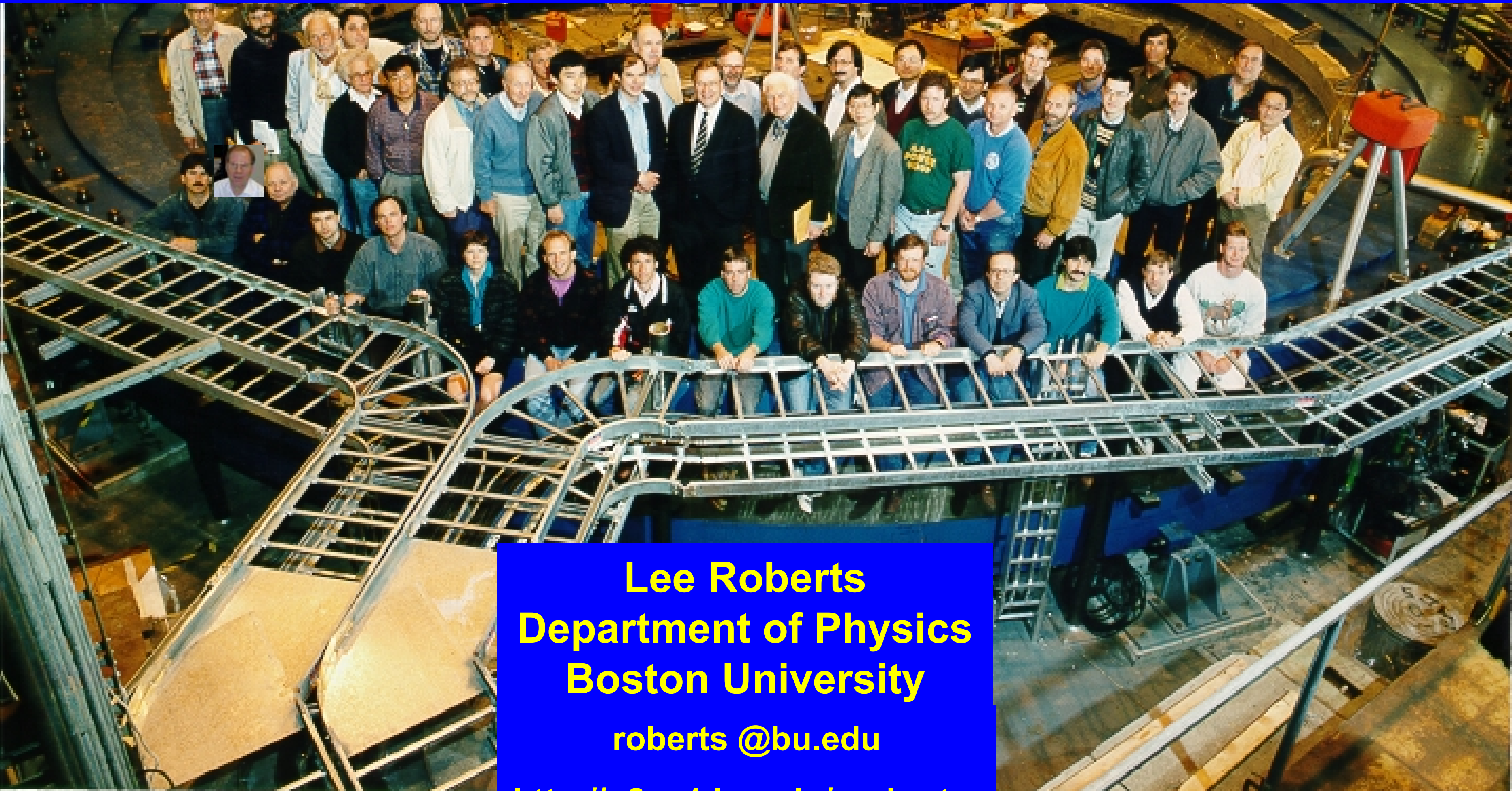
The Electric and Magnetic Dipole moments of the Muon: Results and Future Possibilities



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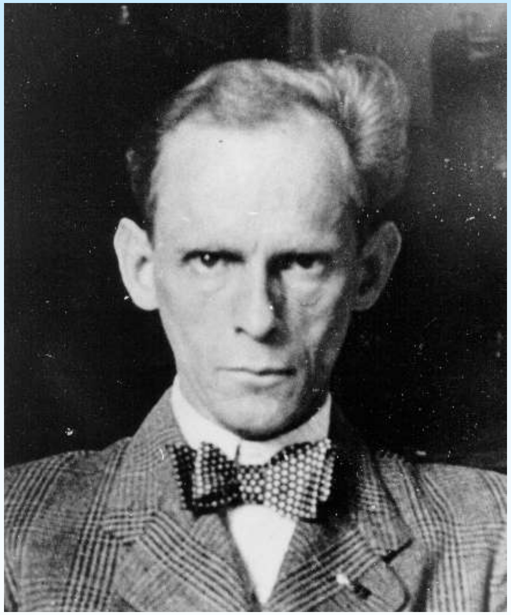
Outline

- Introduction to the muon
- Magnetic (a_μ) and electric (d_μ) dipole moments
  - E821 result and the SM
  - E821 EDM limit
- Limits on CPT/Lorentz Violation in muon spin precession
- Future improvements in a_μ and d_μ?
- Summary and conclusions.

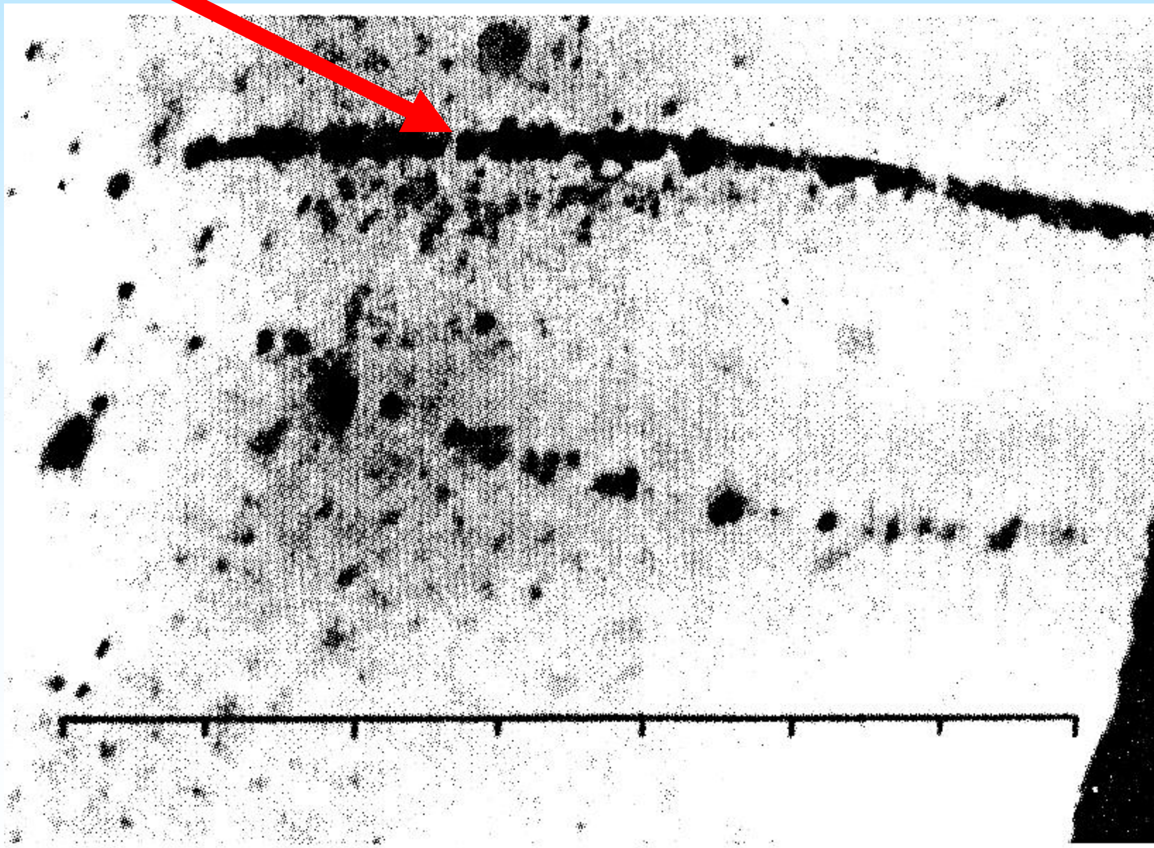
First published observation of the muon came from cosmic rays:

Paul Kunze,

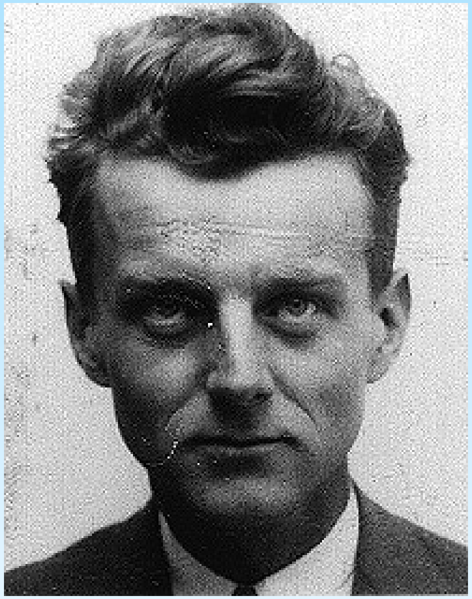
Z. Phys. 83, 1 (1933)



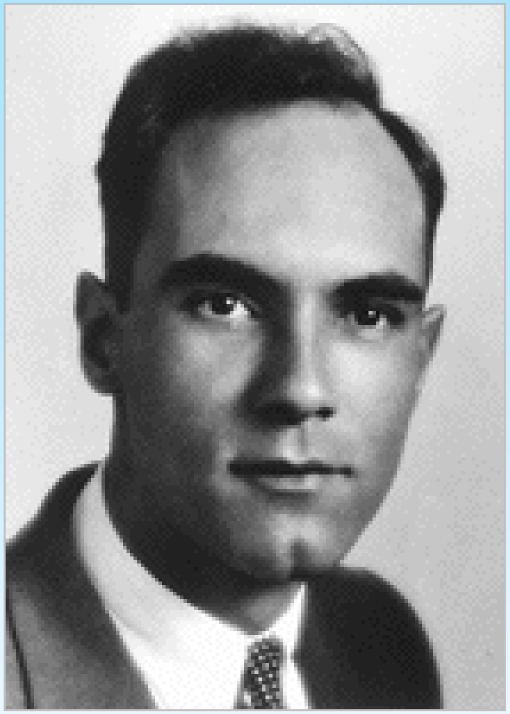
"a particle of uncertain nature"

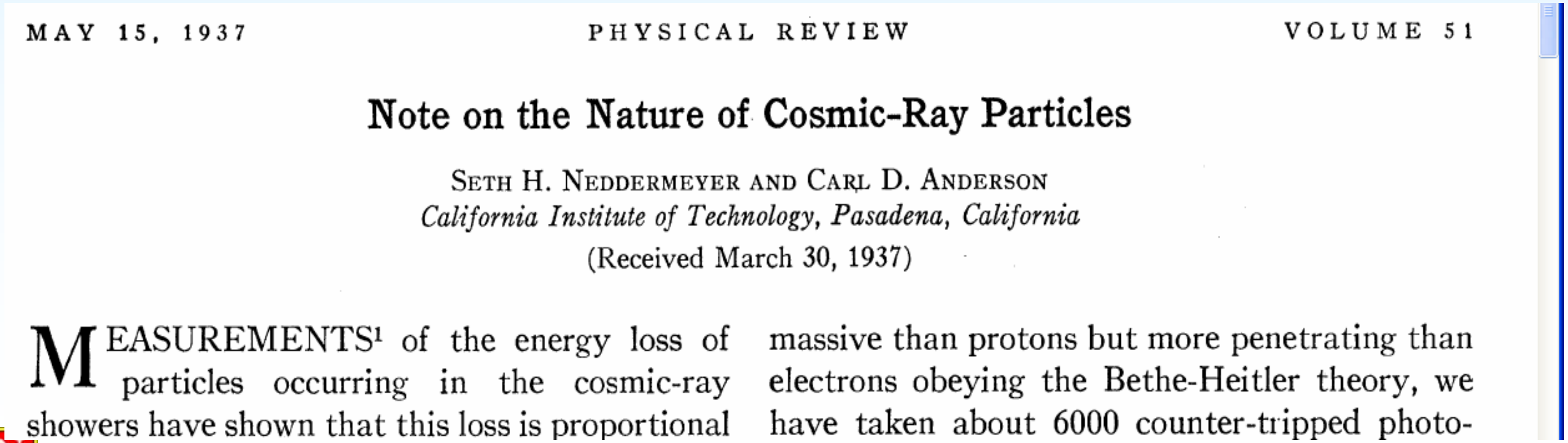


Identified in 1936



Study of cosmic rays by Seth Neddermeyer and Carl Anderson




MAY 15, 1937 PHYSICAL REVIEW VOLUME 51

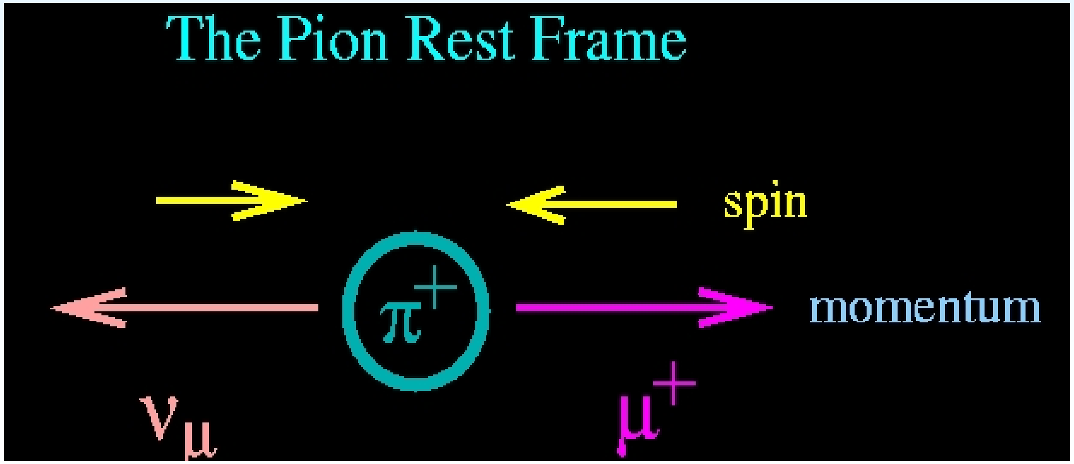
Note on the Nature of Cosmic-Ray Particles

