The Quark-Gluon Plasma at RHIC

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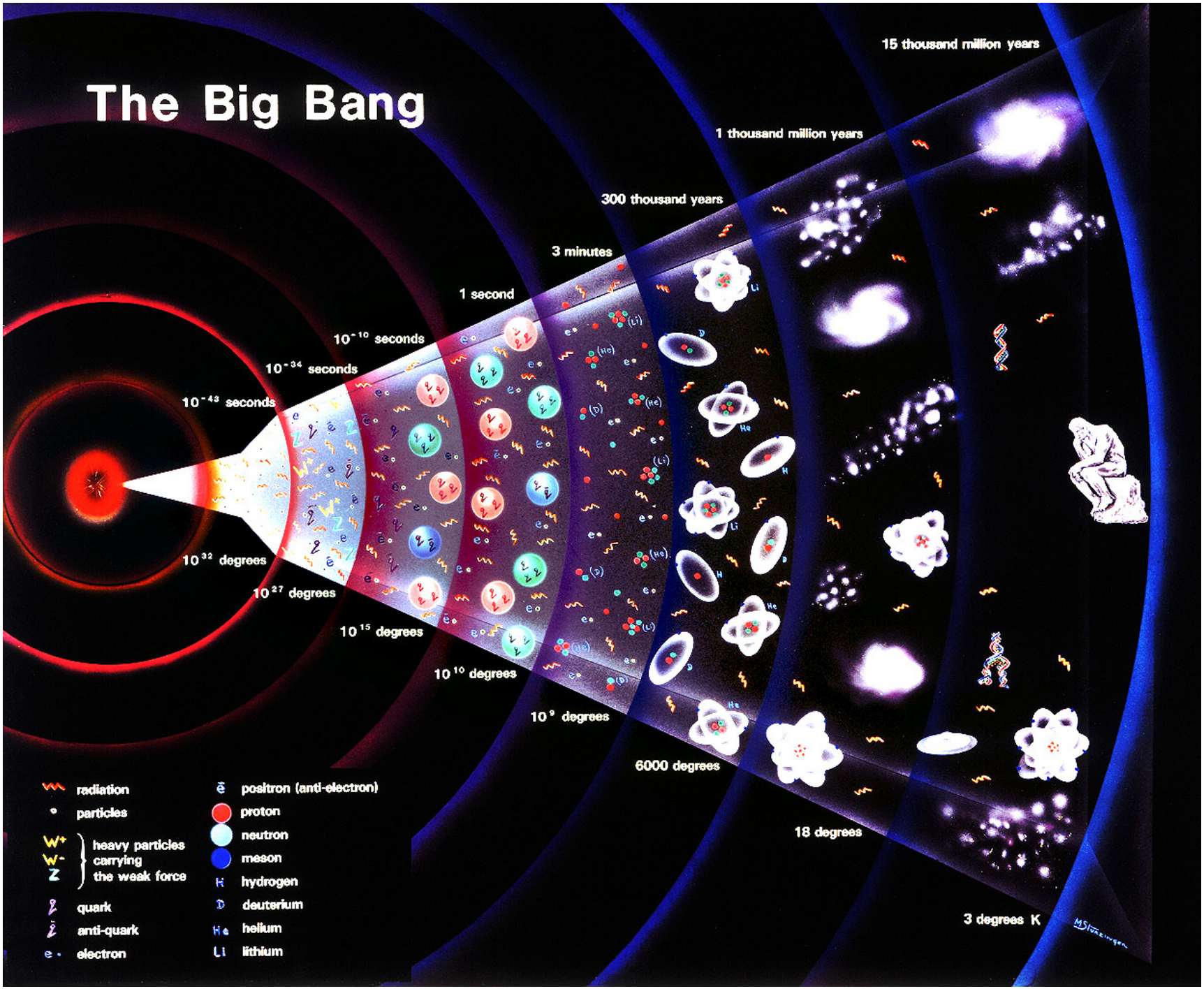
U.H., PANIC'02 (Osaka, 9/30/2002): "Unless a completely different mechanism for creating the elliptic flow can be found, the conclusion seems unavoidable that a thermalized QGP at $e > 10\, e_c$ and $T \geq 2T_c$ is created at RHIC which lives for about 5-7 fm/c before becoming sufficiently dilute to form hadrons. Perturbative mechanisms seem unable to explain the phenomenologically required very short thermalization time scale, pointing to strong non-perturbative dynamics in the QGP even at or above $2T_c$."

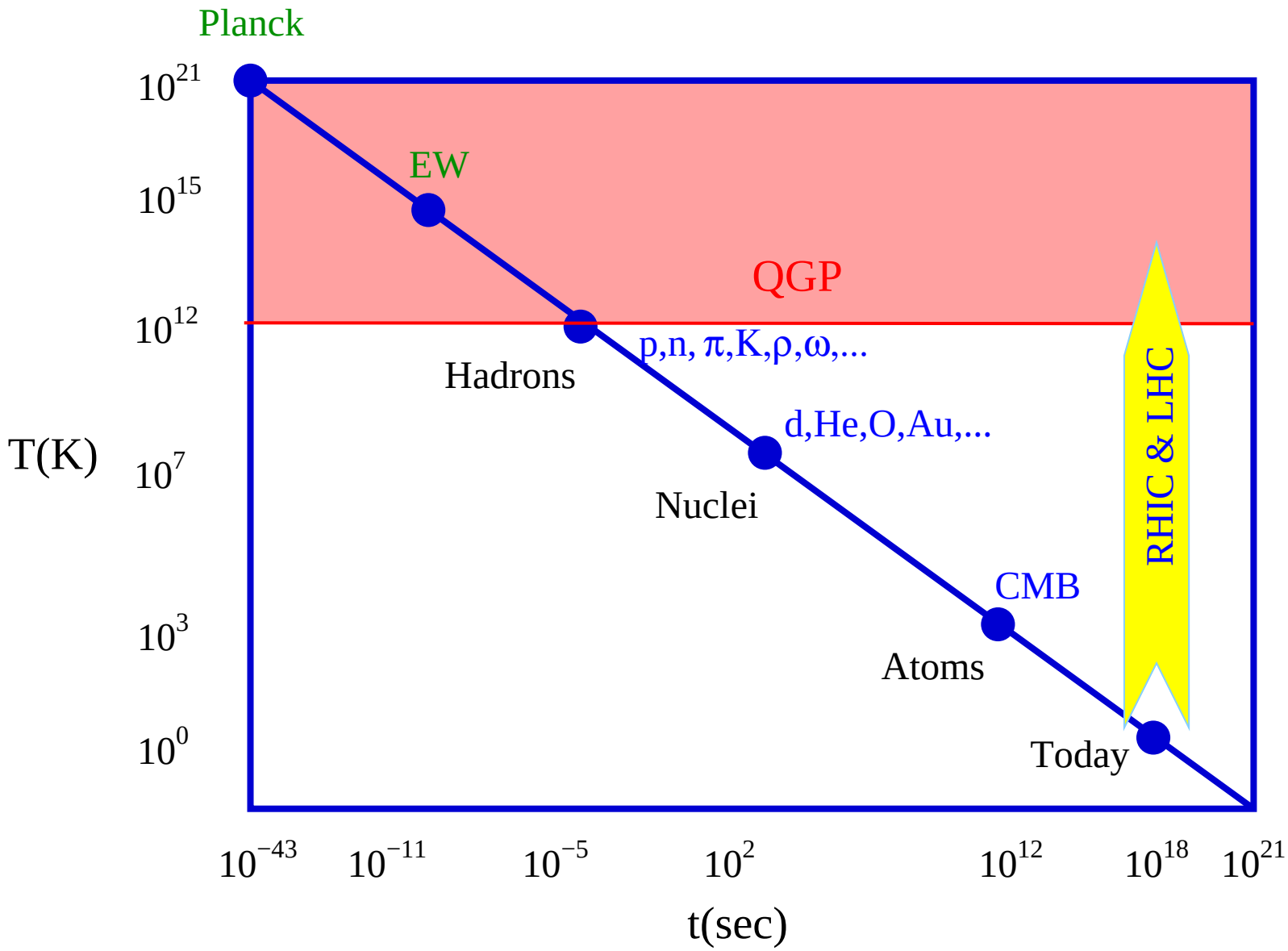
M. Gyulassy, BNL, June 18, 2003 (as quoted in the New York Times):
"It's a Quark-Gluon Plasma. Period."

M. Gyulassy, DNP03 (Tucson, Oct. 31 - Nov. 2, 2003):
"The END of searching for QGP — the BEGINNING of measuring its properties."

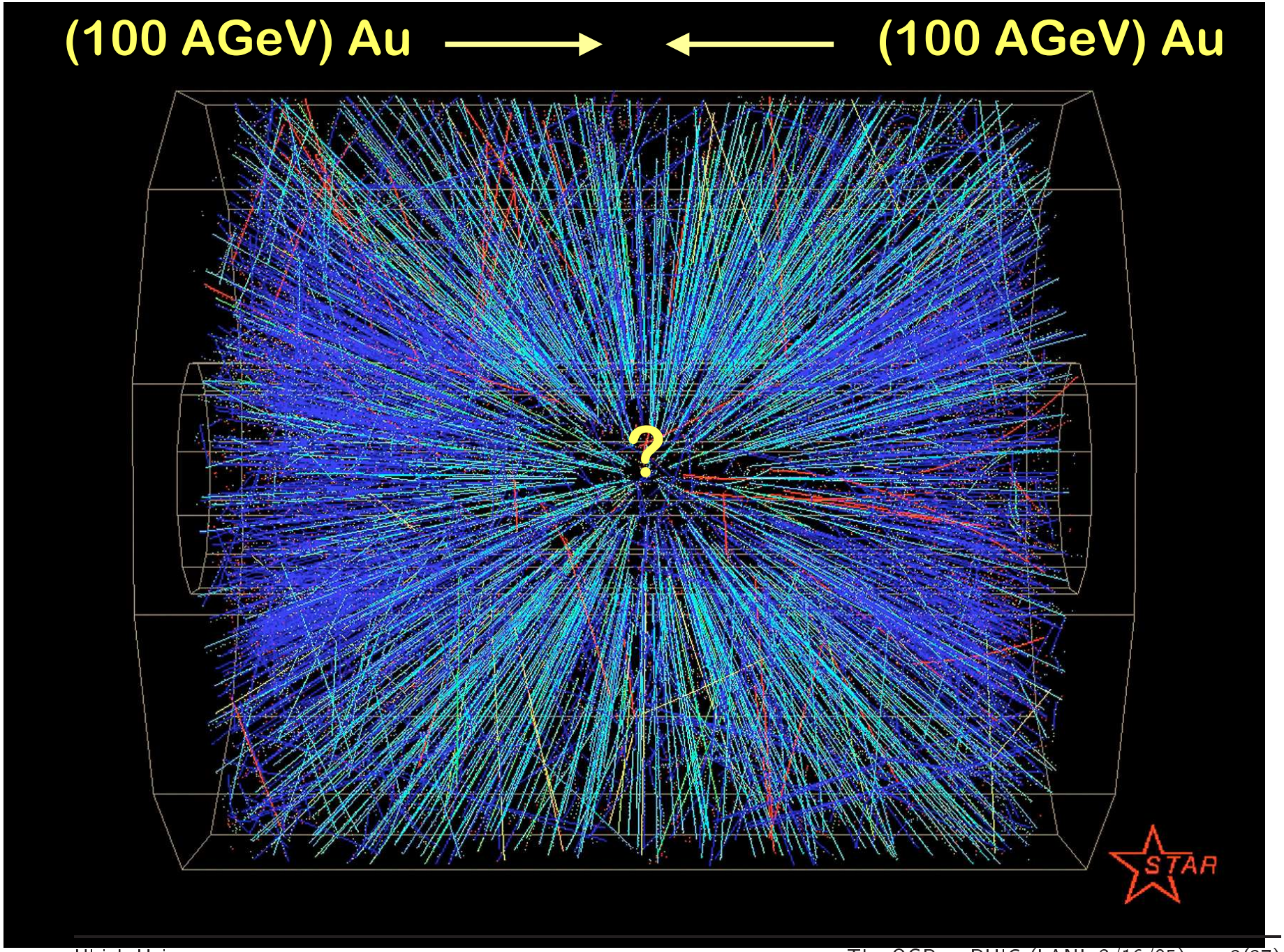
What happened!?


The Big Bang
15 thousand million years
1 thousand million years
300 thousand years
3 minutes
1 second
10^{-10} seconds
10^{-34} seconds
10^{-43} seconds
10^{32} degrees
10^{27} degrees
10^{15} degrees
10^{10} degrees
10^{9} degrees
6000 degrees
18 degrees
3 degrees K
radiation
particles
W+
W-
Z
heavy particles carrying the weak force
quark
anti-quark
e-
electron
positron (anti-electron)
proton
neutron
meson
H
hydrogen
D
deuterium
He
helium
Li
lithium

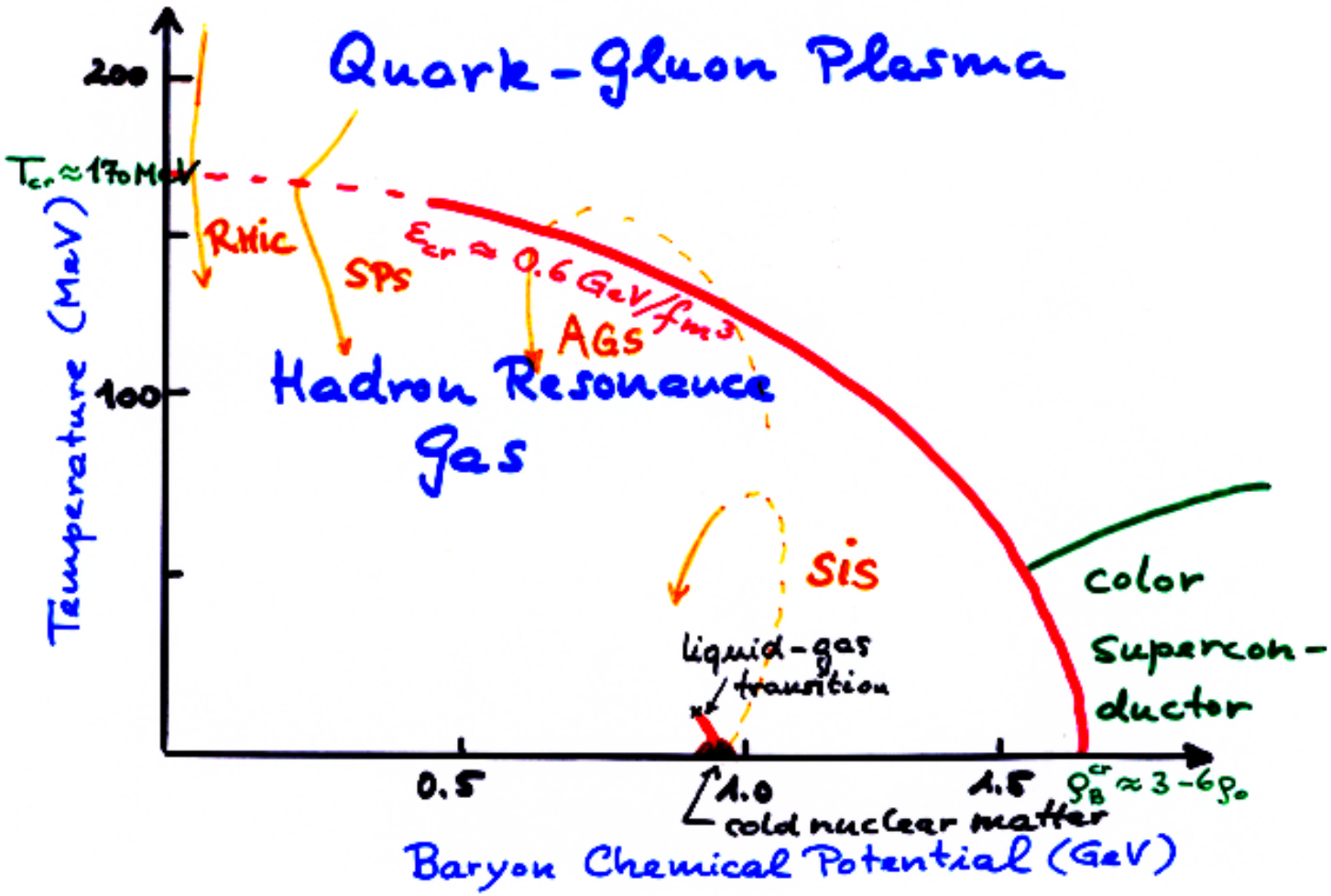
Recreating the Early Universe in the laboratory . . .



