|v_n| \sim \mathcal{O}(1)$

DM, nucl-th/0408044

local $\cos(n\phi)$ **and** $\sin(n\phi)$ **anisotropies** → **use** $v_n \equiv \langle \cos(n\phi) + i\sin(n\phi) \rangle$



averages over 4 spatial wedges

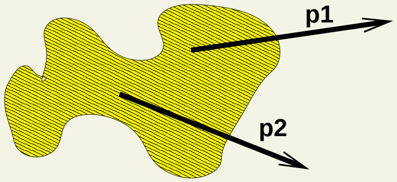


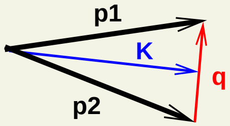
narrow, almost Gaussian peaks - $dN/d\phi \sim \exp[-(\phi-\phi_0)^2/(2\sigma^2)]$

$\Rightarrow |v_n| \sim \mathcal{O}(1)$, $\quad \langle \cos(n\phi) \rangle \equiv \mathrm{Re}\, v_n = \cos(n\phi_0) \cdot |v_2| \rightarrow$ **varies with** x**(!)**

new local scaling: $\quad |v_{k,had}(p_T, x)| \simeq |v_{k,q}(p_T/n_q, x)|^{1/n_q} \neq n_q |v_{k,q}(p_T/n_q, x)|$

HBT essentials





Momentum correlations: **reflect spacetime freezeout**

$$C(\vec{q},\vec{K}) \equiv \frac{N(\vec{p}_1,\vec{p}_2)}{N(\vec{p}_1)N(\vec{p}_2)} \approx 1 + \frac{\left|\int d^4x\ f_{FO}(x,\vec{K})\ e^{iq^\mu x_\mu}\right|^2}{\left[\int d^4x\ f_{FO}(x,\vec{K})\right]^2}$$

[e.g., Pratt, Csörgő & Zimányi, PRC 42, 2646 ('90)]

$\boldsymbol{f_{FO}(x,\vec{p}) \equiv dN/d^4x\ d^3p}$: **7D distribution of last interaction vertices**

Out-side-long coordinates: **special choice of frame**

$$K^\mu \equiv (\tilde{K}^0, K_\perp, 0, 0), \qquad x^\mu \equiv (\tilde{t}, x_O, x_S, x_L) \qquad (\tilde{K}^0 \approx \sqrt{m^2 + K_\perp^2})$$

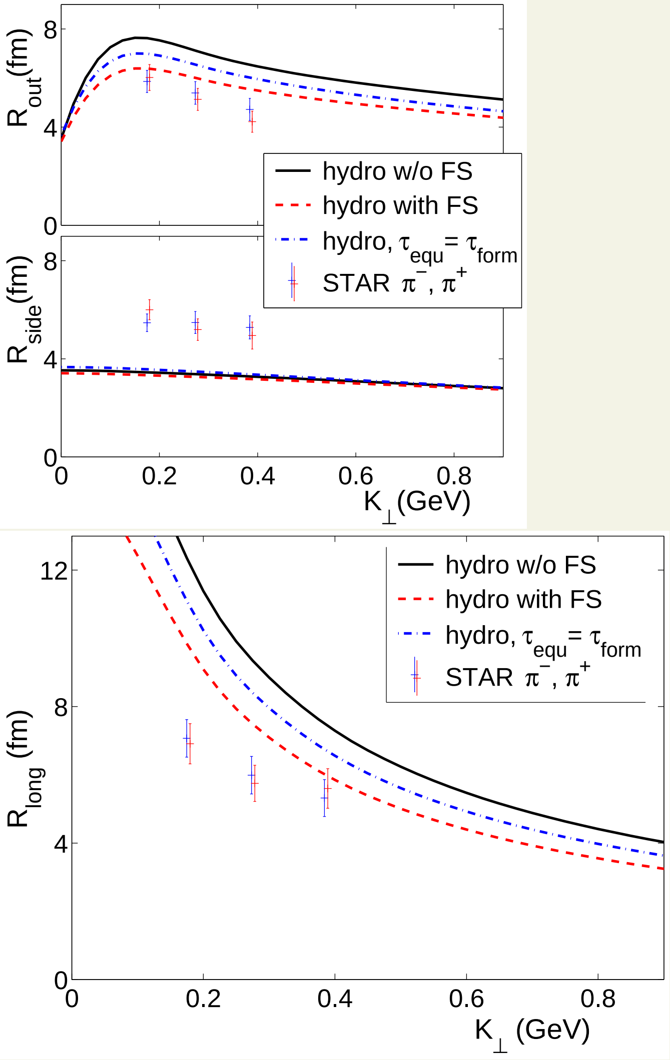
HBT radii:

$$R_O^2 = \langle \Delta x_O^2 \rangle_K + v_\perp^2 \langle \Delta \tilde{t}^2 \rangle_K - 2 v_\perp \langle \Delta x_O \Delta \tilde{t} \rangle_K$$
$$R_S^2 = \langle \Delta x_S^2 \rangle_K, \qquad R_L^2 = \langle \Delta x_L^2 \rangle_K$$

exact for Gaussian source without final-state interactions

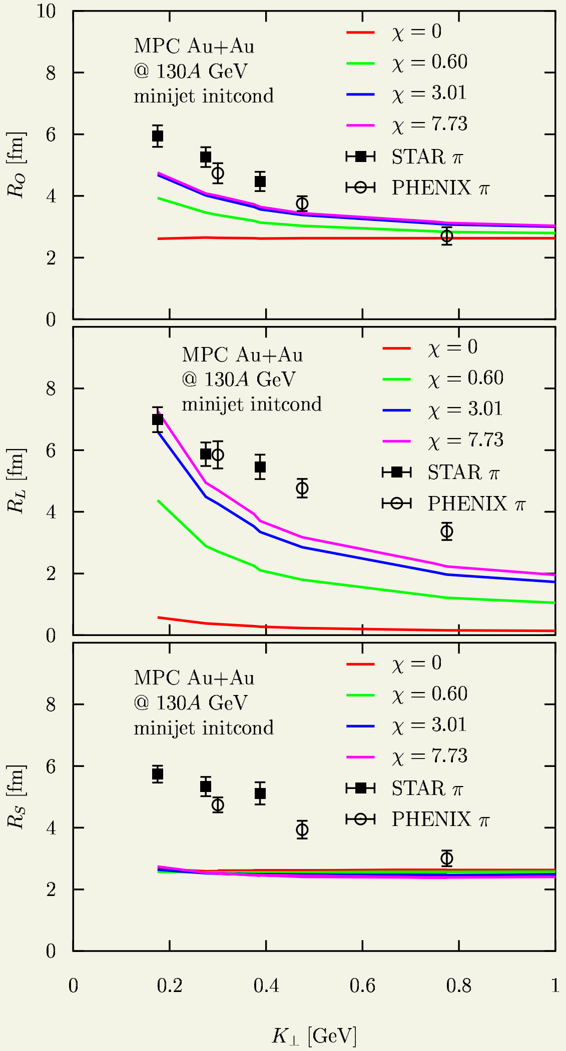
Small R_{side} → PUZZLE

ideal hydro Heinz & Kolb ('02)



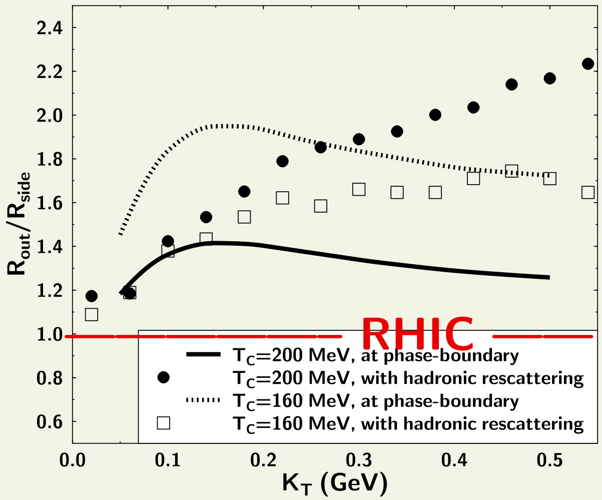
overshoots R_O **&** R_L
while $R_S \approx 4$ **fm only**

cov. transport DM & Gyulassy ('02)



R_O **&** R_L **increase with opacity**
but $R_S \approx 3.5$ **fm stays flat**

hydro+transport
Dumitru, Soff ('01)