SETH H. NEDDERMEYER AND CARL D. ANDERSON
California Institute of Technology, Pasadena, California
(Received March 30, 1937)

MEASUREMENTS[1] of the energy loss of particles occurring in the cosmic-ray showers have shown that this loss is proportional massive than protons but more penetrating than electrons obeying the Bethe-Heitler theory, we have taken about 6000 counter-tripped photo-

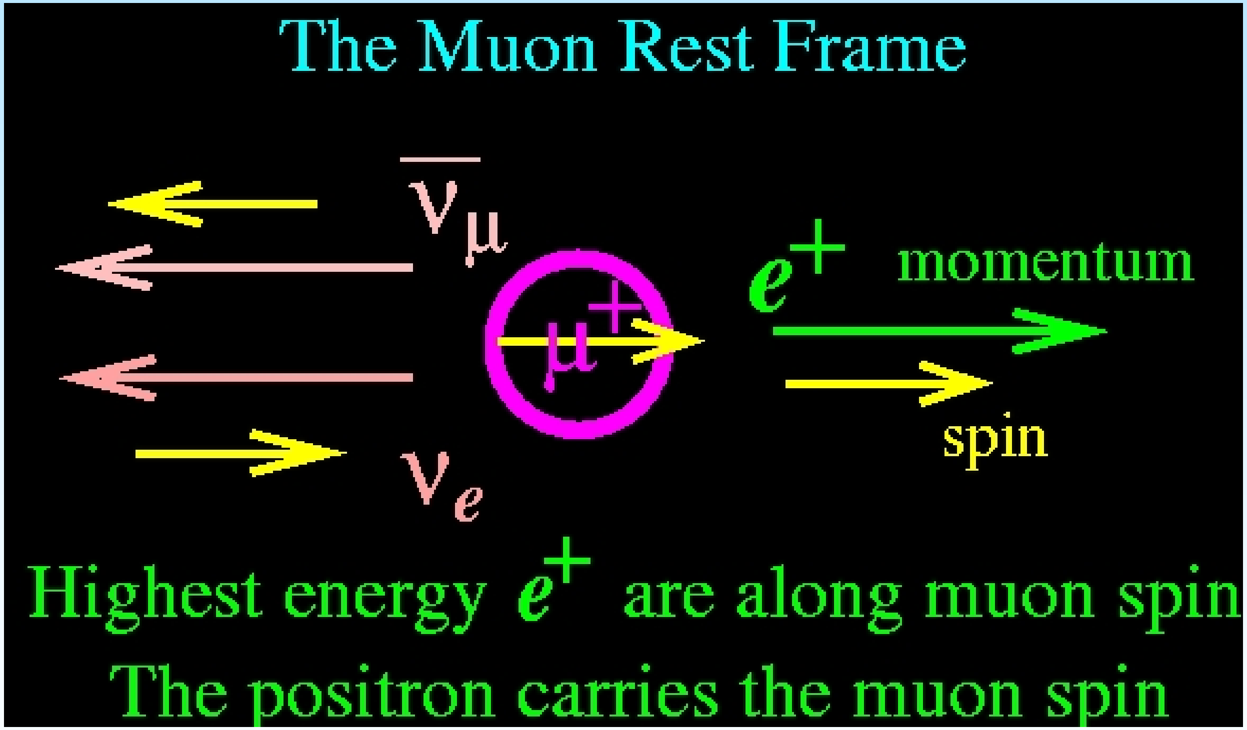
Muon properties:

- Lifetime ~2.2 μs, practically forever
- 2nd generation lepton
- m_μ/m_e = 206.768 277(24)
- produced polarized
  - in-flight decay: both "forward" and "backward" muons are highly polarized
- Paul Scherrer Institut has 10^8 low-energy μ/s in a beam



Death of the Muon

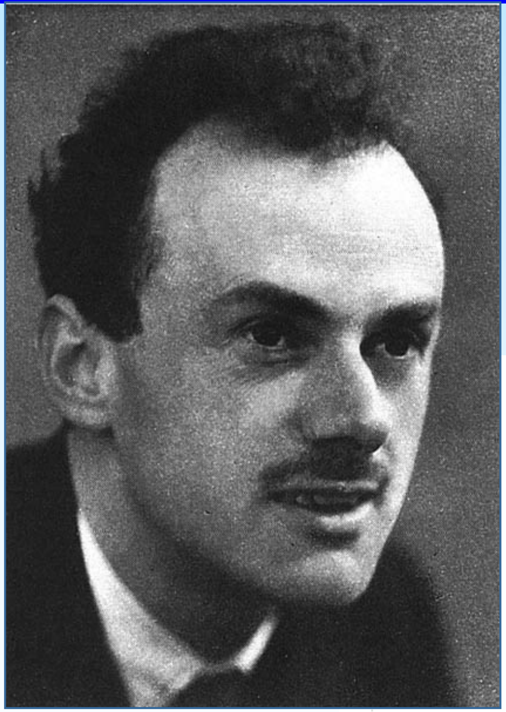
- Decay is self analyzing



What have we learned from the μ's death?

- The strength of the weak interaction
  - i.e. the Fermi constant G_F (more properly G_μ)
- The V - A nature of the weak interaction
- Lepton flavor conservation in μ-decay
- VEV of the Higgs field: $\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$
- Induced form-factors in nuclear μ-capture

Theory of Magnetic and Electric Dipole Moments



The Quantum Theory of the Electron.

By P. A. M. DIRAC, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

Proc. R. Soc. (London) **A117**, 610 (1928)

§ 4. *The Hamiltonian for an Arbitrary Field.*

To obtain the Hamiltonian for an electron in an electromagnetic field with scalar potential A_0 and vector potential $\mathbf{A}$, we adopt the usual procedure of substituting $p_0 + e/c \,.\, A_0$ for p_0 and $\mathbf{p} + e/c \,.\, \mathbf{A}$ for $\mathbf{p}$ in the Hamiltonian for no field. From equation (9) we thus obtain

$$\left[p_0 + \frac{e}{c}A_0 + \rho_1\left(\boldsymbol{\sigma}, \mathbf{p} + \frac{e}{c}\mathbf{A}\right) + \rho_3 mc\right]\psi = 0. \tag{14}$$

This differs from (1) by the two extra terms

$$\boxed{\frac{eh}{c}(\boldsymbol{\sigma}, \mathbf{H}) + \frac{ieh}{c}\rho_1(\boldsymbol{\sigma}, \mathbf{E})}$$

in F. These two terms, when divided by the factor $2m$, can be regarded as the additional potential energy of the electron due to its new degree of freedom. The electron will therefore behave as though it has a magnetic moment $eh/2mc \,.\, \boldsymbol{\sigma}$ and an electric moment $ieh/2mc \,.\, \rho_1\boldsymbol{\sigma}$. This magnetic moment is just that assumed in the spinning electron model. The electric moment, being a pure imaginary, we should not expect to appear in the model. It is doubtful whether

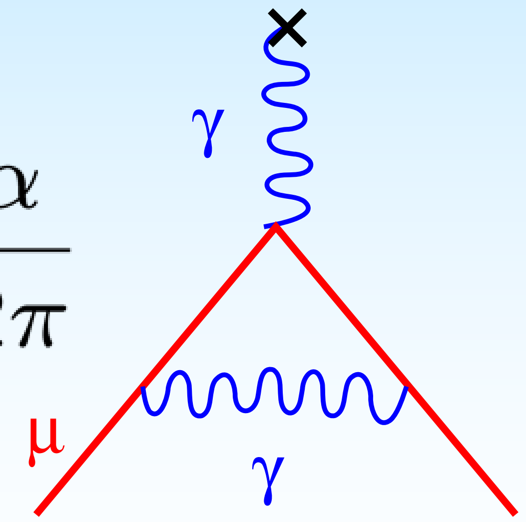
The magnetic dipole moment directed along spin.