. . . with Little Bangs:

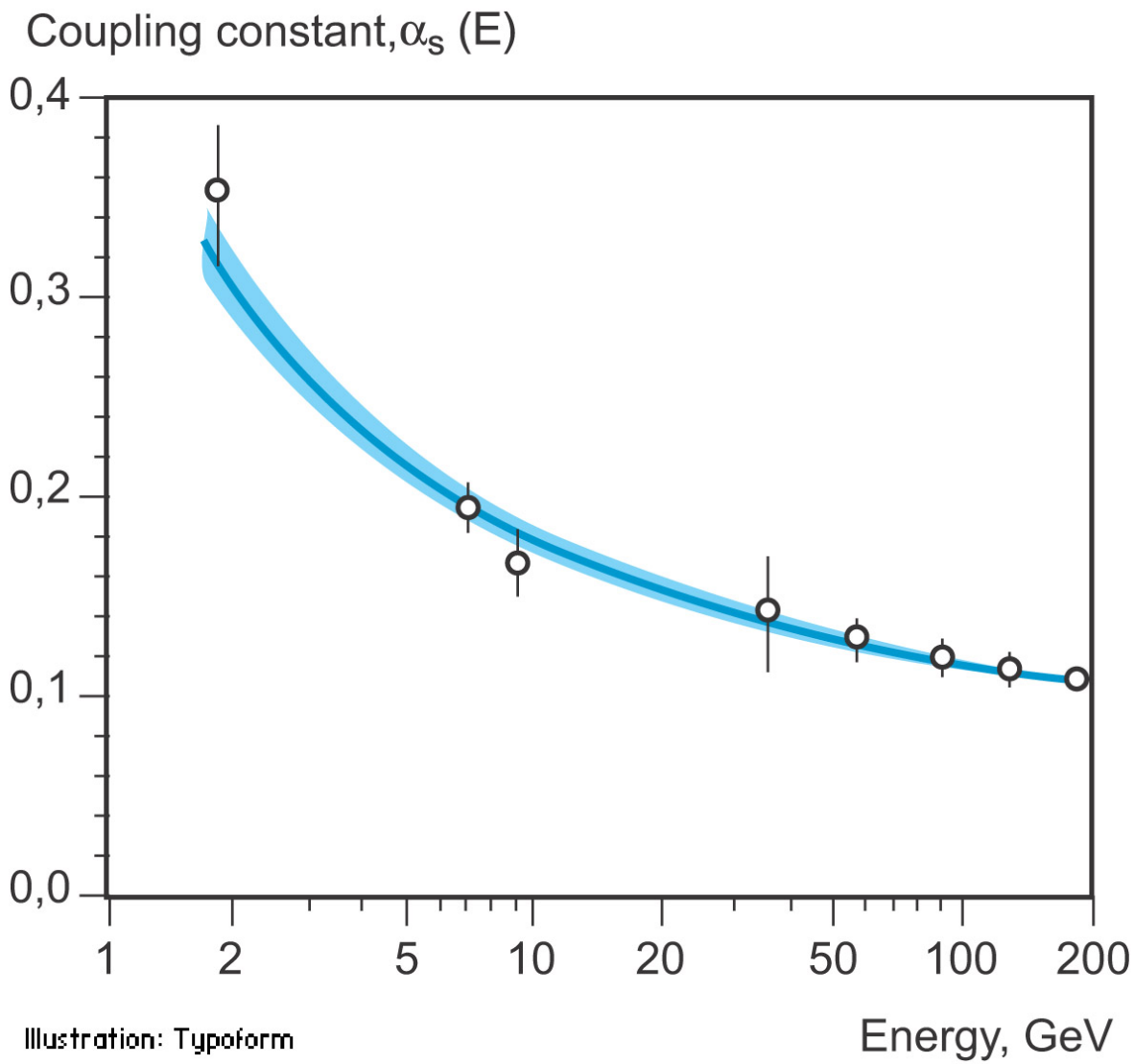


Heavy-Ion Physics = Condensed Matter Physics with strongly interacting particles:



Particle Physics $\leftrightarrow$ Heavy-Ion Physics $\Longleftrightarrow$ Atomic Physics $\leftrightarrow$ Condensed Matter Physics

Strong interactions are different:

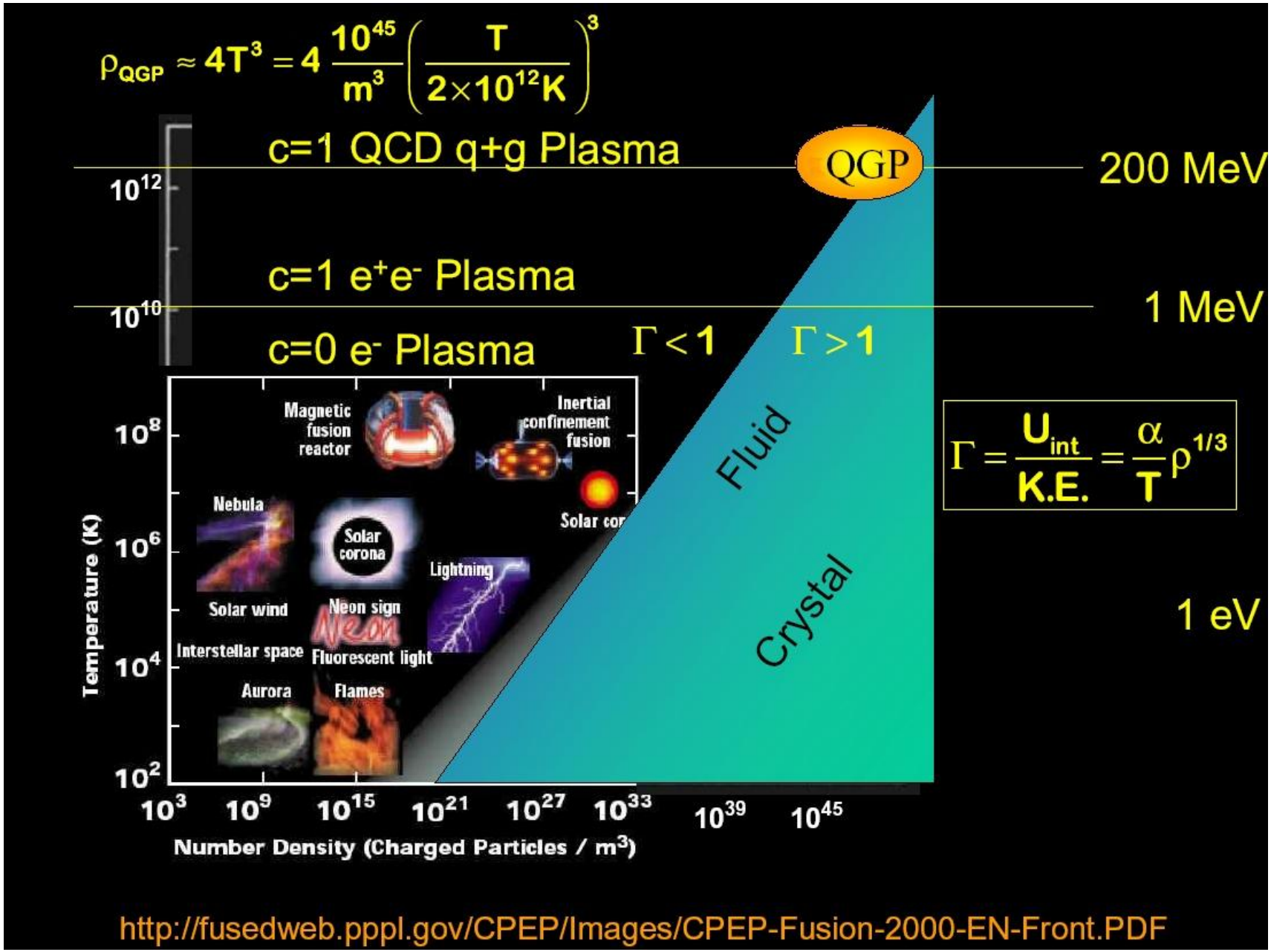


Asymtotic freedom
at high energies

Color confinement
at low energies

$\Longrightarrow$ Expect novel types of bulk behaviour and phase transitions

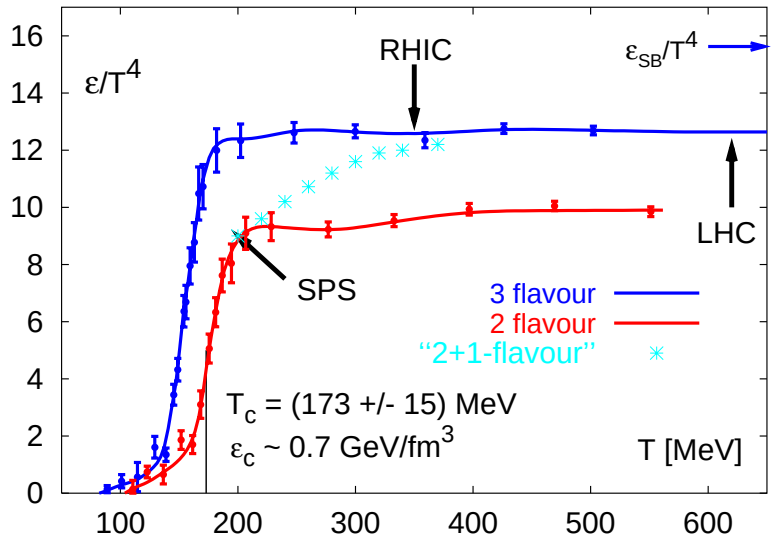
The hottest, densest plasma ever:

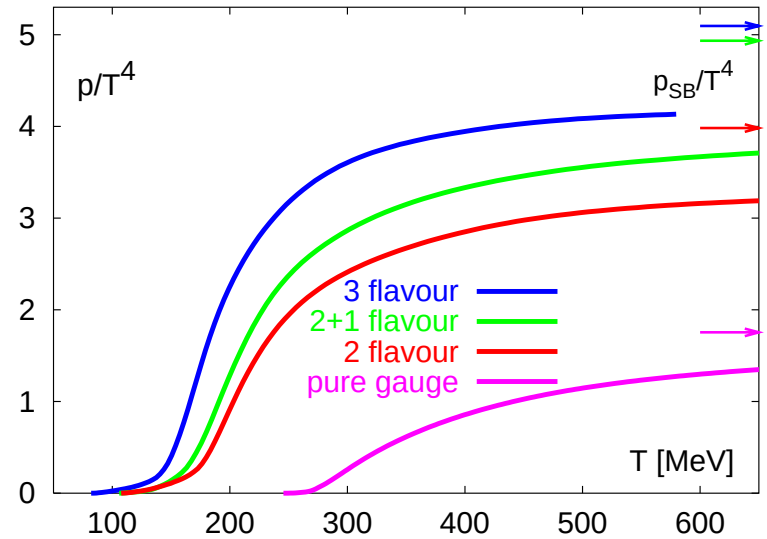


http://fusedweb.pppl.gov/CPEP/Images/CPEP-Fusion-2000-EN-Front.PDF

The QCD equation of state (EOS) at zero baryon density

F. Karsch and E. Laermann, hep-lat/0305025, in "Quark-Gluon Plasma 3"

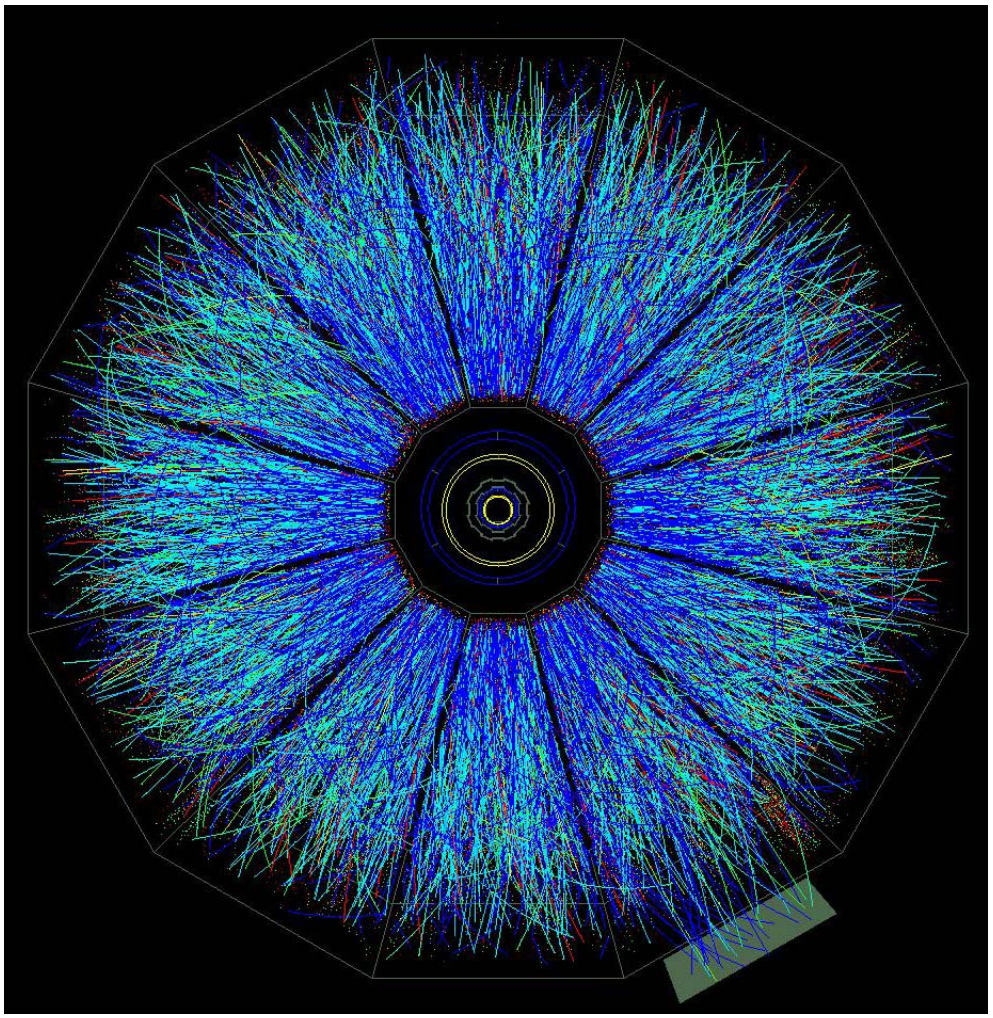




- Critical temperature $T_{\rm cr} = 173 \pm 15\,\text{MeV}$ $(\approx 100\,000 \times T_{\text{center of sun}})$
- Critical energy density $\varepsilon_{\rm cr} \simeq 0.7\,\text{GeV/fm}^3$
- $\varepsilon \approx 0.8\,\varepsilon_{\rm SB}$ for $T \gtrsim 1.3\,T_{\rm cr}$, $\varepsilon \approx 3\,p$ for $T \gtrsim 2\,T_{\rm cr}$

$\Longrightarrow$ Weakly coupled QGP? NO!

How to extract physics from Little Bangs?