R_{out}/R_{side} **shoots above data**

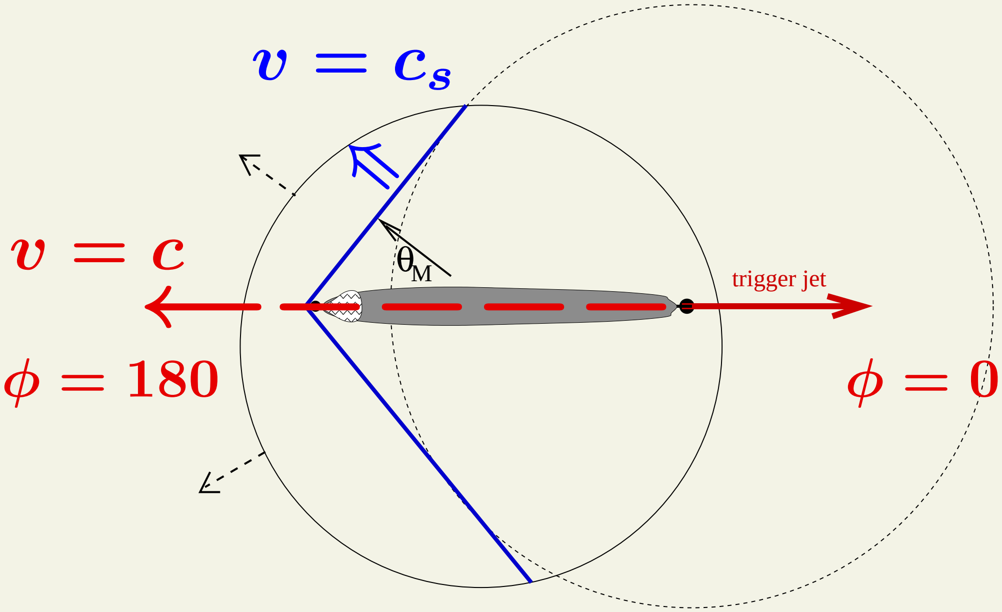
⇒ **late-stage hadronic decoupling not understood**

wrong spacetime evolution, or too simple HBT formula? maybe resonances?

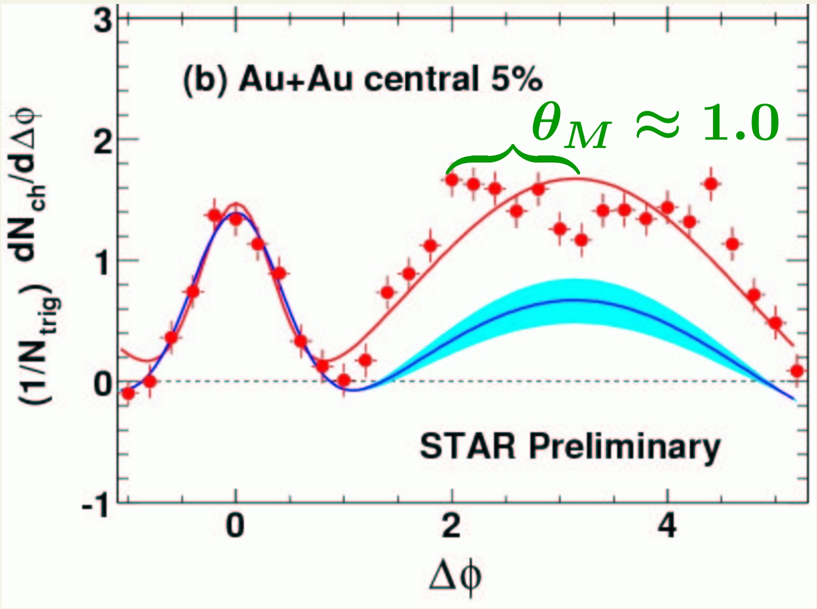
Collective excitations?

Short-wavelength probe could generate collective hydrodynamic response
- exciting, but hotly debated, possibility