$$\vec{\mu}_s = g_s \left(\frac{e\hbar}{2m}\right) \vec{s}$$ **Dirac Theory: $g_s = 2$**

$$\mu = (1 + a)\frac{e\hbar}{2m}$$ Dirac + Pauli moment $$a = \frac{g-2}{2}$$

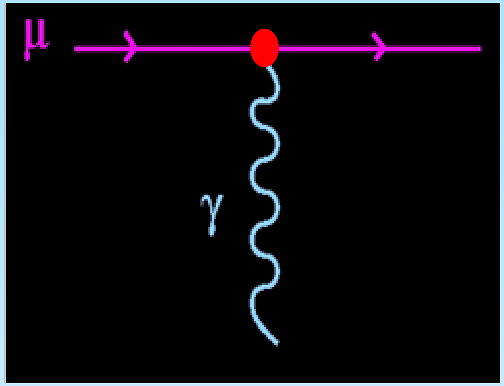
For leptons, radiative corrections dominate the value of $a \simeq 0.00116\ldots$

$$a = \frac{\alpha}{2\pi}$$



Modern Notation:

$$\Gamma_\beta = eF_1\bar{\psi}_R\gamma_\beta\psi_R + \frac{ie}{2m}F_2\bar{\psi}_R\sigma_{\beta\delta}q^\delta\psi_L$$



- Muon Magnetic Dipole Momoment a_μ chiral changing

$$\bar{u}_\mu[eF_1(q^2)\gamma_\beta + \frac{ie}{2m_\mu}F_2(q^2)\sigma_{\beta\delta}q^\delta]u_\mu$$

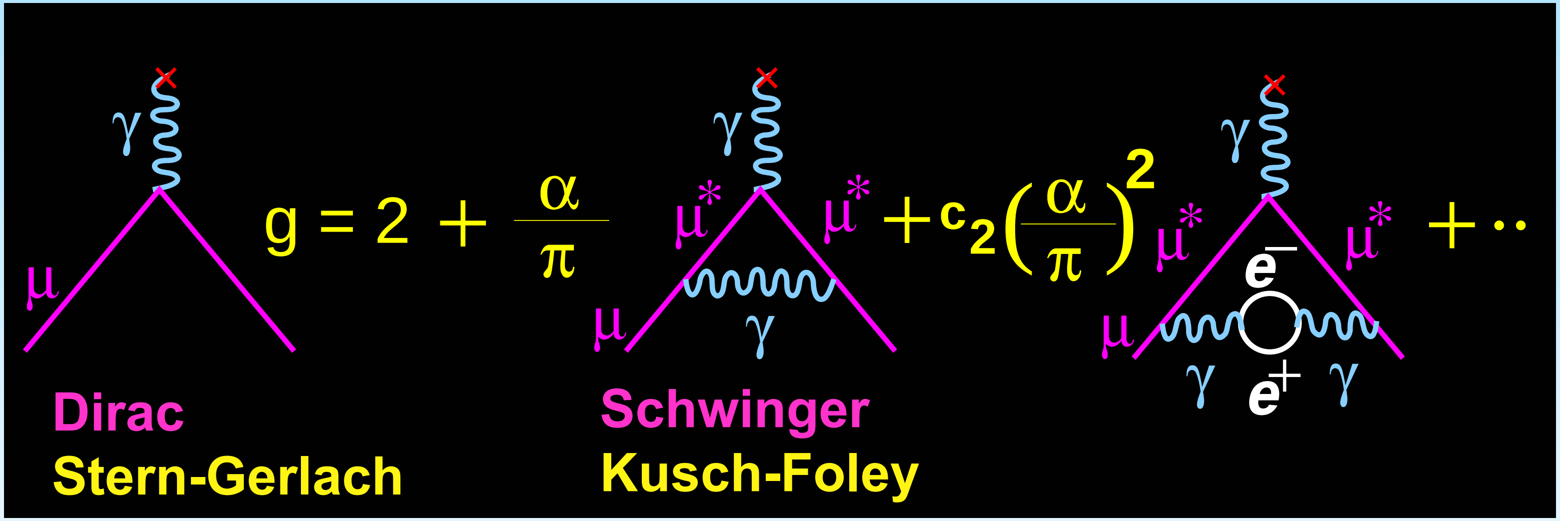
$$F_1(0) = 1 \qquad F_2(0) = a_\mu$$

- Muon EDM

$$\bar{u}_\mu\left[\frac{ie}{2m_\mu}F_2(q^2) - F_3(q^2)\gamma_5\right]\sigma_{\beta\delta}q^\delta u_\mu$$