- hadron yield ratios (chemistry)
- hadron spectra (thermal radiation)
- collective flow (radial flow, flow anisotropies)
- hard direct probes (jets, charm, direct electromagnetic radiation)
- 2-particle correlations (HBT interferometry with femtometer spatial and yoctosecond temporal resolution)

Big Bang vs. Little Bang:

Three Pillars of Big Bang Theory:

- Hubble expansion $v(d) \sim d$
  ($\rightarrow$ cosmic acceleration)
- primordial nucleosynthesis
  ($\rightarrow \Omega_{\rm mat}$)
- cosmic microwave background
  ($\rightarrow \Omega_{\rm CDM}, \Omega_\Lambda$)

Little Bang analogues:

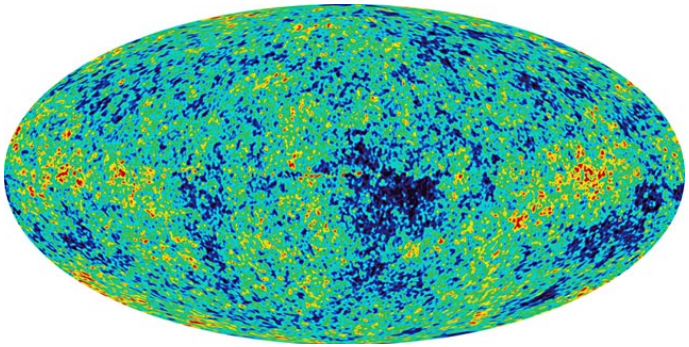
- Hubble flow $v(r) \sim r$
- primordial hadrosynthesis
  ($\rightarrow T_{\rm cr} \approx 170\,$MeV)
- thermal hadron emission
  ($\rightarrow \tau_{\rm therm} < 1\,\frac{\rm fm}{c},\ e_0 > 15\,\frac{\rm GeV}{\rm fm^3}$)

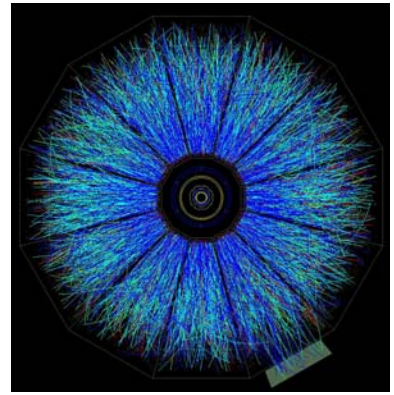
Key differences:

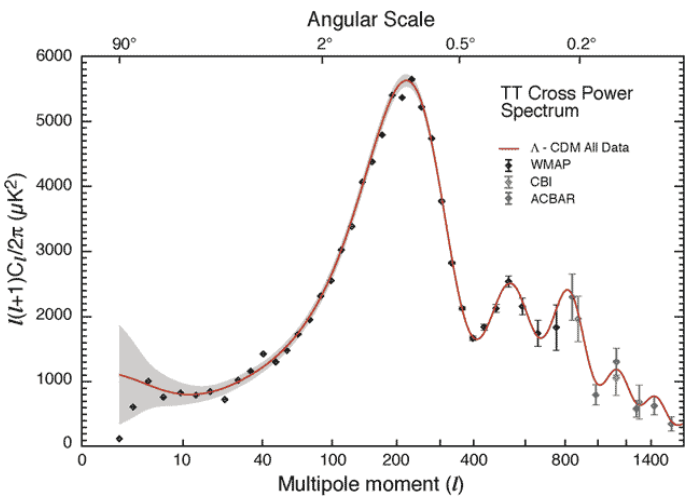
Expansion in 4 d,
controlled by gravity

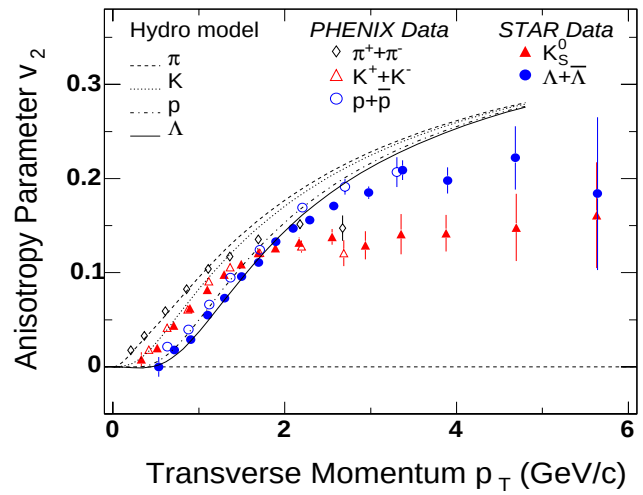
Expansion in 3 d,
controlled by thermal pressure

Precision pays!









spectrum anisotropies $\Longrightarrow$ inflation, dark matter, dark energy, . . .

flow anisotropies $\Longrightarrow$ thermalization, QGP viscosity, EOS, . . .

Collective Flow - the “Bang”

Collective flow tests the Equation of State:

Hydrodynamic equations, ideal fluid limit:

($\dot{f}$ = time derivative in local rest frame, $\partial\cdot u$ = local expansion rate)

$$\dot{n}_B = -n_B\,(\partial\cdot u)$$

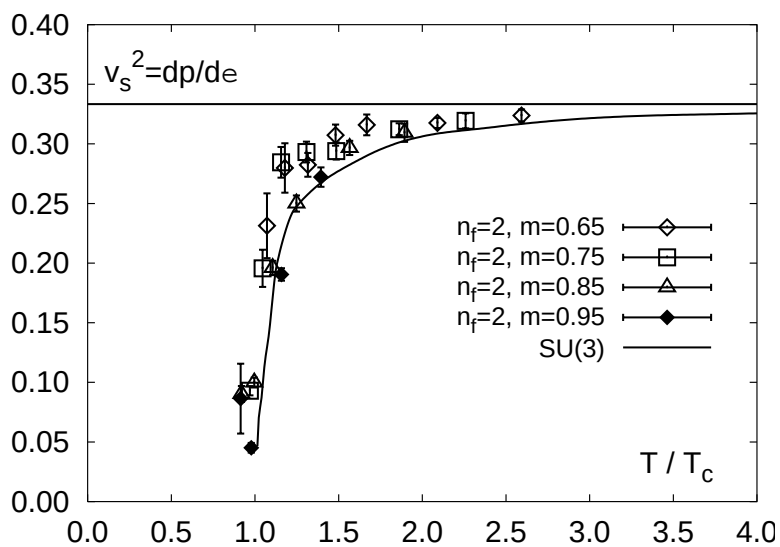
$$\dot{\varepsilon} = -(\varepsilon + p)\,(\partial\cdot u)$$

$$\dot{u}^\mu = \frac{\nabla^\mu p}{\varepsilon + p}$$

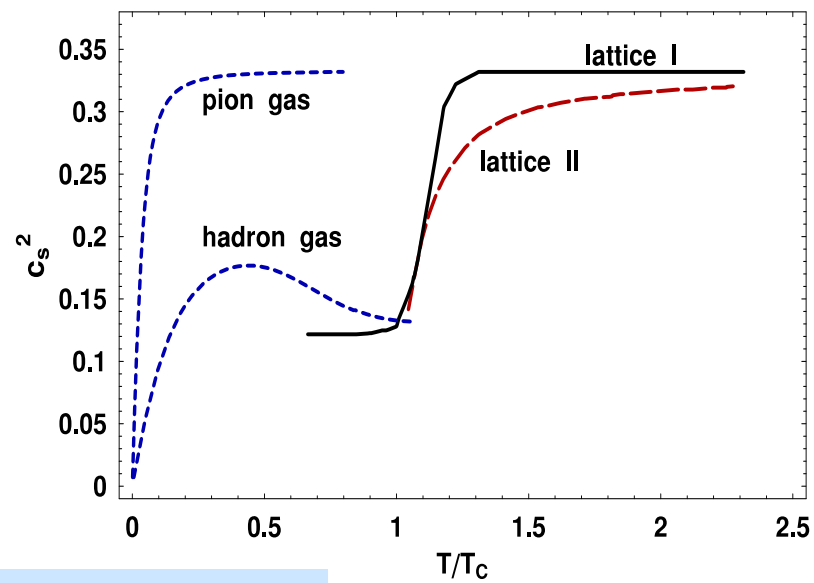
- flow driven by pressure gradients $\nabla^\mu p$
- acceleration $\frac{\nabla^\mu p}{\varepsilon+p}$ closely related to

speed of sound $c_s^2 = \frac{\partial p}{\partial \varepsilon}$

Karsch+Laermann, hep-lat/0305025



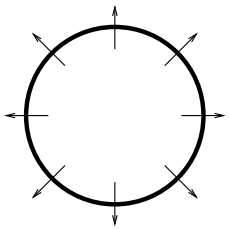
Chojnacki et al., nucl-th/0410036



"Softest point" near $T = T_{\rm cr}$.

"Flavors" of transverse flow:

Radial flow:

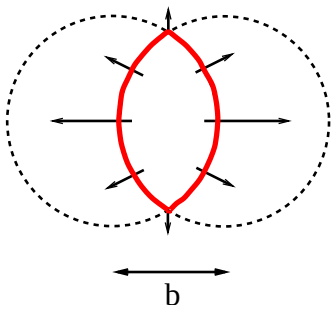


→ the only type of transverse flow in $b = 0$ collisions between equal spherical nuclei

→ integrates pressure history over entire expansion stage

→ observable via effect of $\langle v_\perp \rangle$ on slope of $m_\perp$ spectra

Elliptic flow ($b \neq 0$ or collisions between deformed nuclei, e.g. U+U):



→ peaks at midrapidity

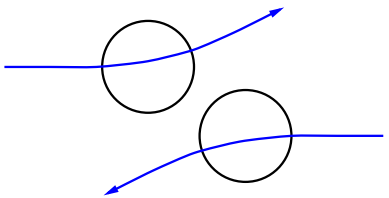
→ requires spatial deformation of reaction zone at thermalization

→ magnitude of signal probes degree and time of thermalization

→ shuts itself off as dynamics reduces deformation (H. Sorge)

→ sensitive to Equation of State during first $\sim 5\,\mathrm{fm}/c$

Directed flow ($b \neq 0$, $y \neq 0$):



→ generated very early while nuclei penetrate each other

→ dominated by early non-equilibrium processes

→ becomes weaker with increasing collision energy

Flow = unavoidable consequence of thermalization!

QGP $\Longrightarrow$ an (approximately) thermalized system of quarks and gluons

$\Longrightarrow$ thermal pressure gradients $\Longrightarrow$ **collective flow**

Hydrodynamics – the natural tool to study flow:

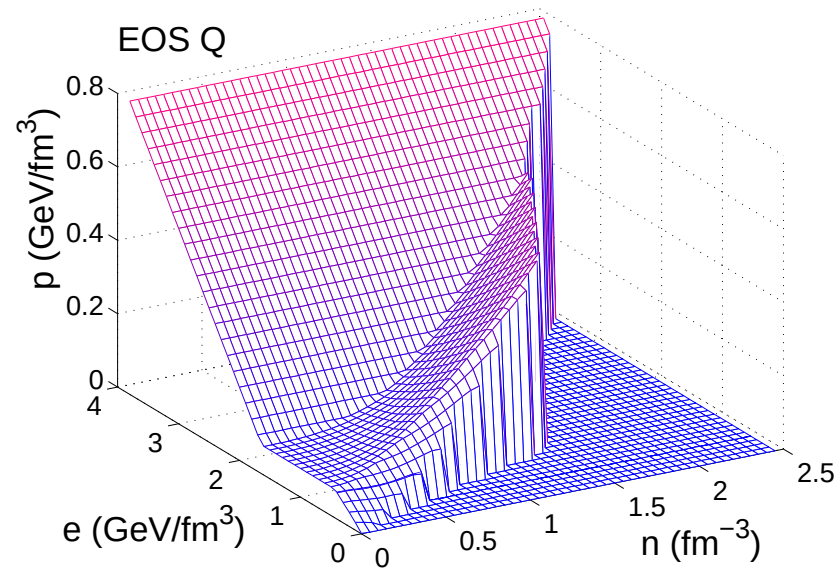
Relativistic Hydrodynamics:	Conservation of energy, momentum and baryon number	$\partial_\mu T^{\mu\nu} = 0$ $\partial_\mu j^\mu = 0$

with energy momentum tensor $T^{\mu\nu}(x) = (e(x)+p(x))\, u^\mu(x) u^\nu(x) - g^{\mu\nu} p(x)$
and baryon current $j^\mu(x) = n(x)\, u^\mu(x)$

Equation of state:

- EOS I: ultrarelativistic ideal gas, $p = \frac{1}{3}\, e$
- EOS H: hadron resonance gas, $p \sim 0.15\, e$
- EOS Q: Maxwell construction between EOS I and EOS H

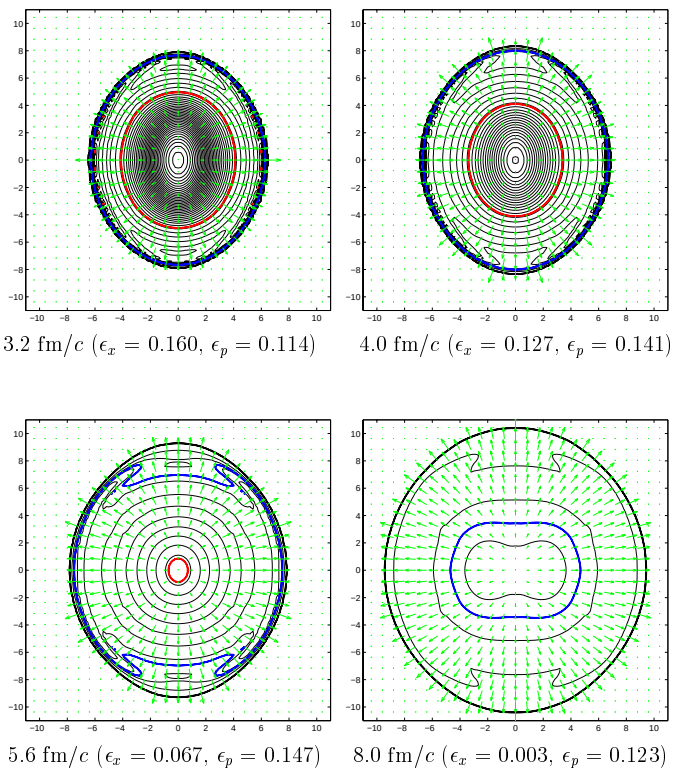
critical temperature $T_{\text{crit}} = 0.164$ GeV
$\Rightarrow$ bag constant $B^{1/4} = 0.23$ GeV
latent heat $\Delta e = 1.15$ GeV/fm^3



Implement exact longitudinal boost invariance for simplicity ($Y \approx 0$ only)

Radial and elliptic flow from hydrodynamics:

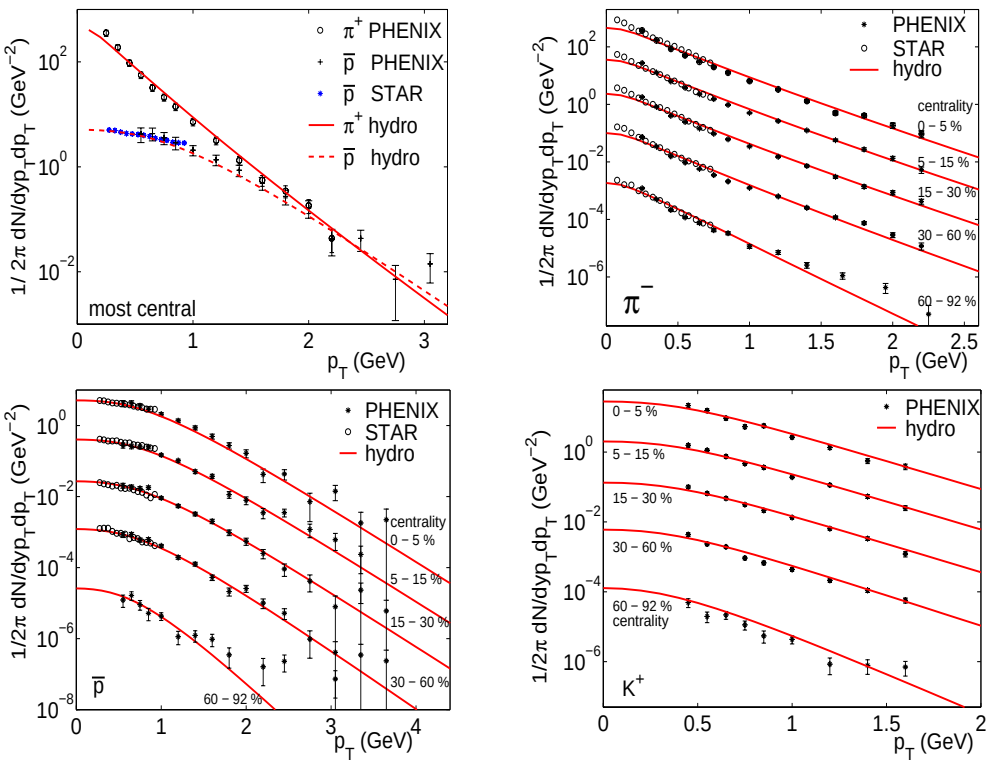
Au+Au at $b = 7$ fm



- when do elliptic and radial flow develop?
- how is elliptic flow related to the time-dependent spatial deformation $\epsilon_x = \dfrac{\langle\langle y^2 - x^2 \rangle\rangle}{\langle\langle y^2 + x^2 \rangle\rangle}$?
- how do radial and elliptic flow depend on the EOS?
- what is the source deformation at freeze-out?
- is there enough time before freeze-out to change sign of ϵ_x?