"sonic boom" Stöcker '04, Casselderey-Solana et al



azimuthal correlations F. Wang [STAR] '04

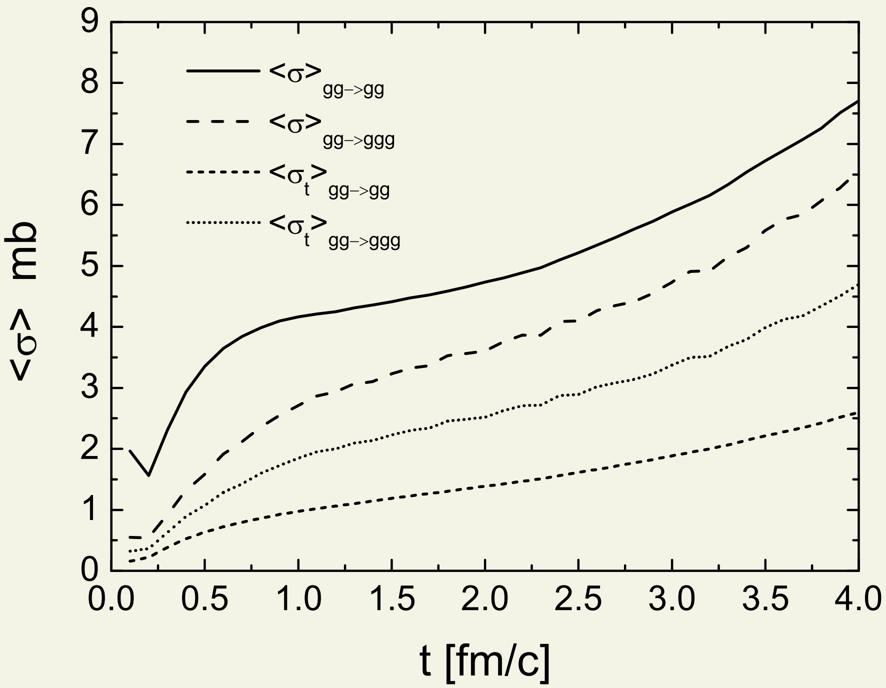


Mach cone: $\cos\theta_M = c_s/c$ $\Rightarrow$ $c_s^2 \approx 0.25 - 0.3 \cdot c^2$ ...

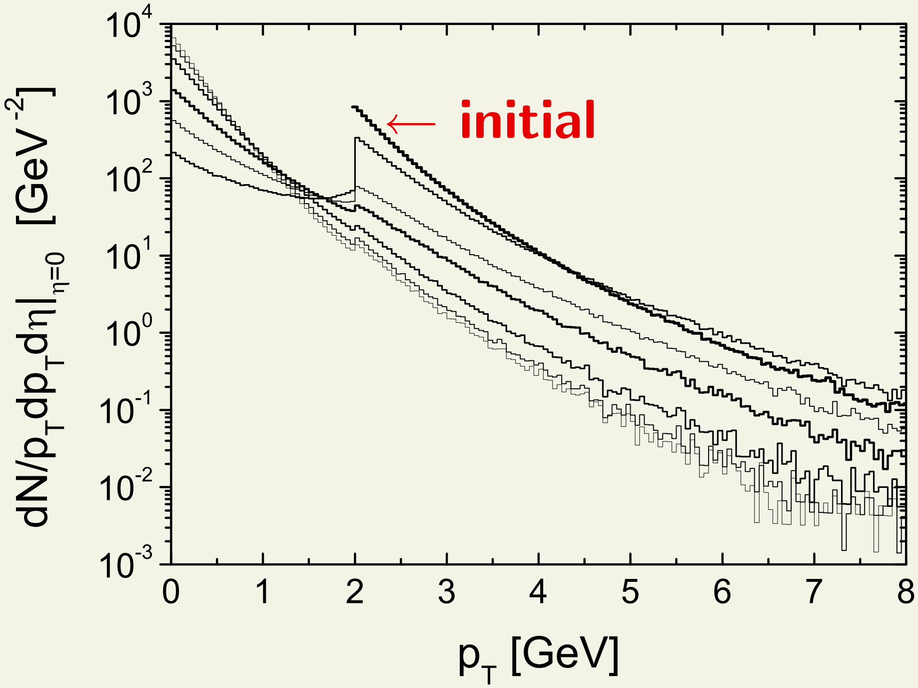
ideal parton gas: $c_s^2 = c^2/3$

Greiner & Xu '04: **claim thermalization time-scale $\tau \sim 2 - 3$ fm/c**

$2 \to 2$, $2 \to 3$ transport cross sections



spectra vs. time



inel. roughly doubles σ_{tr} ← OK

rapid cooling via $2 \to 3$, because assumed there is nobody below 2 GeV(!)

$\Rightarrow$ mostly phasespace driven, not collective "pressure" $\Rightarrow$ expect little v_2

in contrast DM & Gyulassy**: low-pT region initially filled**