$$F_2(0) = a_\mu \qquad F_3(0) = d_\mu;\ \text{EDM}$$

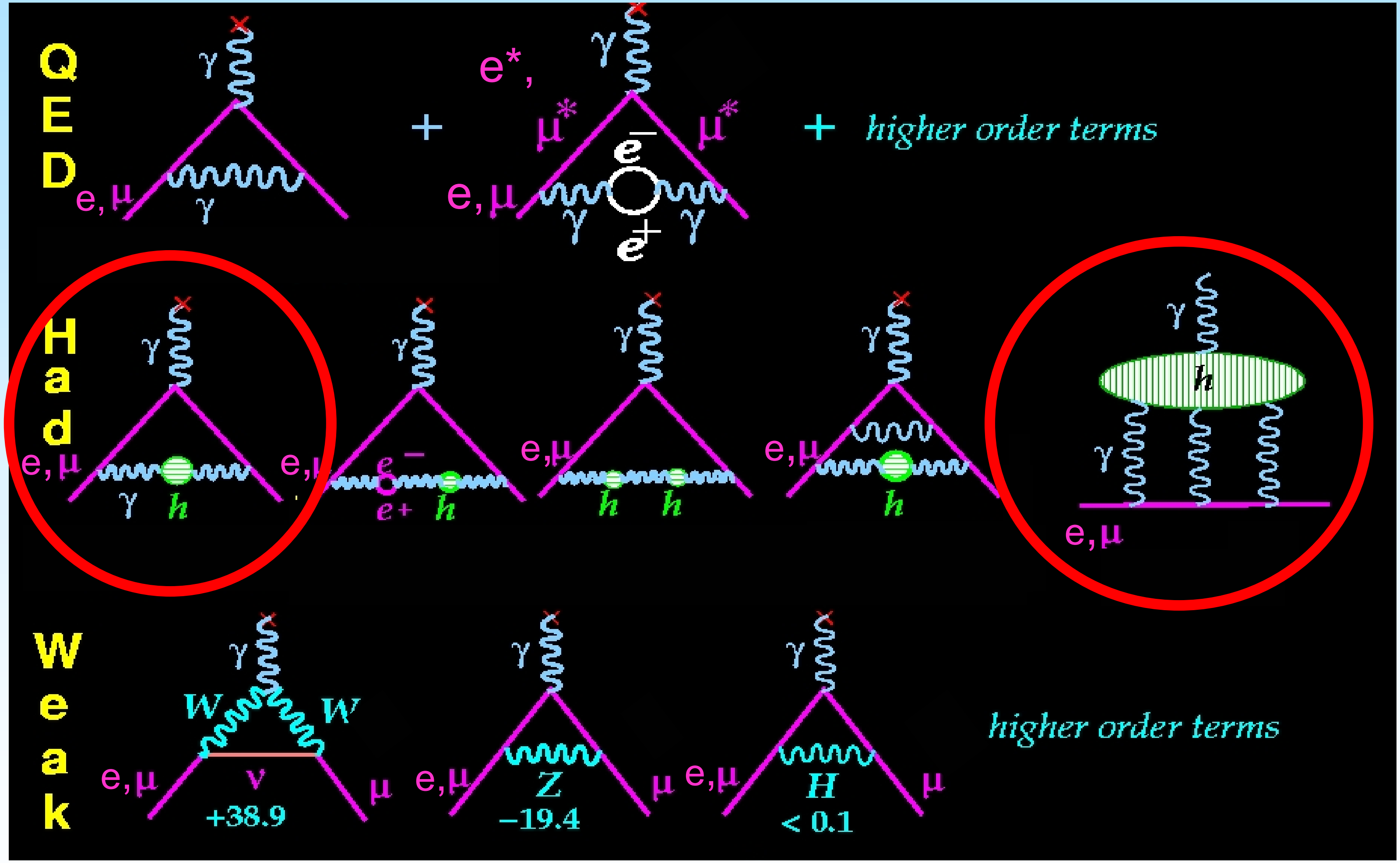
Radiative corrections change g



$$a(\mathrm{QED}) = \frac{1}{2}\frac{\alpha}{\pi} + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

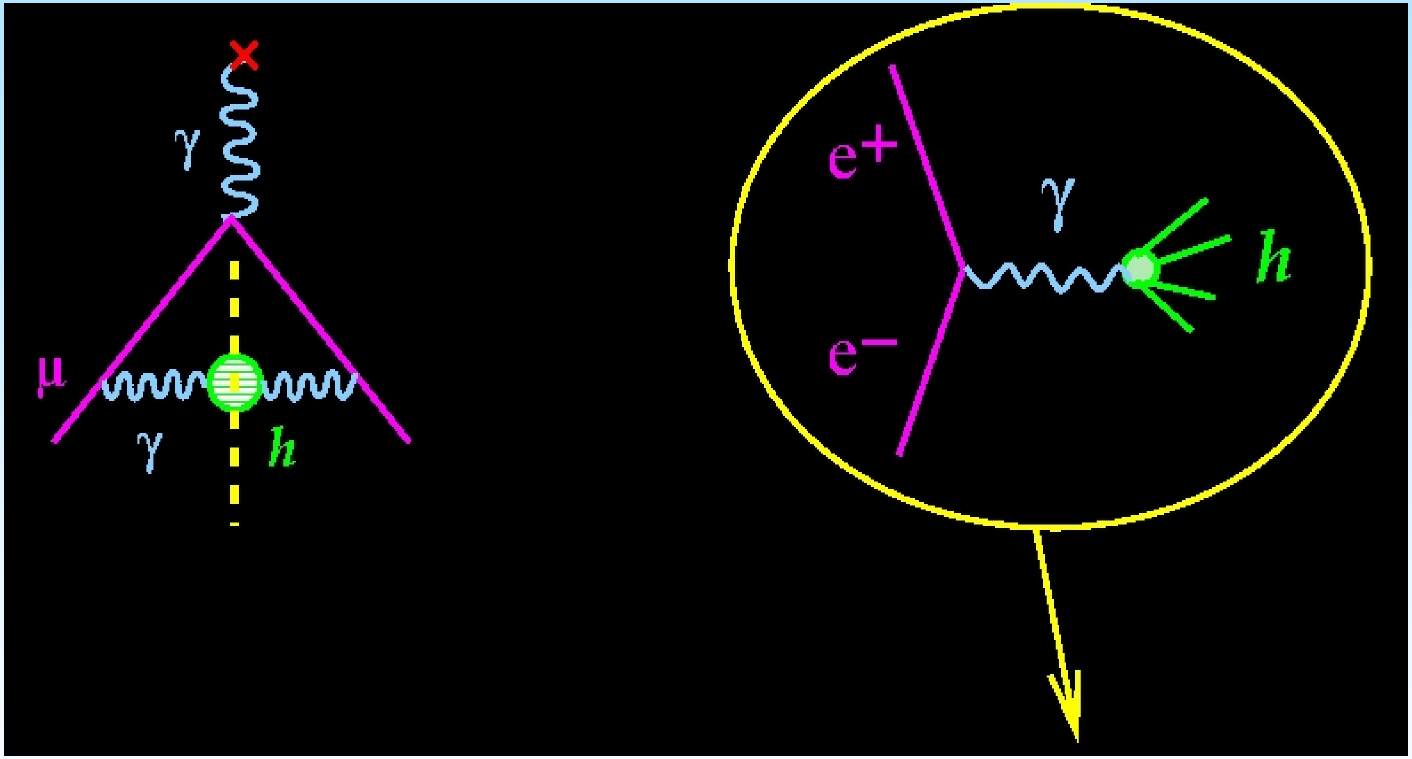
The SM Value for electron and muon anomalies

$$a_\mu(\mathrm{SM}) = a_\mu(\mathrm{QED}) + a_\mu(\mathrm{hadronic}) + a_\mu(\mathrm{weak})$$



e vrs. μ : relative contribution of heavier things $\left(\frac{m_\mu}{m_e}\right)^2 \simeq 42,000$

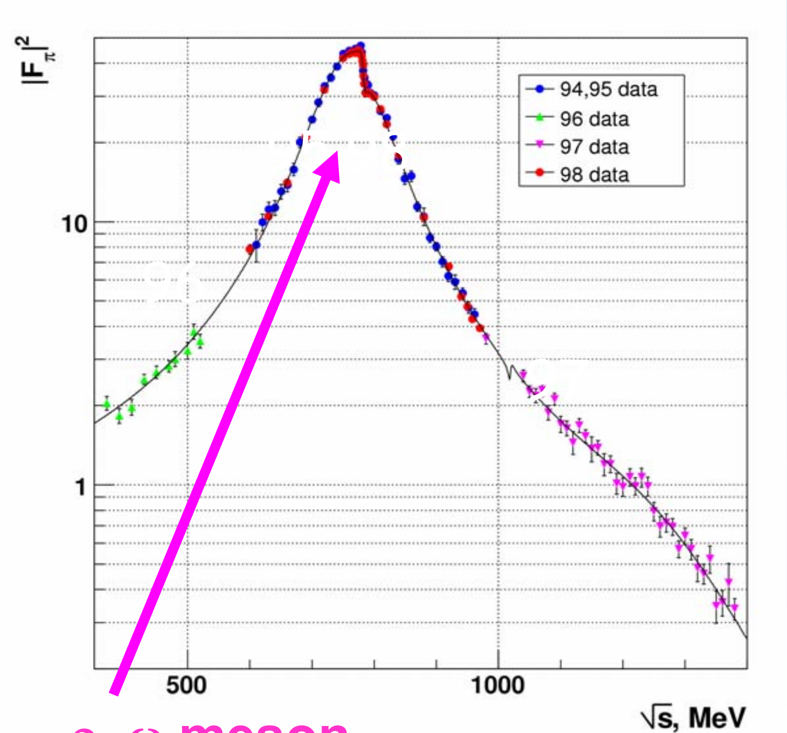
Lowest Order Hadronic from e^+e^- annihilation using analyticity and the optical theorem:



$$a_\mu(\text{had}) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) \left(\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}\right)$$