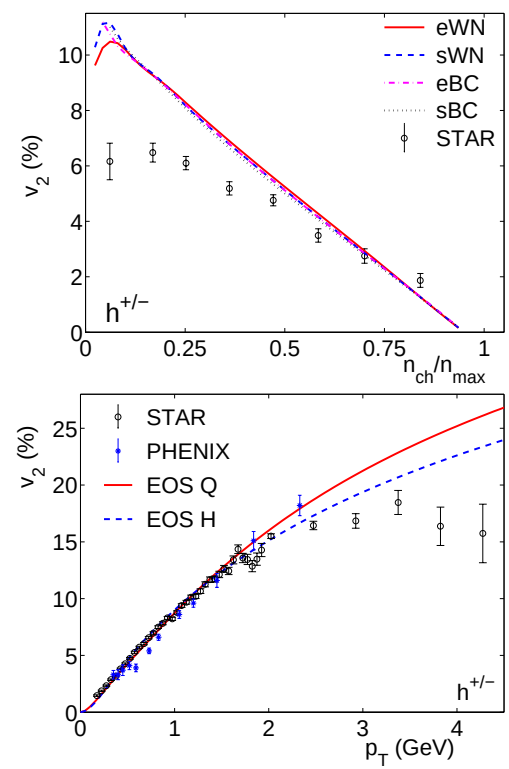
Successes of hydrodynamics at RHIC:

Single particle spectra from central and peripheral Au+Au @ 130 A GeV (STAR, PHENIX):



Model parameters fixed with π, $\bar{p}$ spectra at $b = 0$; all other spectra predicted (UH & P.Kolb, hep-ph/0204061).

Centrality and momentum dependence of elliptic flow v_2 (STAR, PHENIX, PHOBOS):



$$v_2 = \langle \cos(2\phi) \rangle$$

What is fitted, what is predicted?

Au+Au @ 130 A GeV:

$\tau_{\rm eq} = 0.6\,{\rm fm}/c$, $e_{\rm max}(b{=}0) = 24.6\,{\rm GeV/fm}^3$, $\langle e\rangle(\tau{=}1\,{\rm fm}/c) = 5.4\,{\rm GeV/fm}^3$

$T_{\rm max}(b{=}0) = 340\,{\rm MeV}$, $T_{\rm chem} = T_{\rm had} = 165\,{\rm MeV}$, $T_{\rm dec} = 130\,{\rm MeV}$

All fit parameters are fixed in central ($b{=}0$) collisions:

- Glauber model $\Rightarrow$ shape of initial transverse entropy and baryon density profiles $s(\boldsymbol{r}, \tau_{\rm eq}), n_B(\boldsymbol{r}, \tau_{\rm eq})$
  $\Rightarrow$ free parameters $s_0(\tau_{\rm eq})$, $n_0(\tau_{\rm eq})$, soft/hard fraction
- Measured p/π ratio $\Rightarrow$ fixes n_0/s_0
- Total charged multiplicity $dN_{\rm ch}/dy \Rightarrow$ fixes product $\tau_{\rm eq} \cdot s_0(\tau_{\rm eq})$
- soft/hard fraction $\Rightarrow$ fixed through centrality dependence of $dN_{\rm ch}/dy$
- Shape of π, p spectra $\Rightarrow$ fixes decoupling temperature $T_{\rm dec}$ and radial flow $\langle v_\perp\rangle$
- Final radial flow $\langle v_\perp\rangle \Rightarrow$ "fixes" $\tau_{\rm eq}$ [upper limit] (flow needs time and pressure to develop)
- Equation of State $\Rightarrow$ compute $e_0 = e_{\rm max}(b{=}0)$, $T_{\rm max}(b{=}0)$ from s_0, n_0

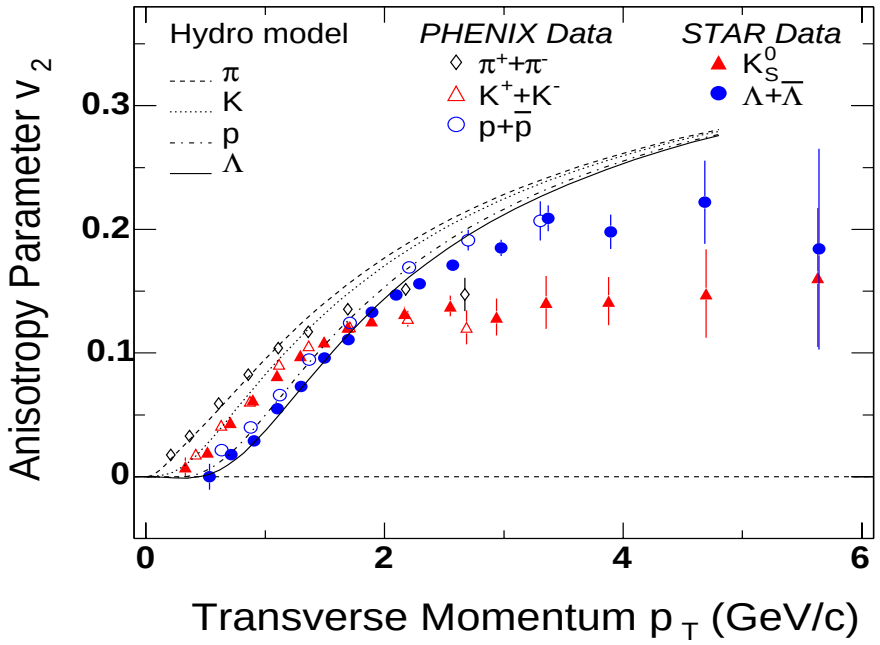
Predictions (no additional parameters!):

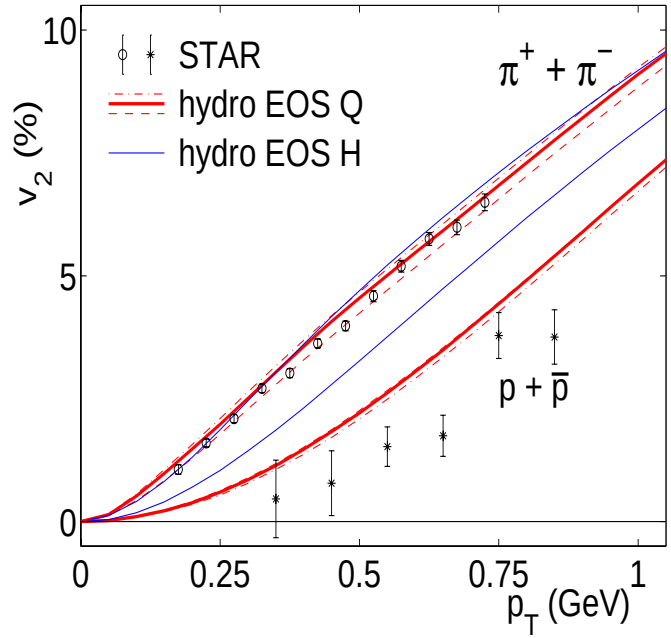
- All hadron spectra other than p, π in $b{=}0$ collisions
- All hadron spectra and elliptic flow coefficients for non-central collisions at any impact parameter

Final radial flow $\langle v_\perp\rangle > 0.5\,c \Longrightarrow$ bang!

Rest mass dependence of differential elliptic flow (the "fine structure")

STAR Coll., PRL 87, 182301 (2001) and PRL 92, 052302 (2004); PHENIX Coll., PRL 91, 182301 (2003)





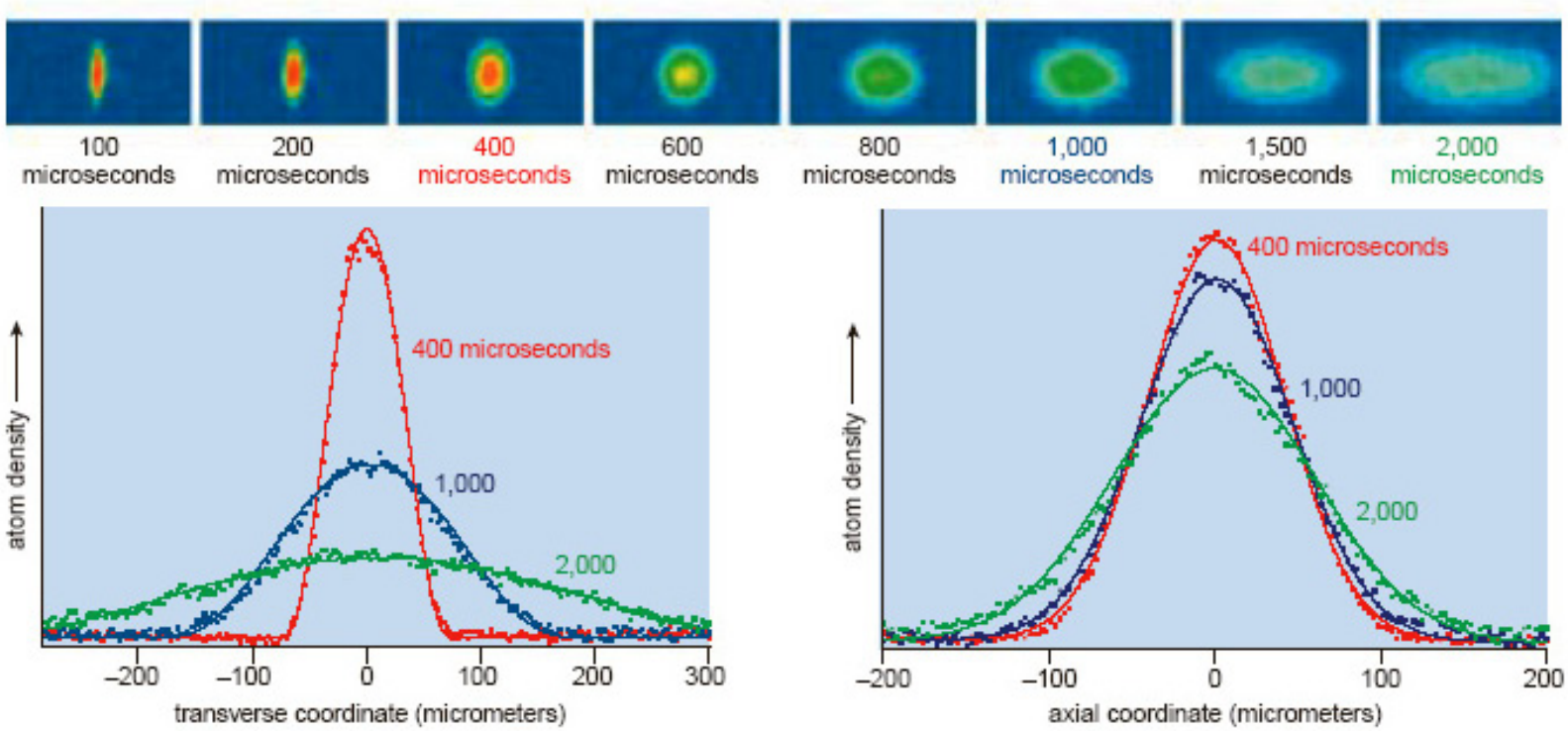
Data follow the hydrodynamically predicted rest mass dependence of $v_2(p_\perp)$ out to $p_\perp \sim 1.5$ GeV for mesons and out to $p_\perp \sim 2.3$ GeV for baryons

$\Longrightarrow$ bulk of matter ($> 99\%$ of all particles) behaves hydrodynamically!

Note: mass-splitting of v_2 ("fine structure") sensitive to EOS!

Elliptic collective flow of strongly coupled atoms at $T = 10^{-6}$ K:

J.E.Thomas et al., Am. Scientist 92 (2004) 238



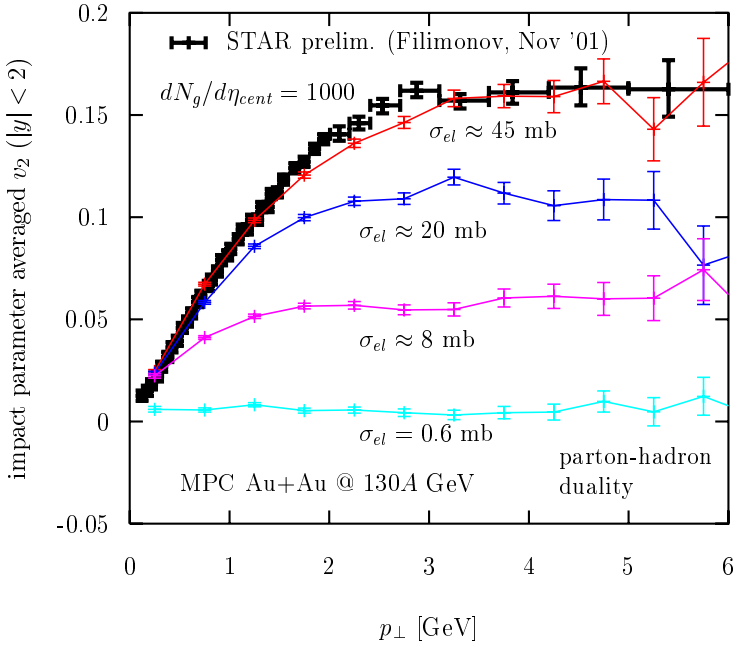
Interaction strength can be tuned (Feshbach resonance):

Strong interaction: elliptic collective flow

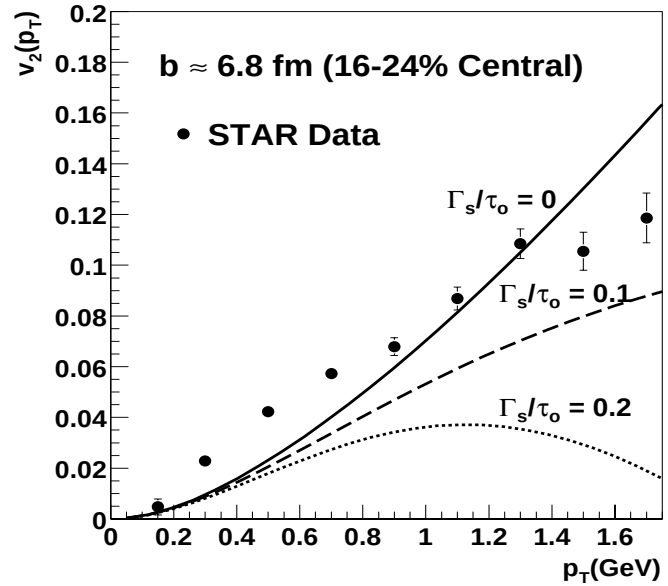
Weak interaction: ballistic expansion with aspect ratio $\to 1$