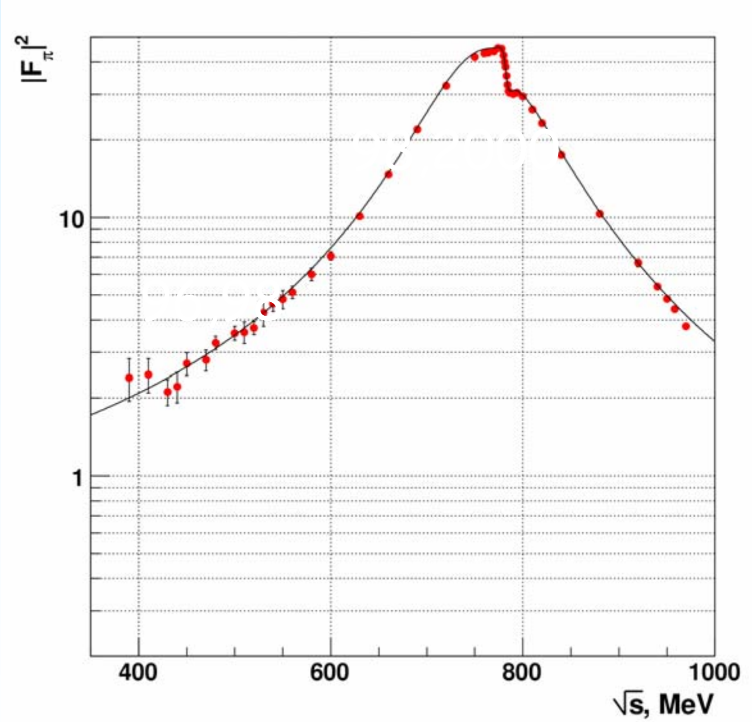
Two experiments at the Budker Insitute at Novosibirsk have measured R(s) to better than a percent. KLOE at Frascati has also measured R, and BaBar has a large data set that is being analyzed with a blind analysis.

CMD-2



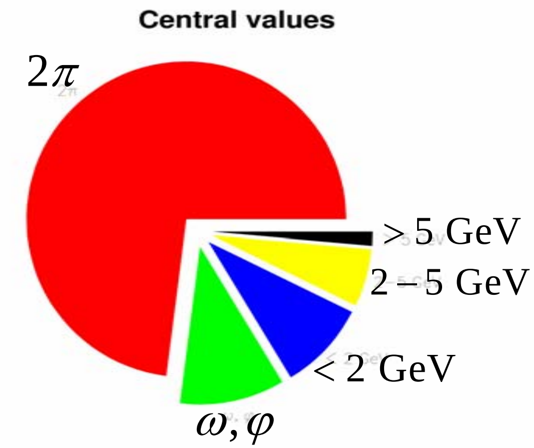
ρ–ω meson interference

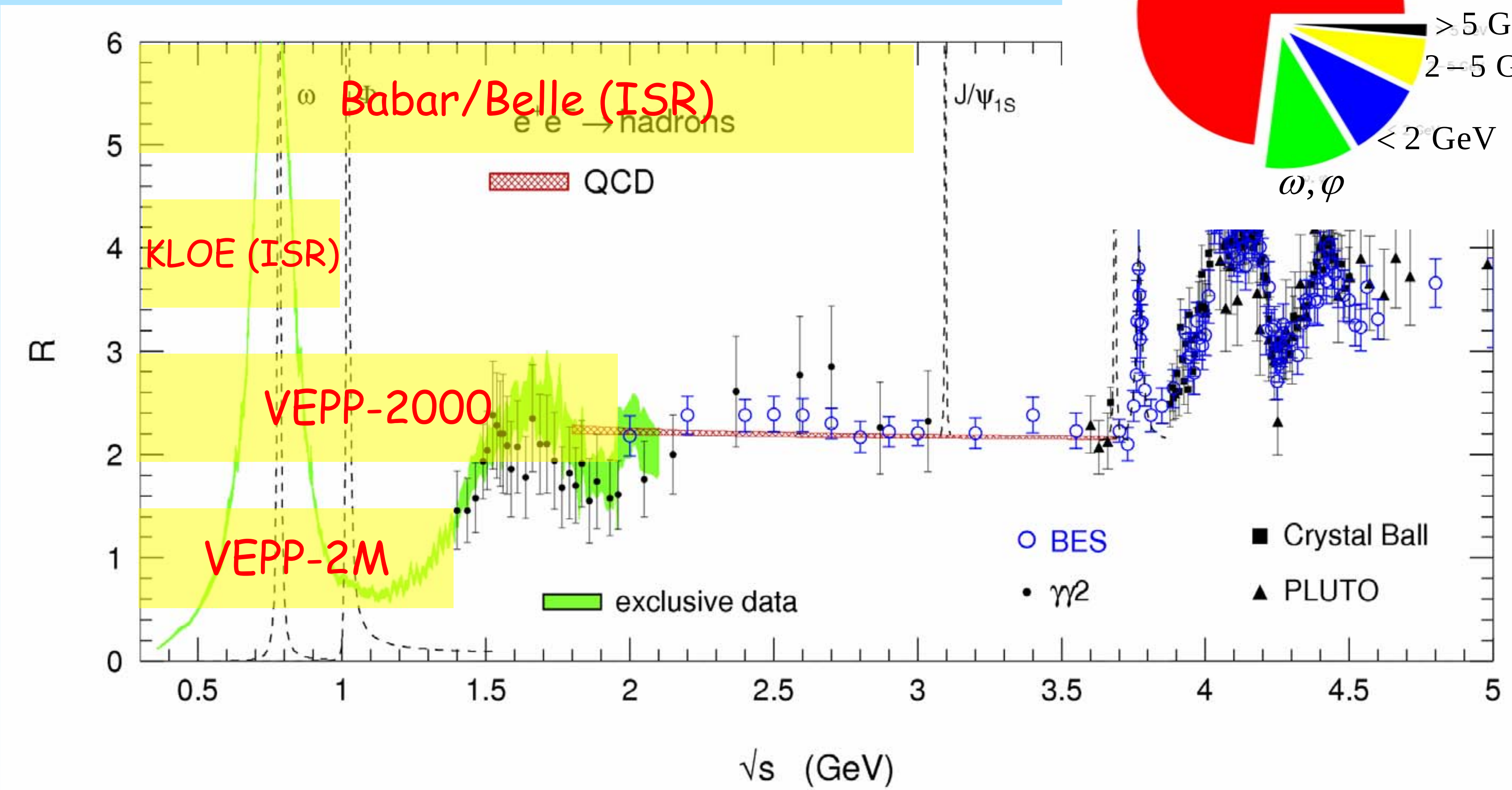
SND



F_π from $e^+e^- \to \pi\pi$

R(s) measurements at low s



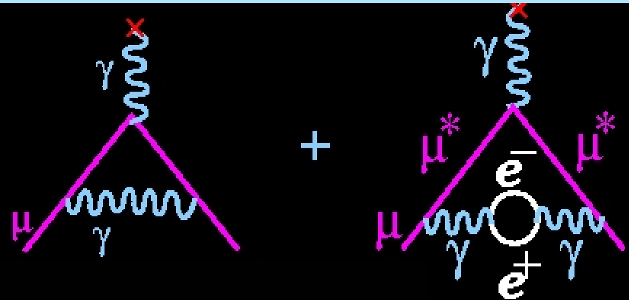


At low s the cross-section is measured independently for each final state

from Davier/Höcker

The SM Value for the muon anomaly (10^{-10})

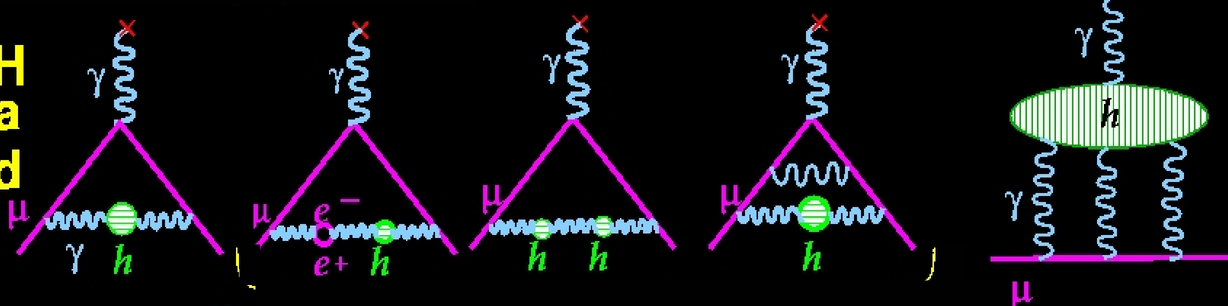
QED



$$+\cdots+C_5\left(\frac{\alpha}{\pi}\right)^5$$

11 658 471.809 (.016)

Had

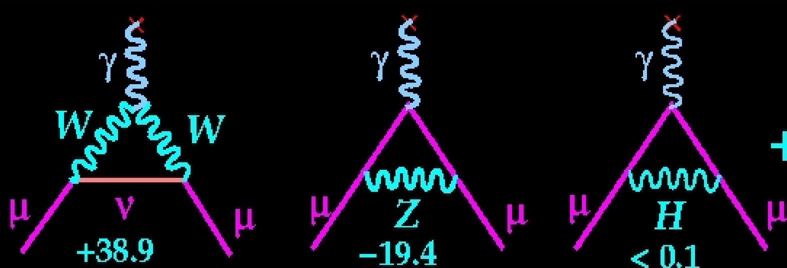


690.1 (4.7)

−9.79 (.09)

10 (2)

Weak

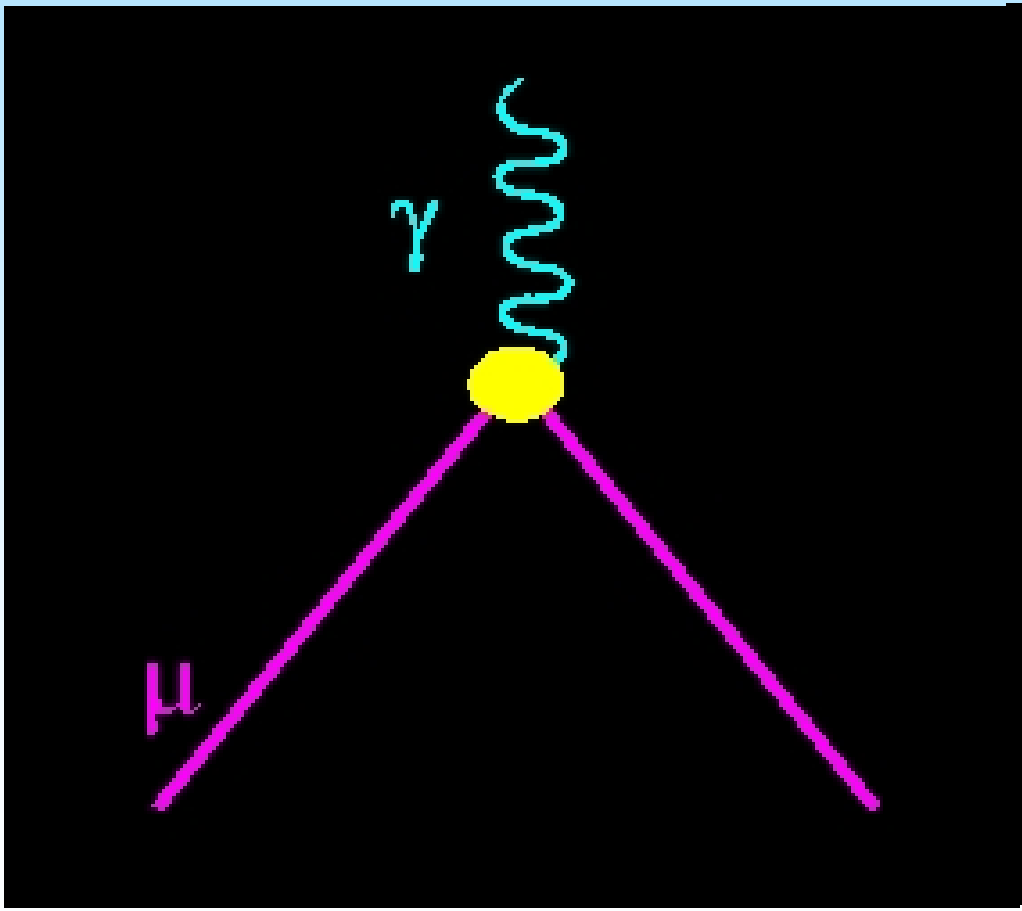


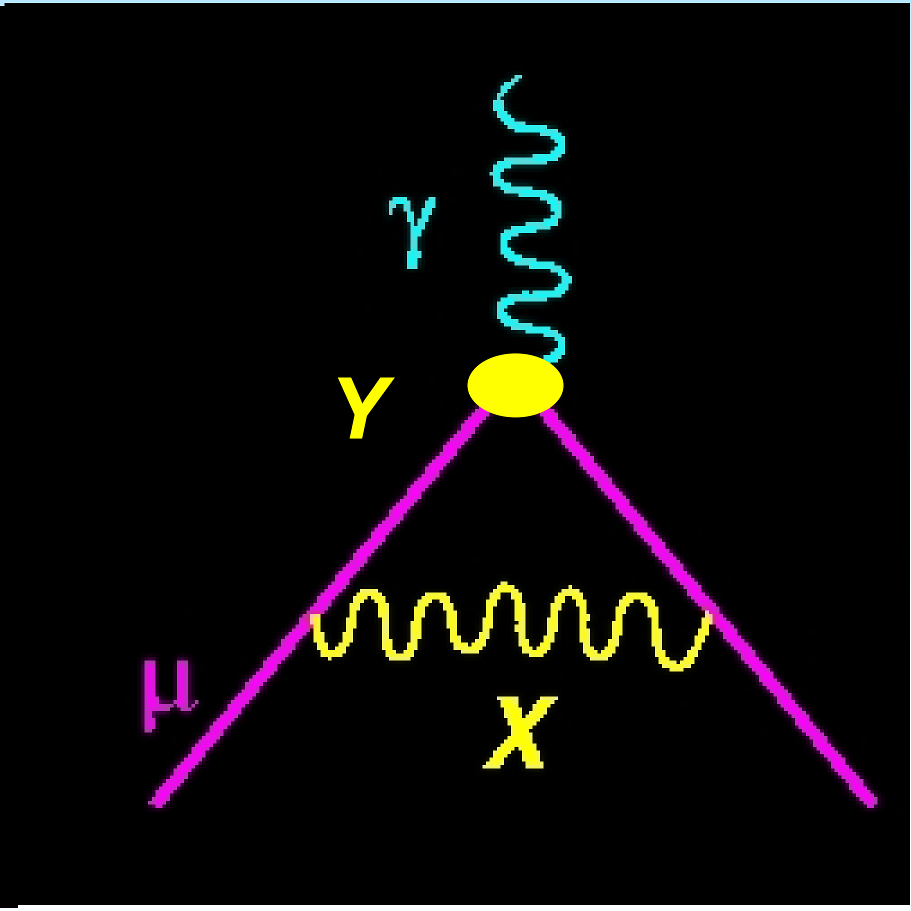
$+\,+\cdots$

15.4 (.2)