Breakdown of hydrodynamics at high $p_\perp$: upper limits for the QGP viscosity

D. Molnár and M. Gyulassy, NPA 697 (2002) 495



D. Teaney, PRC 68 (2003) 034913



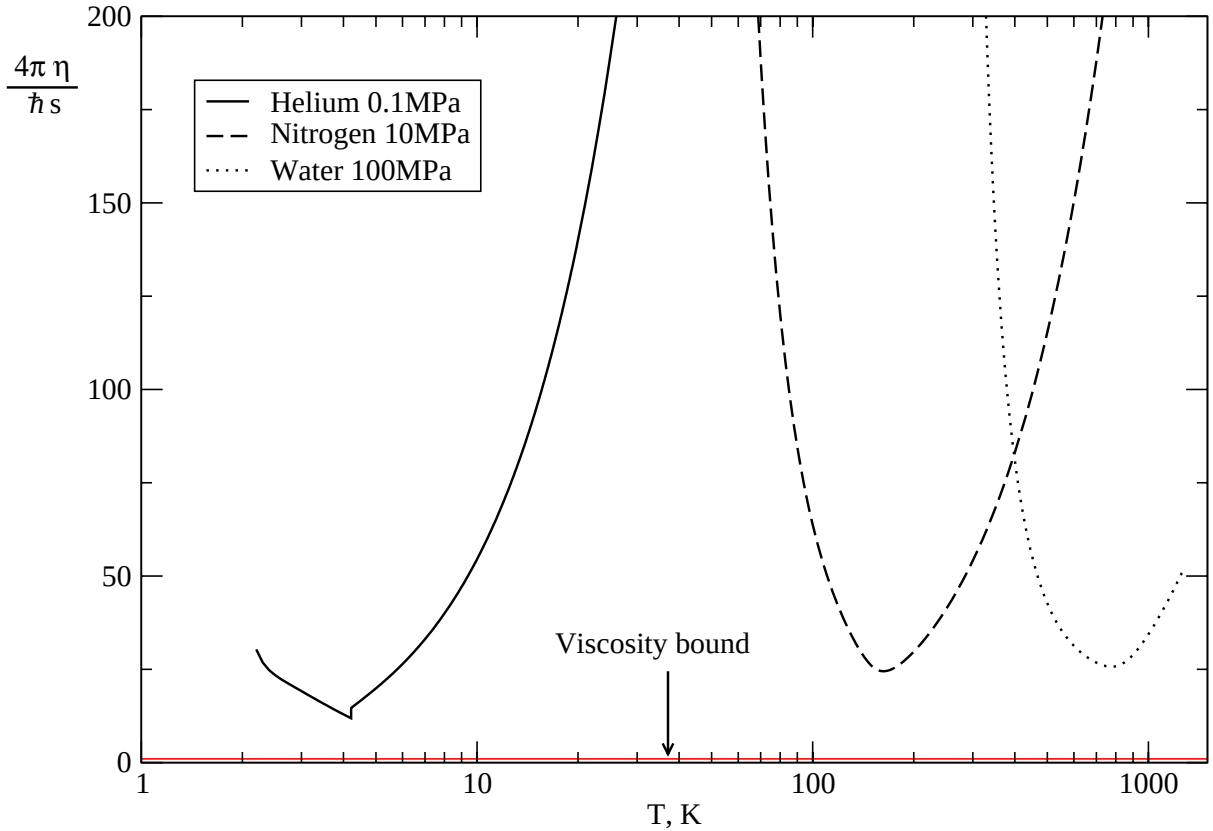
$$\Gamma_s = \frac{4}{3}\eta/(T\cdot s)$$

- For sufficiently (very) large $\sigma_{\rm el}$, $v_2(p_\perp)$ from covariant parton transport model MPC follows hydrodynamic curve at low $p_\perp$ and reproduces observed saturation at high $p_\perp$
- Similar pattern is seen in viscous hydrodynamics: viscous corrections increase $\sim p_\perp^2$
- v_2 data suggest $\frac{\Gamma_s}{\tau} < 0.1$, close to minimum viscosity $\frac{\eta}{s} = \frac{\hbar}{4\pi}$ (Son et al. 2002)

QGP – the most ideal fluid ever observed!

adS/CFT universal lower viscosity bound conjecture: $\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$

Kovtun, Son, Starinets, hep-th/0405231



Upper limit for QGP viscosity from Teaney's estimate is close to this bound!

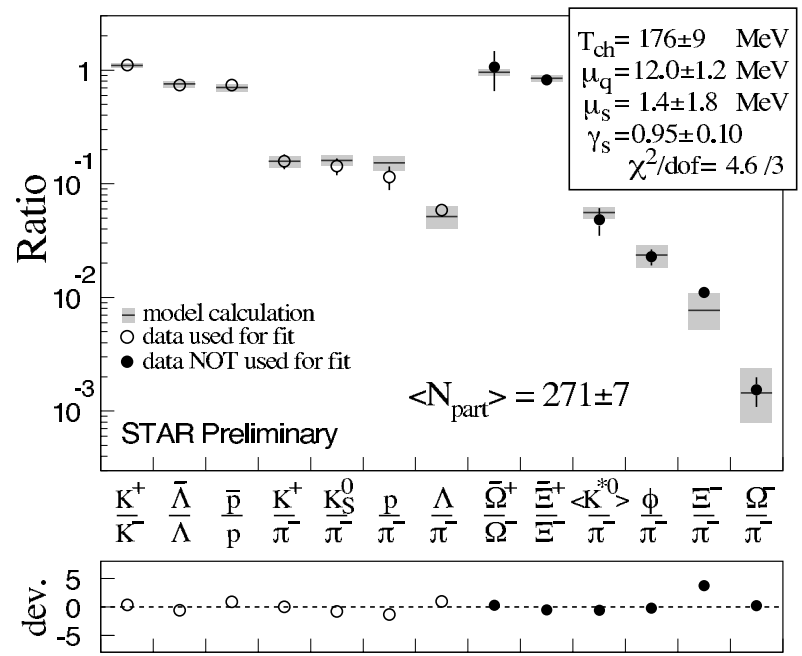
More quantitative constraints on η require viscous hydrodynamics code.

Primordial Hadrosynthesis – Measuring $T_{\rm cr}$

Chemical Freeze-out at $T_{\rm had} \simeq 170$ MeV

Central Au+Au @ 130 A GeV

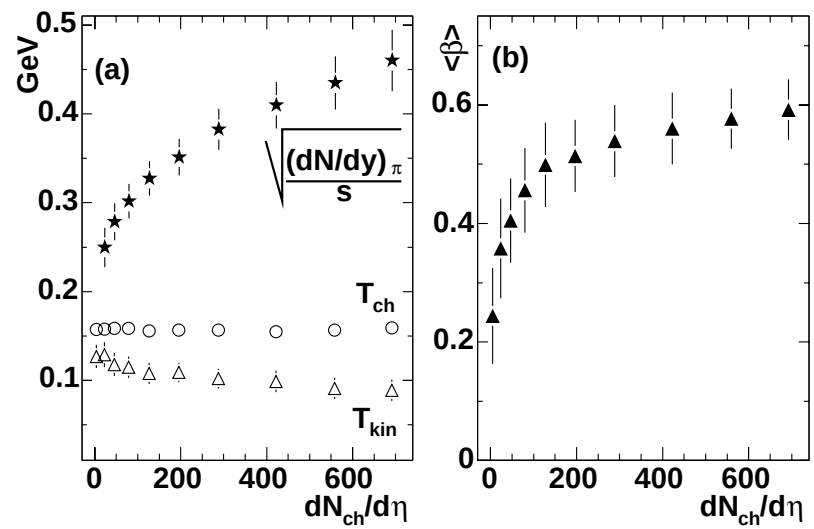
(STAR Coll., G. van Buren, QM2002)



Abundance ratios of stable hadrons decouple in maximum entropy state of "apparent chemical equilibrium" with $T_{\rm chem} \simeq T_{\rm had} \simeq 170$ MeV.

$T_{\rm chem}$ insensitive to expansion rate:

STAR Coll., PRL 92 (2004) 112301



$\Longrightarrow$ **phase transition!**

Note: Hadron abundances are in statistical, not in kinetic chemical equilibrium!

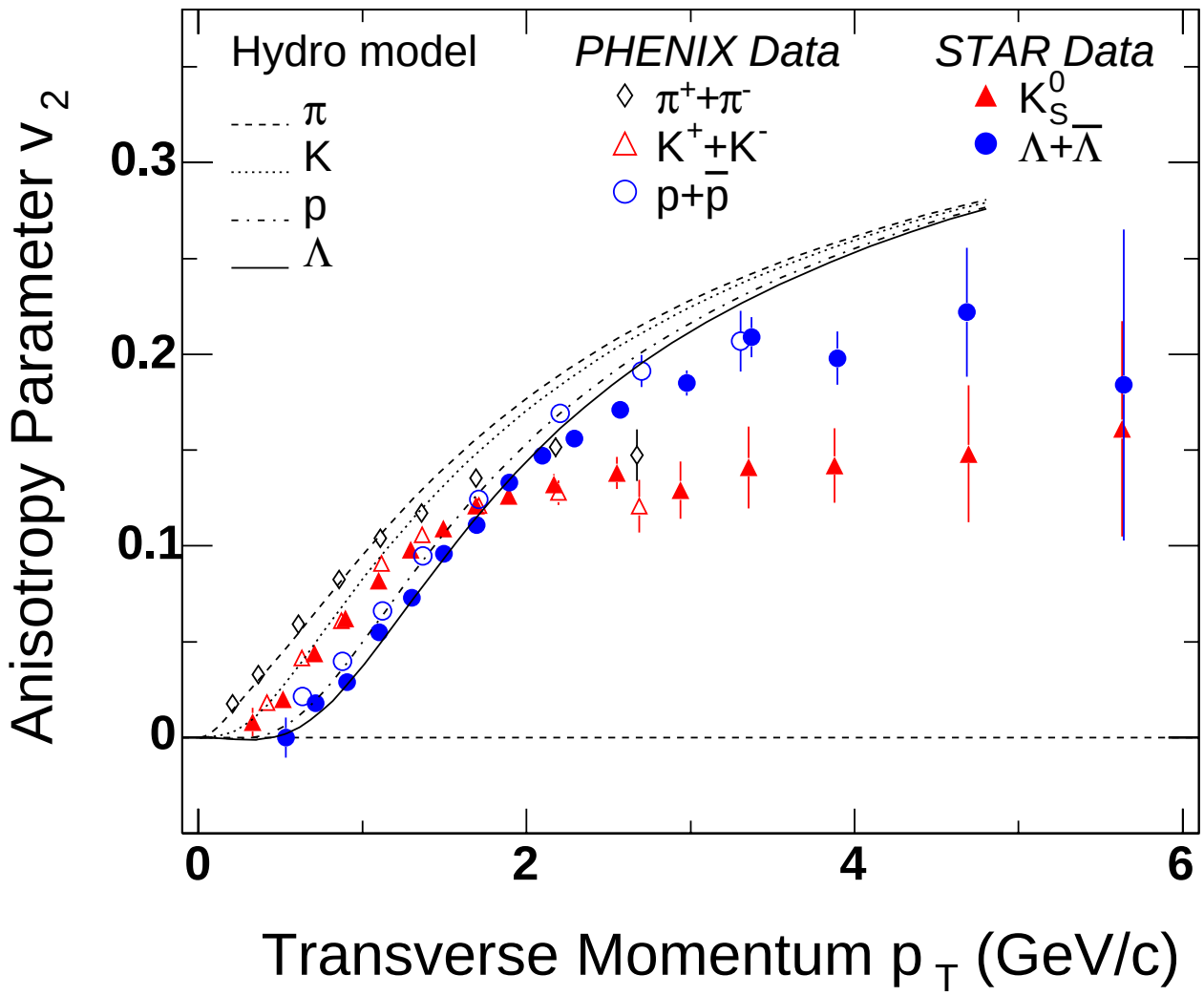
Requires pre-hadronic phase with large strangeness correlation volume.

Quark coalescence –

a sign of color deconfinement?

At intermediate p_T, mesons and baryons behave differently:

STAR Coll., PRL 87, 182301 (2001) and PRL 92, 052302 (2004); PHENIX Coll., PRL 91, 182301 (2003)



Parton coalescence:

Ko & Lin '02, Hwa & Yang '02, Greco et al. '03, Fries et al. '03, Molnár & Voloshin '03, Lin & Molnár '03

- Picture: - **coalescence of massive "dressed" valence quarks**
  - **no dynamical gluons**

- Basic equations: $qq \to \text{meson}$, $qqq \to \text{baryon}$

$$E\frac{dN_M(\boldsymbol{p})}{d^3p} = \int \frac{d\sigma^\mu p_\mu}{(2\pi)^3} \int d^3q \qquad |\psi_{\boldsymbol{p}}(\boldsymbol{q})|^2 \qquad f_\alpha(\boldsymbol{p}_\alpha, x) f_\beta(\boldsymbol{p}_\beta, x)$$

$$E\frac{dN_B(\boldsymbol{p})}{d^3p} = \int \frac{d\sigma^\mu p_\mu}{(2\pi)^3} \int d^3q_1 d^3q_2 |\psi_{\boldsymbol{p}}(\boldsymbol{q}_1, \boldsymbol{q}_2)|^2 f_\alpha(\boldsymbol{p}_\alpha, x) f_\beta(\boldsymbol{p}_\beta, x) f_\gamma(\boldsymbol{p}_\gamma, x)$$

hadron yield space-time wave-fn. quark distributions

assumes: rare process, weak binding, factorizable 2-body and 3-body density matrix, smooth spacetime distributions, 3D hypersurface (sudden approx.)

Can dominate over fragm. for $p_\perp < 4{-}5$ GeV [Greco et al., Fries et al., PRL90 ('03)]

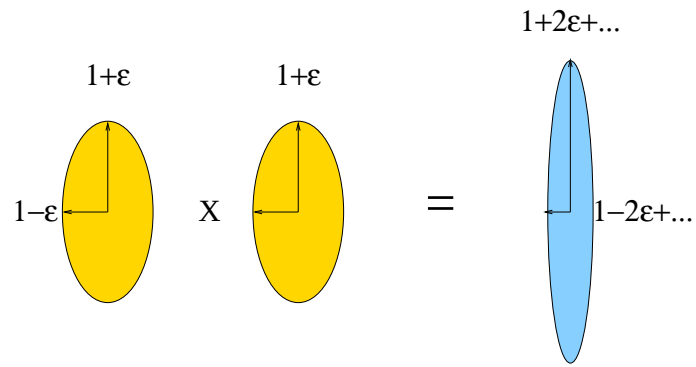
Quark number scaling at intermediate $p_\perp$: coalescence

D. Molnár and S. Voloshin, PRL 91 (2003) 092301

Narrow wave function limit $(q = 0)$: $\frac{dN_M}{d\phi} \propto \left(\frac{dN_q}{d\phi}\right)^2, \quad \frac{dN_B}{d\phi} \propto \left(\frac{dN_q}{d\phi}\right)^3$

$$v_2^M(p_\perp) \approx v_2^a\left(\frac{p_\perp}{2}\right) + v_2^{\bar{a}}\left(\frac{p_\perp}{2}\right)$$

$$v_2^B(p_\perp) \approx v_2^a\left(\frac{p_\perp}{3}\right) + v_2^b\left(\frac{p_\perp}{3}\right) + v_2^c\left(\frac{p_\perp}{3}\right)$$



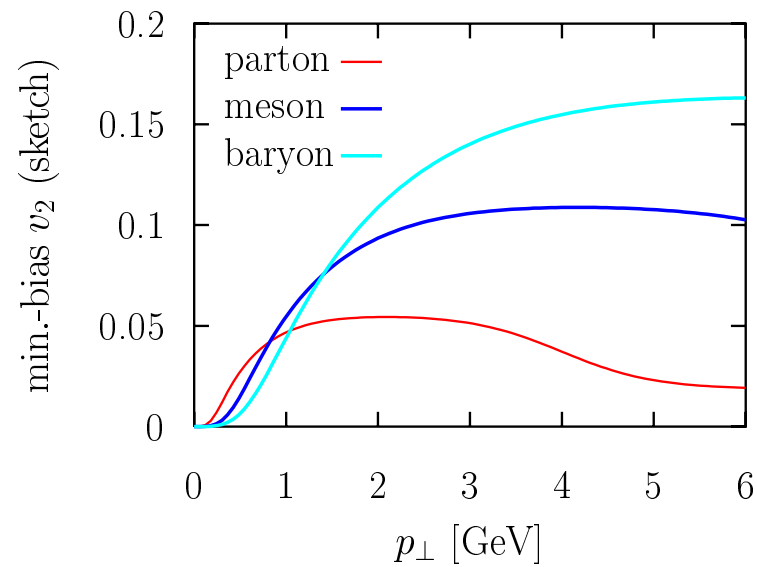
⇒ **Hadron v_2 amplified at high $p_\perp$:**

If all quark flavors have same v_2:

$3\times$ for baryons

$2\times$ for mesons

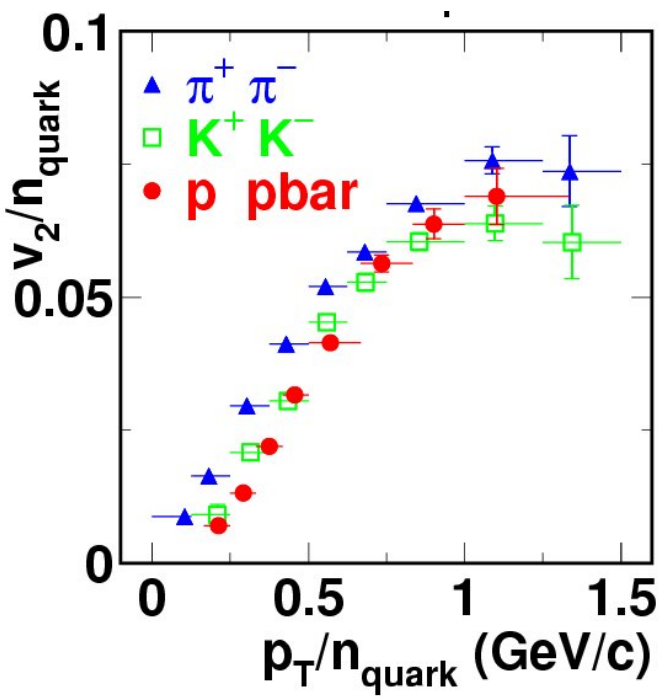
$$v_2^h(p_\perp) \approx n \times v_2^q(p_\perp/n)$$



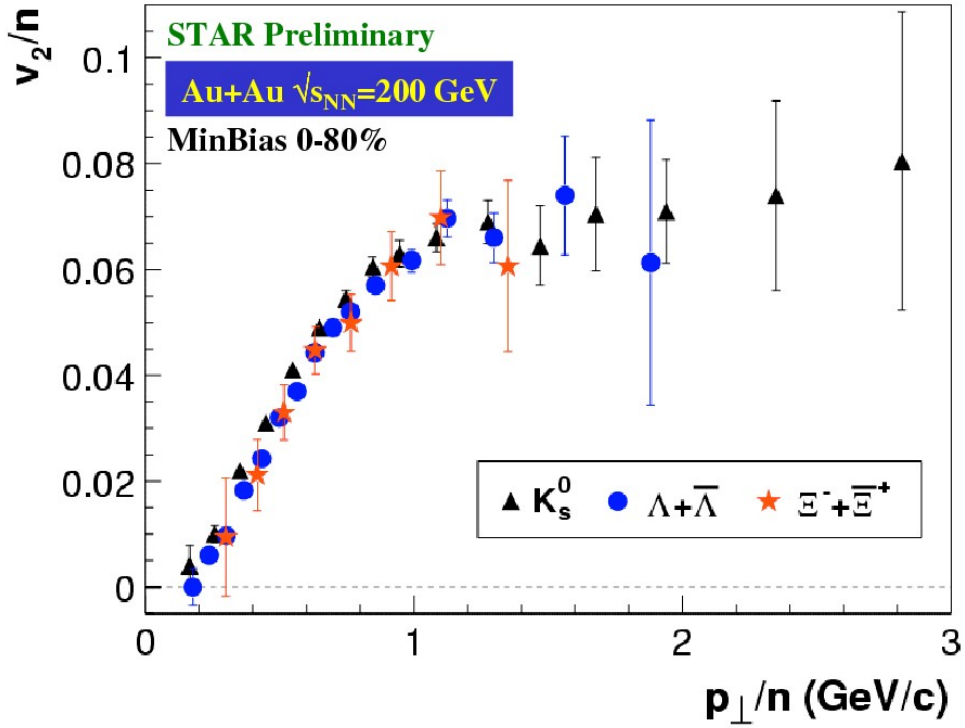
Note: Parton v_2 can be extracted only because it breaks away from hydro at high p_T!

Experimental extraction of parton elliptic flow

PHENIX Coll., PRL 91 ('03) 182301



J. Castillo (STAR Coll.), nucl-ex/0403027



- Coalescence predictions confirmed for $\pi, K, K_0, p, \Lambda, \Xi$
- RHIC data indicate $v_2^q \approx v_2^s$
- Parton elliptic flow follows hydro to $p_{T,\mathrm{break}} \approx 750\,\mathrm{MeV}$, saturates at $\approx 7\%$ in min. bias. collisions

Quark number scaling indicates dynamical role for deconfined quarks!

JET –

Jet Emission Tomography of the QGP

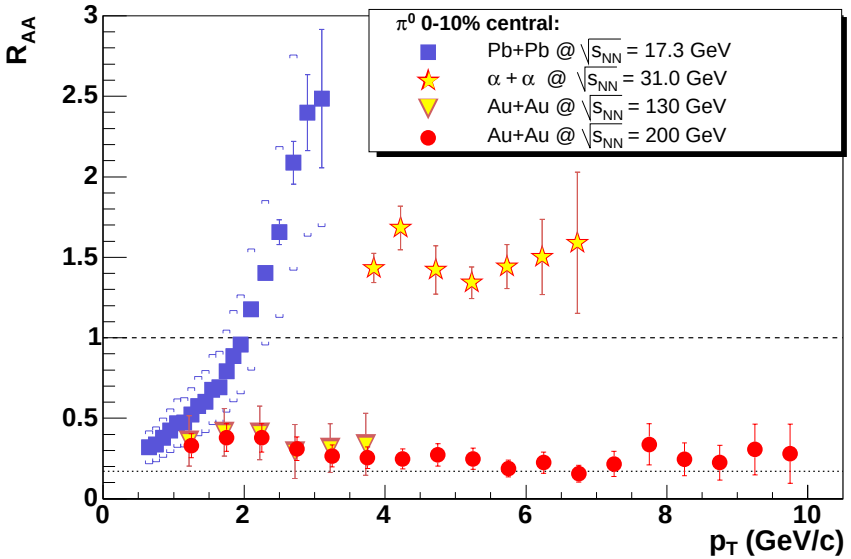
Suppression of high p_T hadron production in Au+Au:

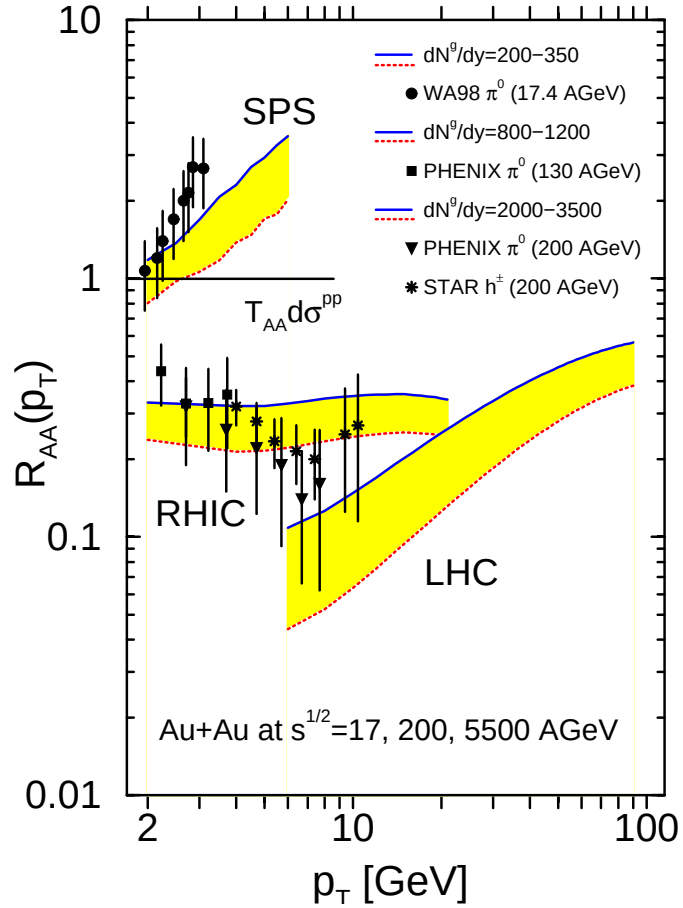
$$R_{AA}(p_T;b) = \frac{\frac{dN_{AA}}{dp_T}(b)}{N_{\rm coll}(b)\frac{dN_{pp}}{dp_T}}$$

M. Gyulassy and I. Vitev, PRL 89 (2002) 252301

Au+Au at $\sqrt{s} = 130$ and $200\ A$ GeV

PHENIX Coll., PRL 88 (2002) 022301; PLB 561 (2003) 82



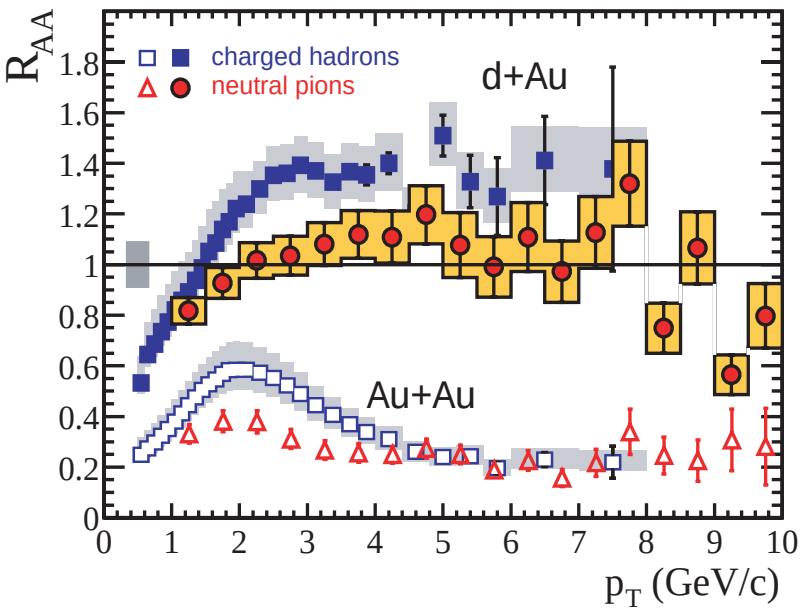


$\Longrightarrow \quad \frac{dN_g}{dy} = 1000 \pm 200$

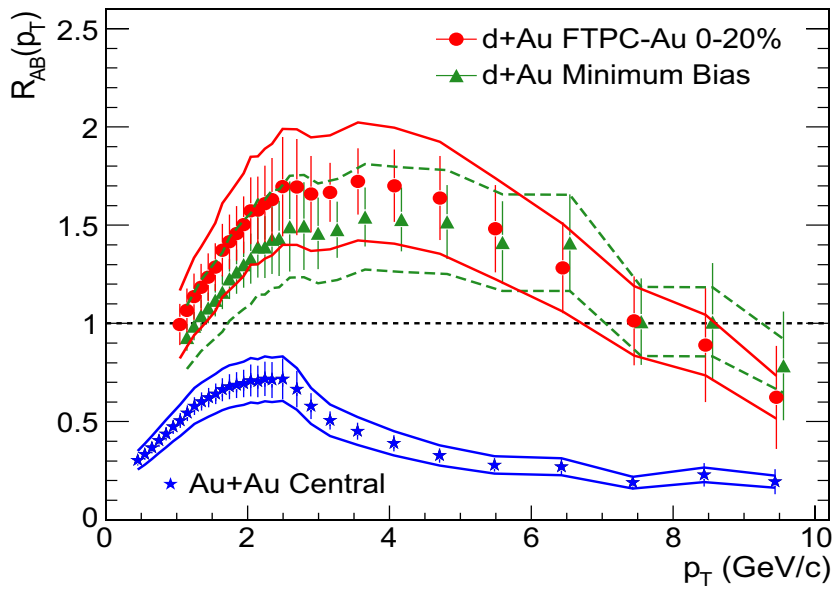
$\Longrightarrow \quad \langle e \rangle (\tau_0 = 0.2\,\mathrm{fm}) \approx 20\,\mathrm{GeV/fm^3}$!

No high p_T suppression in d+Au:

PHENIX Coll., PRL, nucl-ex/0306021



STAR Coll., PRL, nucl-ex/0306024



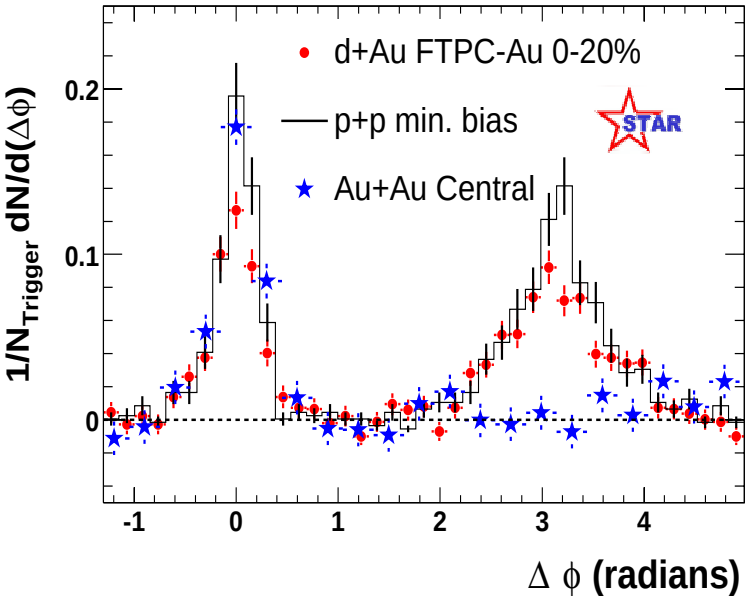
- as collision centrality increases, R_{AA} increases in d+Au (Cronin effect) but decreases in Au+Au

- high-p_T suppression absent in d+Au $\Longrightarrow$ suppression in Au+Au not due to nuclear wavefunction (e.g. CGC) but a final state effect

Jet quenching in central Au+Au collisions:

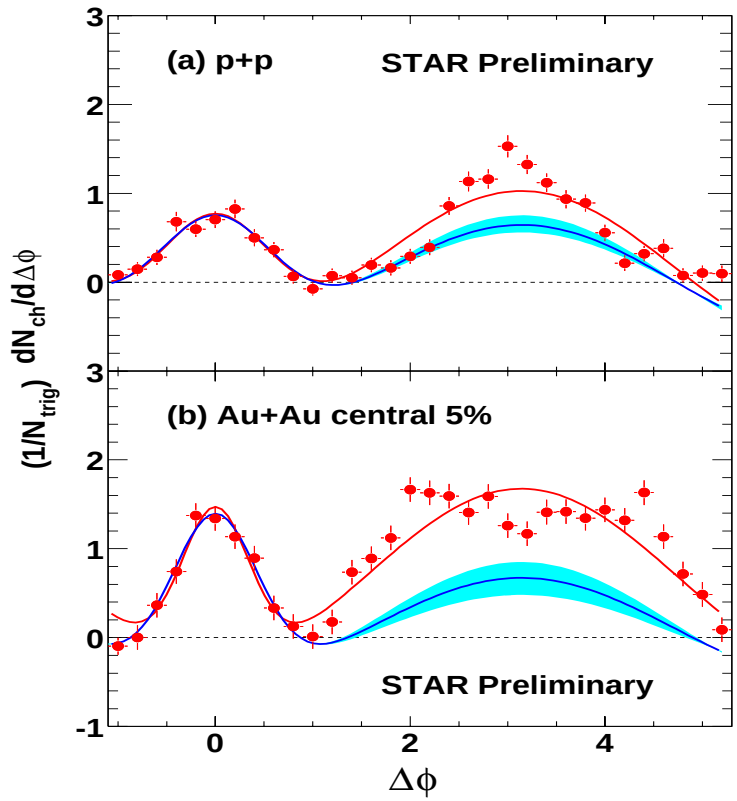
STAR Coll., PRL 91 (2003) 072304

$p_T^{\rm assoc} > 2\,{\rm GeV}$



STAR Coll., F. Wang, Quark Matter 2004

$0.15 < p_T^{\rm assoc} < 4\,{\rm GeV}$

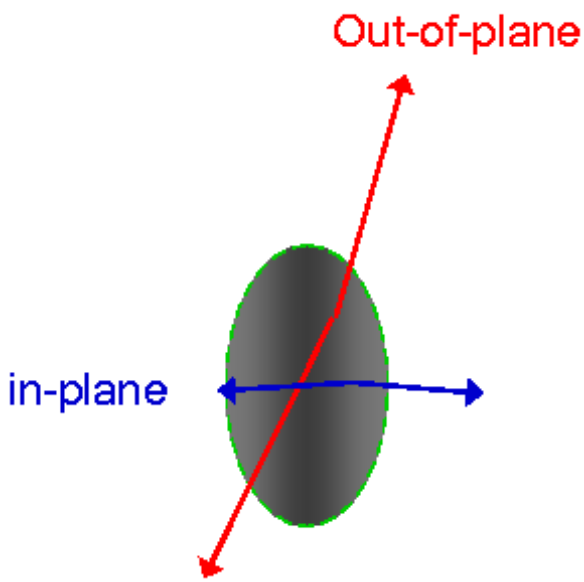


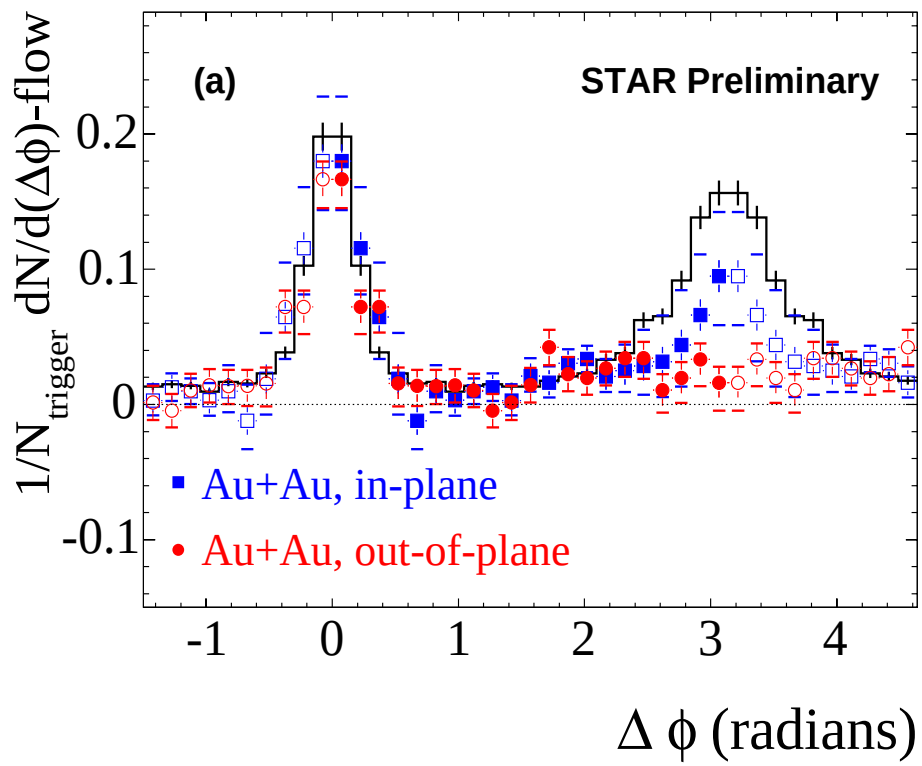
- trigger particle for near-side jet has $4 < p_T < 6\,{\rm GeV}$
- away-side jet ($p_T > 2\,{\rm GeV}$) visible in p+p and d+Au, but fully quenched in central Au+Au
- energy of quenched jet appears as additional multiplicity of low-p_T particles opposite to trigger particle
- $\Longrightarrow$ "thermalization" of intermediate-p_T jets!

Path length dependence of parton energy loss:

STAR Coll., nucl-ex/0403018, Quark Matter 2004

Emission angle dependence:

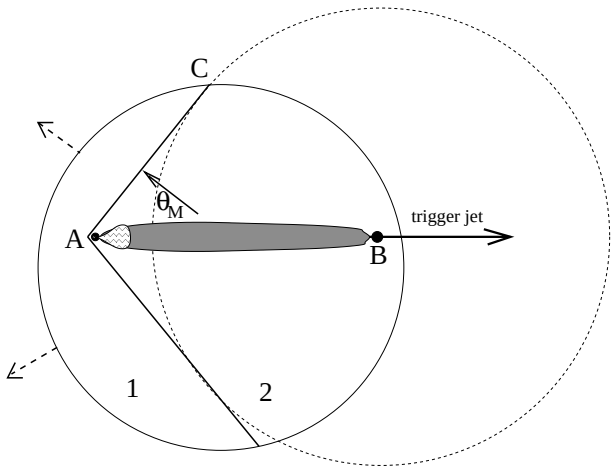




- medium opaque for colored particles
- energy loss increases strongly with path length

Evidence for a "sonic boom"?!

Shuryak et al., hep-ph/0411315

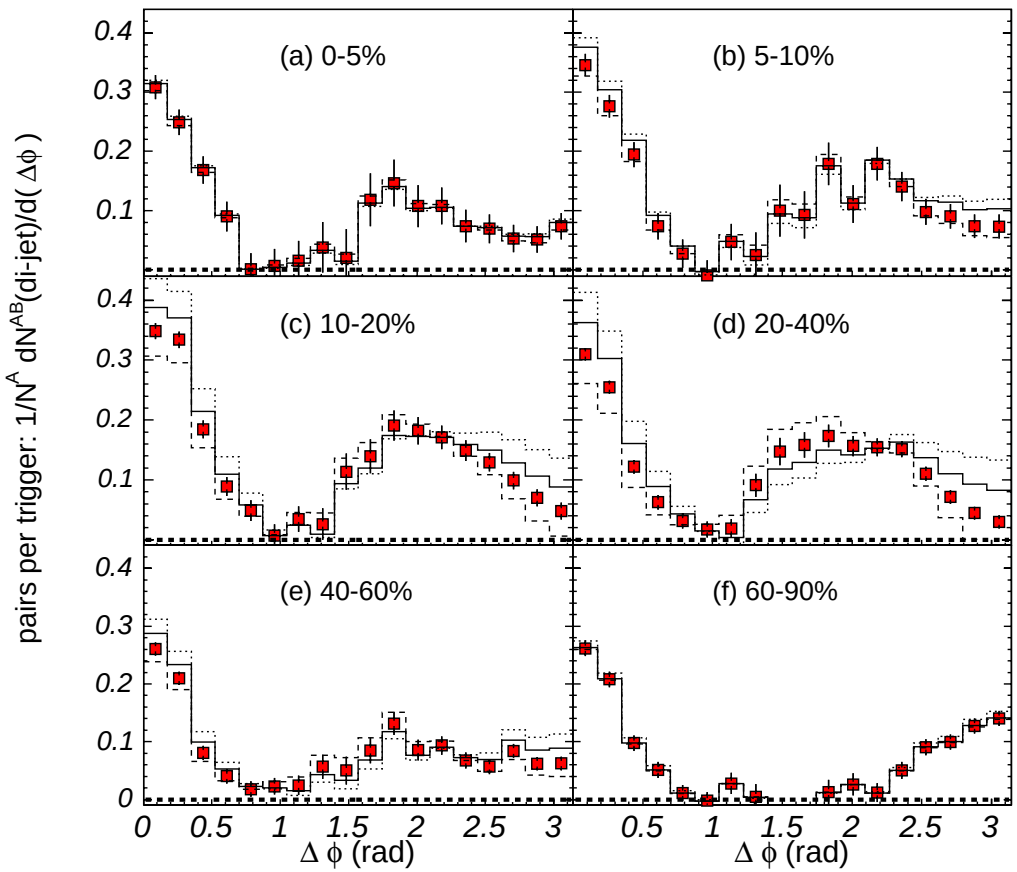


Away-side jet creates Mach cone at $\cos\theta_{\rm M} = \frac{c_s}{c}$

$\Longrightarrow$ $\theta_{\rm M} \approx 63^\circ \approx 1.1\,\text{rad}$

PHENIX Coll., B. Jacak, ICPAQGP 2005

$1 < p_T^{\rm assoc} < 2.5\,\text{GeV} < p_T^{\rm trigger} < 4\,\text{GeV}$



Summary

Key discoveries at RHIC at all wavelengths:

- strong radial and elliptic flow (also at SPS) (predicted)
- elliptic flow exhausts the ideal fluid limit (new) (predicted)
- strangeness enhancement and statistical hadronization at $T_c \approx 170\,\mathrm{MeV}$ (also at SPS) (predicted)
- suppression of high-p_T hadron production in Au+Au vs. Cronin enhancement in d+Au (opposite centrality dependence in Au+Au and d+Au) (new) (predicted)
- jet quenching by parton energy loss with energy redistribution to low p_T ("jet thermalization") (new) (predicted)
- universality of $\frac{v_2}{n_{\mathrm{val.}}}\left(\frac{p_T}{n_{\mathrm{val.}}}\right)$ (quark coalescence, elliptic flow is partonic) (new)

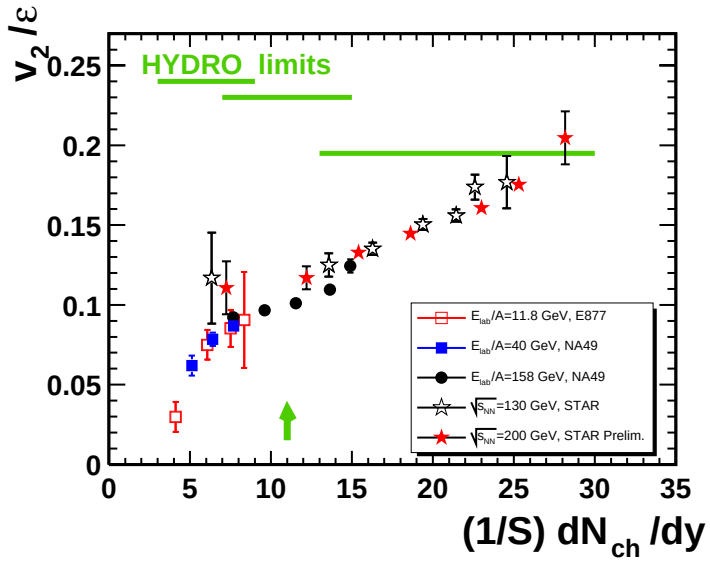
The picture begins to take shape . . .

- $v_2 \implies$ short thermalization time $\tau_{\rm therm} < 1\,{\rm fm}/c \implies$ matter initially in QGP state, $e_0 > 10\,e_{\rm cr}$, $T_0 \sim 2\,T_{\rm cr}$, lives $\sim 5-7\,{\rm fm}/c$ before hadronizing
- Bulk of matter created at RHIC = almost ideal fluid with low viscosity $\implies$ QGP = strongly coupled plasma
- $T_{\rm cr} \approx 170\,{\rm MeV}$ measured (statistical hadronization), consistent/w LQCD
- strangeness enhancement, quark number scaling of v_2, R_{CP} of identified hadrons at intermediate p_T indicate quark deconfinement
- jet emission tomography (JET) $\implies$ QGP is color opaque
- JET and hydro yield independent, consistent estimates for initial energy density $e_{\rm init} \approx 15\,e_{\rm cr} \approx 100\,e_{\rm nucl.mat.}$
- some puzzles remain (HBT, v_2 at high p_T), detailed properties of QGP still to be explored (charm flow, thermal photons & dileptons, precision JET, . . .)

Supplements

Limits of ideal fluid dynamics: smaller, less dense systems

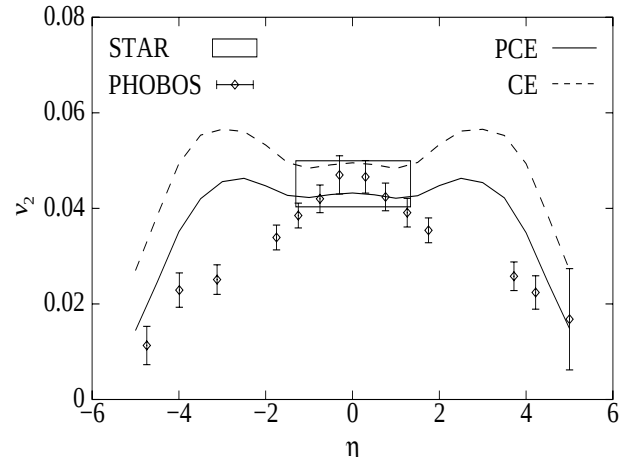
STAR, PRC 66 ('02) 034904; NA49, PRC 68 ('03) 034903



- $\frac{v_2^{\rm measured}}{v_2^{\rm hydro}}$ scales with $\frac{1}{S}\frac{dN_{\rm ch}}{dy} \propto s_{\rm init}$
- $e_{\rm init} > 10\,{\rm GeV/fm}^3$ needed for v_2 to saturate before hadronization and exhaust ideal hydro limit!
- hydrodynamics predicts non-monotonic v_2/ϵ: between AGS and RHIC it decreases, due to softening of EOS by quark-hadron transition (Kolb, Sollfrank, UH, PRC 62 (2000) 054909)
- data show instead monotonous increase of v_2/ϵ with $\sqrt{s}$ $\Rightarrow$ the inability of viscous hadronic phase to build elliptic flow kills the phase transition signature!

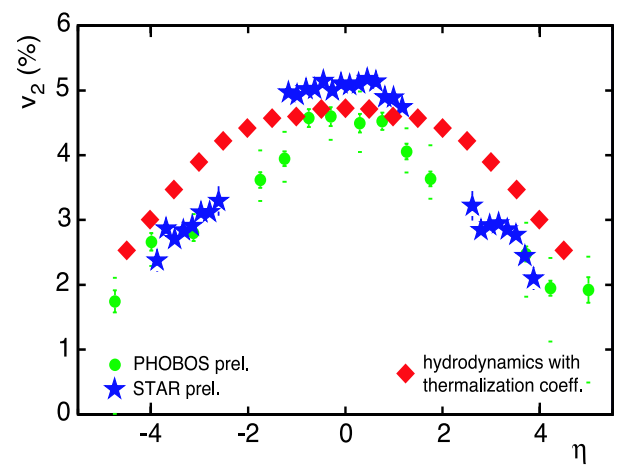
3d hydro:

T. Hirano, PRC 65 ('02) 011901; 66 ('02) 054905



hydro corrected with thermalization coefficient:

U.H., P. Kolb, J. Phys. G30 ('04) S1229

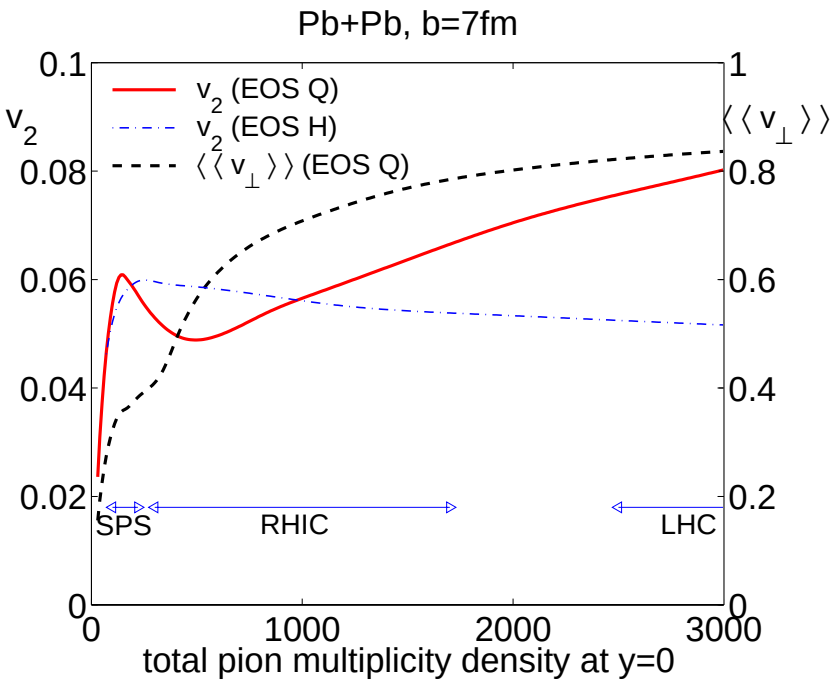


Breakdown of ideal hydro: the viscous hadron fluid

Excitation function of elliptic flow:

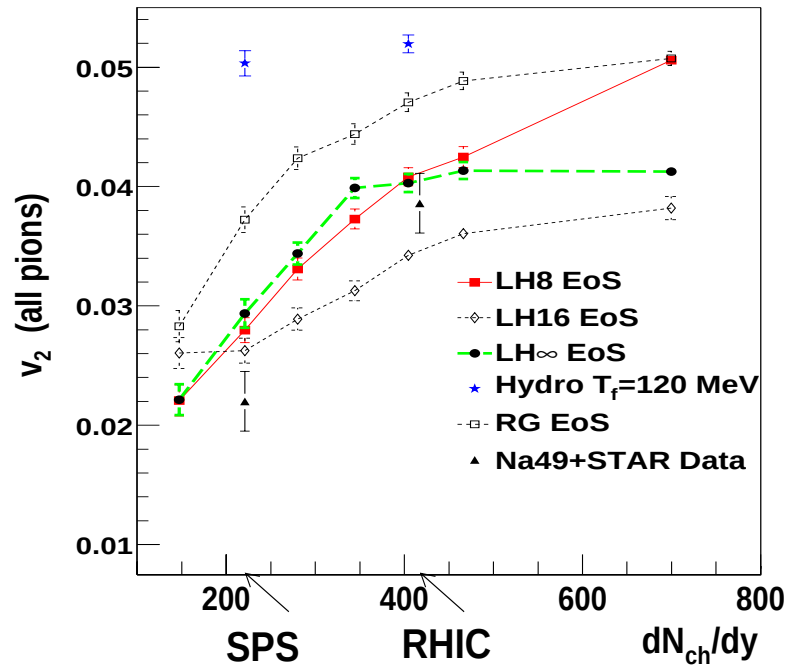
Ideal hydro

P. Kolb, J. Sollfrank, U.H., PRC 62 ('00) 054909



Hydro + RQMD

D. Teaney, J. Lauret, E. Shuryak, nucl-th/0110037

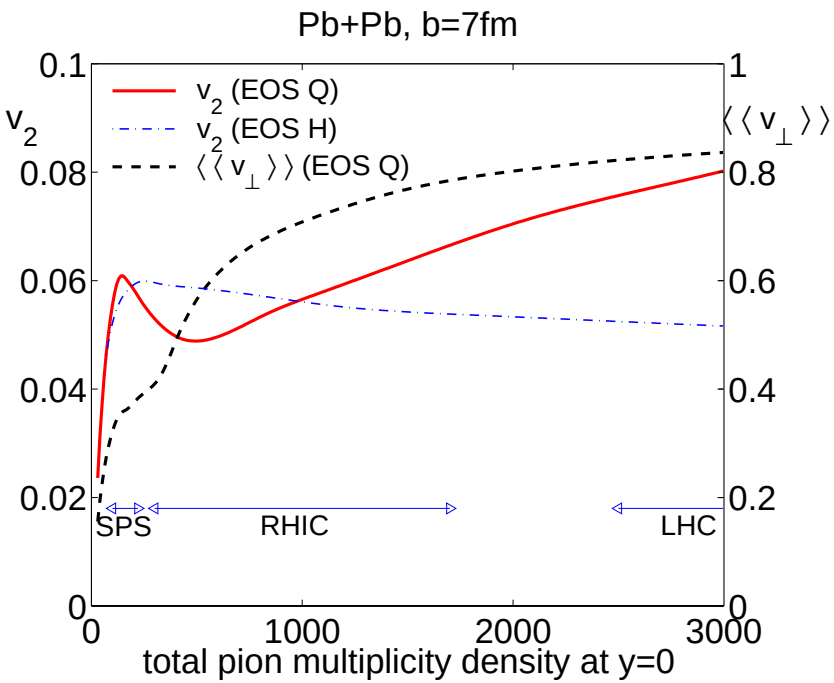


Hadron resonance gas is very viscous and does not respond strongly to spatial eccentricity $\Longrightarrow$ non-monotonic behaviour of v_2 resulting from dip in c_s^2 near phase-transition is erased!

"Saturation" of v_2: first hint of real "phase transition"?

Ideal hydro

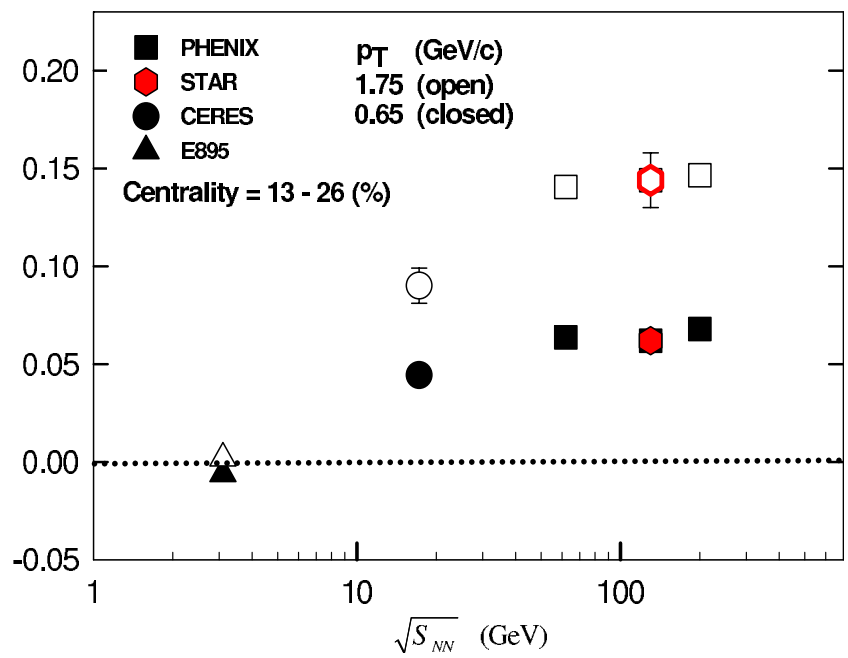
P. Kolb, J. Sollfrank, U.H., PRC 62 ('00) 054909



AGS, SPS and RHIC data

PHENIX Coll., nucl-ex/0411040

$(A{\approx}200) + (A{\approx}200)$, 13-26% centrality



"Saturation" of v_2 between $\sqrt{s_{NN}}{=}63$ and 200 GeV $\Longleftrightarrow$ signature of softening of EOS near phase transition?! (Sorge, PRL 82 (1999) 2048)

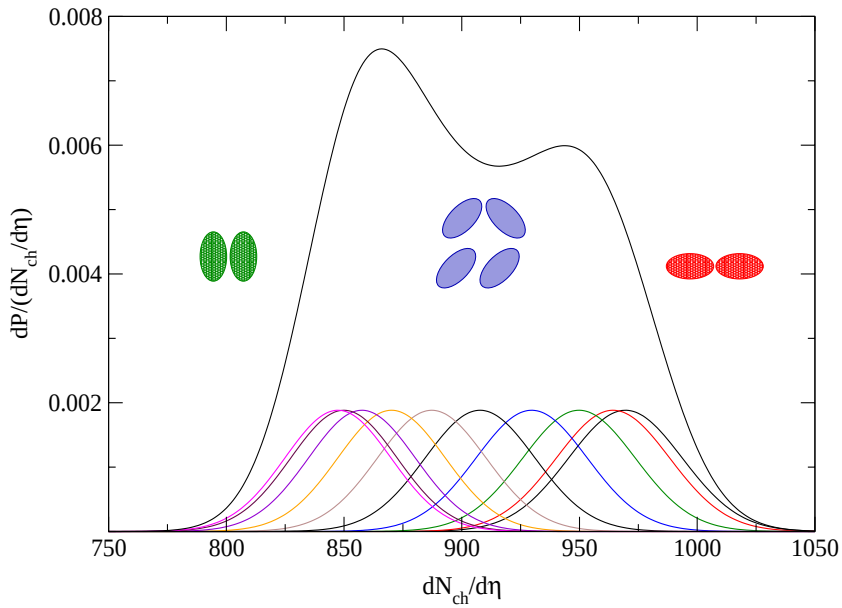
Beware! Many similar "phase transition signatures" have had a short shelf life . . .

But certainly worth investigating in more detail! (e.g. with U+U . . .)

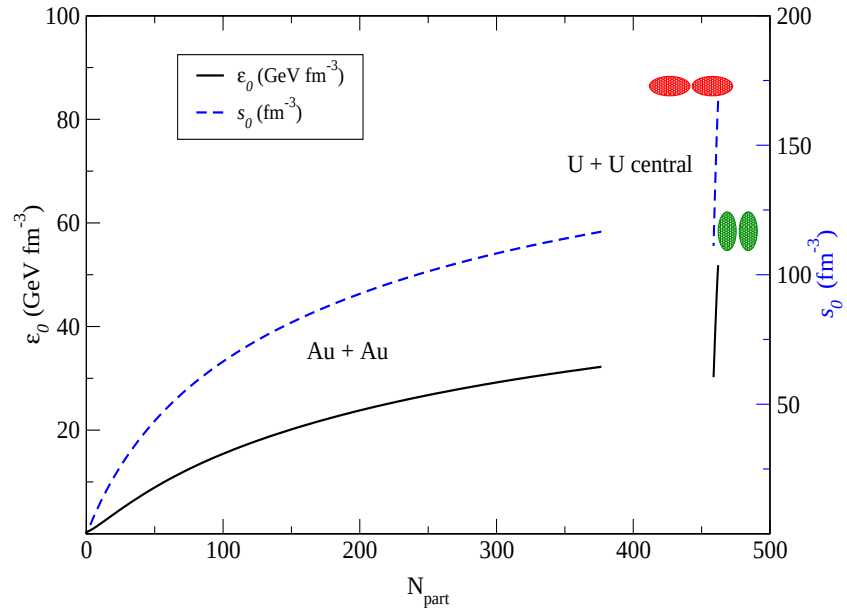
The ultimate test of hydrodynamics at RHIC: fully aligned ("zero spectators") U+U at 200 A GeV

A.J. Kuhlman, U.H., PRL, in press (nucl-th/0411054)

Multiplicity distribution for full overlap U+U collisions at 200 A GeV



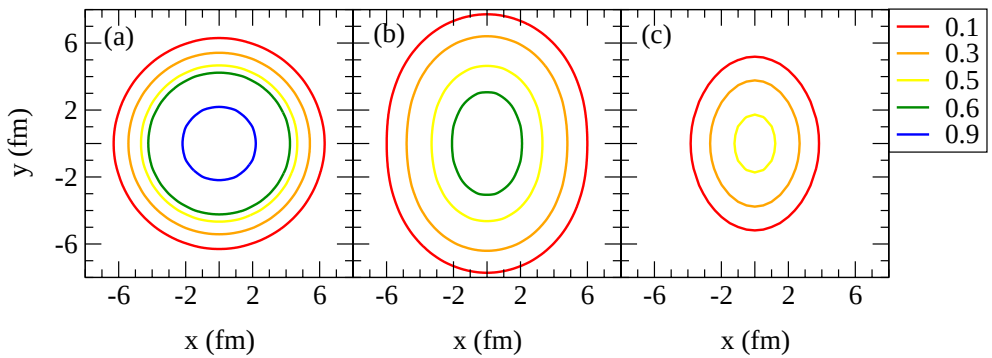
Initial maximal energy and entropy density for Au+Au and U+U collisions



- $dN_{\rm ch}/d\eta$ in head-on-head collisions $\approx 14\%$ larger than in side-on-side collisions
- e_0 in head-on-head U+U collisions $\approx 65\%$ larger than in side-on-side U+U and b=0 Au+Au!
- ϵ_x(side-on-side U+U) $\approx$ ϵ_x(b=7 fm Au+Au), at 30% higher e_0

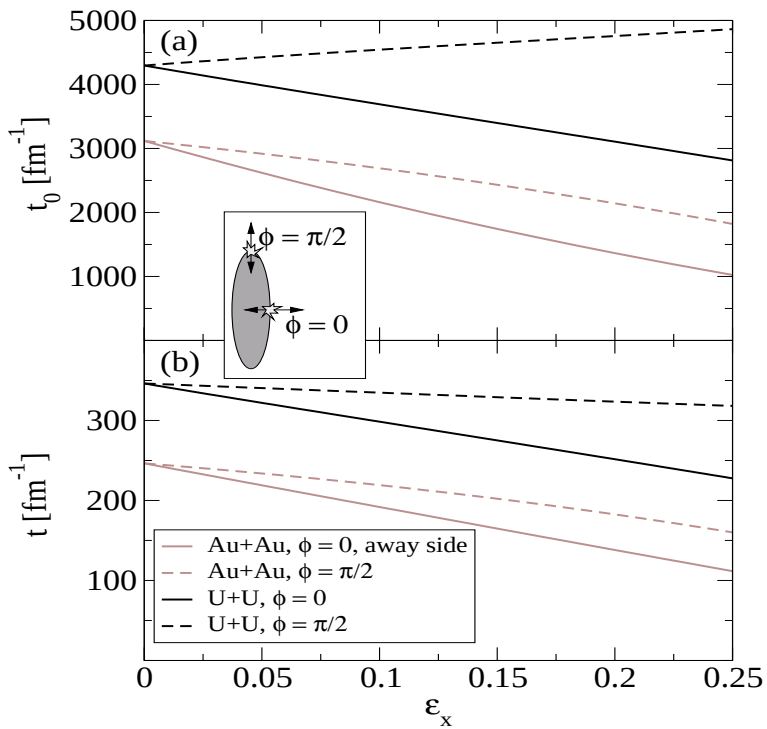
Full-overlap U+U vs. semi-peripheral Au+Au:

Initial entropy density contours at 200 A GeV



Bigger, denser deformed fireballs $\Longrightarrow$ larger in-plane vs. out-of-plane contrast for JET:

Parton energy loss figure of merit: $t \equiv \int_{\tau_0}^{\infty} d\tau \; \rho(\boldsymbol{r}_\perp(\tau), \tau)(\tau - \tau_0)$



static fireball

longitudinally expanding fireball

Viscous relativistic hydrodynamics (Israel & Stewart 1979)

Include shear viscosity η, neglect bulk viscosity (massless partons) and heat conduction ($\mu_B \approx 0$); solve

$$\partial_\mu T^{\mu\nu} = 0$$

with modified energy momentum tensor

$$T^{\mu\nu}(x) = \big(e(x)+p(x)\big)u^\mu(x)u^\nu(x) - g^{\mu\nu}p(x) + \pi^{\mu\nu}.$$

$\pi^{\mu\nu}$ = traceless viscous pressure tensor which relaxes locally to 2η times the shear tensor $\nabla^{\langle\mu}u^{\nu\rangle}$ on a microscopic kinetic time scale τ_π:

$$D\pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right)$$

where $D \equiv u^\mu\partial_\mu$ is the time derivative in the local rest frame.

Kinetic theory relates η and τ_π, but for a strongly coupled QGP neither η nor this relation are known $\Longrightarrow$ treat η and τ_π as independent phenomenological parameters. For consistency: $\tau_\pi\theta \ll 1$ ($\theta = \partial^\mu u_\mu =$ local expansion rate).

(1+1)-d viscous hydrodynamic equations

(Muronga & Rischke 2004, Chaudhuri & Heinz 2005)

Azimuthally symmetric transverse dynamics with long. boost invariance: Use (τ, r, ϕ, η) coordinates and solve

- hydrodynamic equations for $T^{\tau\tau} = (e+\mathcal{P})\gamma_r^2 - \mathcal{P}, \quad T^{\tau r} = (e+\mathcal{P})\gamma_r^2 v_r$ (with "effective pressure" $\mathcal{P} = p - r^2\pi^{\phi\phi} - \tau^2\pi^{\eta\eta}$) together with
- kinetic relaxation equations for $\pi^{\phi\phi}, \ \pi^{\eta\eta}$:

$$\frac{1}{\tau}\partial_\tau\Big(\tau T^{\tau\tau}\Big) + \frac{1}{r}\partial_r\Big(r(T^{\tau\tau}+\mathcal{P})v_r\Big) = -\frac{p+\tau^2\pi^{\eta\eta}}{\tau},$$

$$\frac{1}{\tau}\partial_\tau\Big(\tau T^{\tau r}\Big) + \frac{1}{r}\partial_r\Big(r(T^{\tau r}v_r+\mathcal{P})\Big) = +\frac{p+r^2\pi^{\phi\phi}}{r},$$

$$\Big(\partial_\tau + v_r\partial_r\Big)\pi^{\eta\eta} = -\frac{1}{\gamma_r\tau_\pi}\left[\pi^{\eta\eta} - \frac{2\eta}{\tau^2}\left(\frac{\theta}{3} - \frac{\gamma_r}{\tau}\right)\right],$$

$$\Big(\partial_\tau + v_r\partial_r\Big)\pi^{\phi\phi} = -\frac{1}{\gamma_r\tau_\pi}\left[\pi^{\phi\phi} - \frac{2\eta}{r^2}\left(\frac{\theta}{3} - \frac{\gamma_r v_r}{r}\right)\right].$$

Close equations with EOS $p(e)$ where $e = T^{\tau\tau} - v_r T^{\tau r}$ and $v_r = T^{\tau r}/(T^{\tau\tau}+\mathcal{P})$.