11 659 178.3 (4.8)

Since a_μ represents a sum over all physics, it is sensitive to a wide range of potential new physics



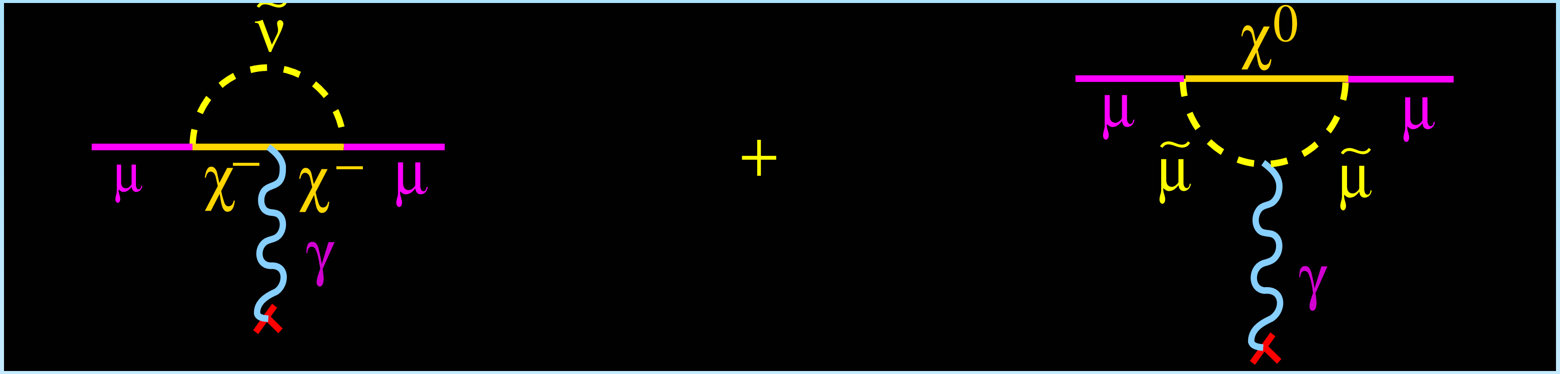


a_μ is sensitive to a wide range of new physics

- substructure

$$\delta a_\mu(\Lambda_\mu) \simeq \frac{m_\mu^2}{\Lambda_\mu^2}$$

a_μ is sensitive to a wide range of new physics

- 

- SUSY (with large tanβ)

$$a_\mu(\mathrm{SUSY}) \simeq \frac{\alpha(M_Z)}{8\pi \sin^2\theta_W} \frac{m_\mu^2}{\tilde{m}^2} \tan\beta \left(1 - \frac{4\alpha}{\pi} \ln \frac{\tilde{m}}{m_\mu}\right)$$

$$\simeq (\mathrm{sgn}\mu)\, 13 \times 10^{-10} \tan\beta \left(\frac{100\ \mathrm{GeV}}{\tilde{m}}\right)^2$$

- many other things (extra dimensions, etc.)

Spin Motion in a Magnetic Field

Momentum turns with ω_C, cyclotron frequency

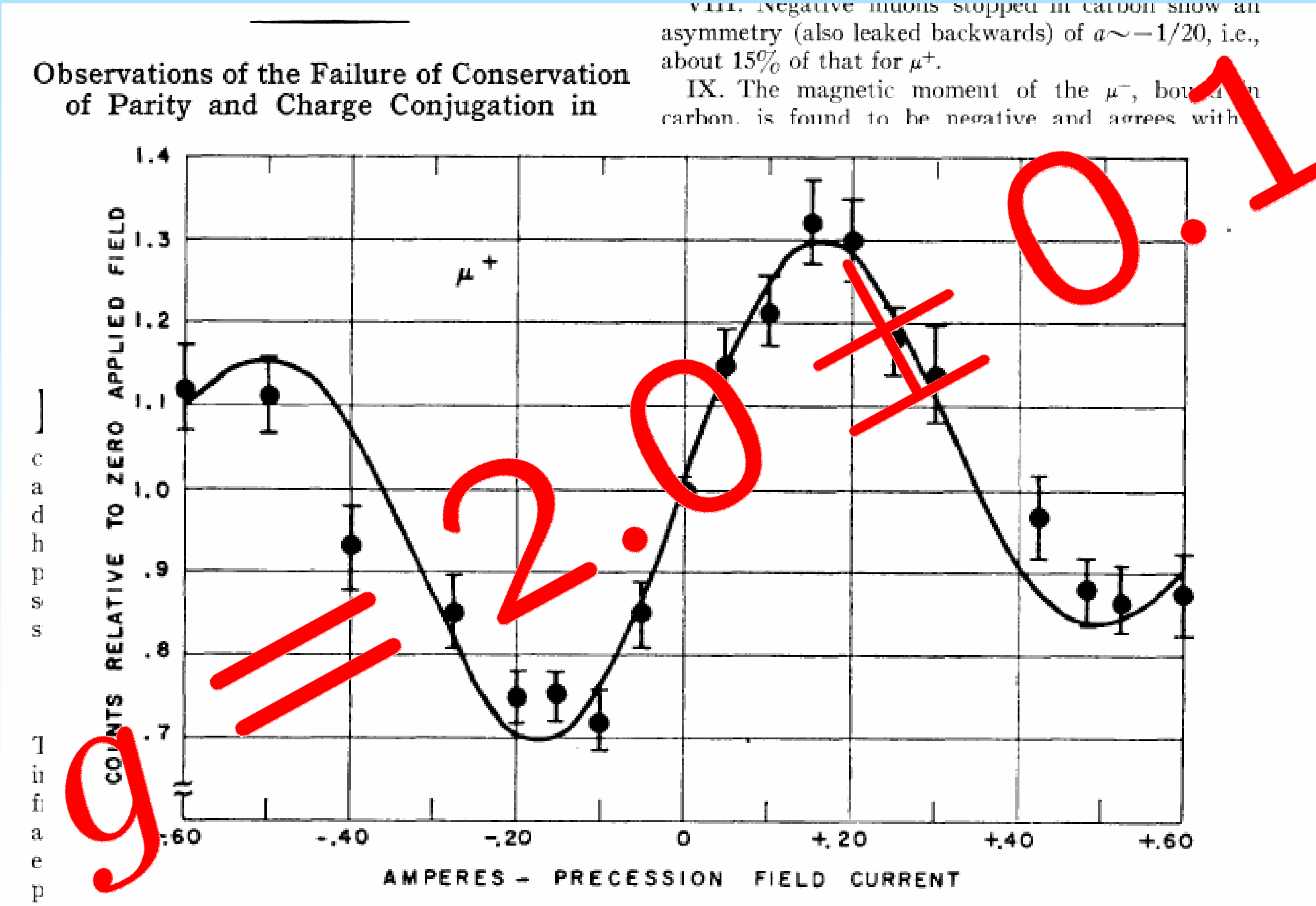
Spin turns with ω_S

$$\omega_C = \frac{eB}{mc\gamma} \qquad \omega_S = \frac{geB}{2mc} + (1-\gamma)\frac{eB}{\gamma mc}$$

Spin turns relative to the momentum with ω_a

$$\omega_a = \omega_S - \omega_C = \left(\frac{g-2}{2}\right)\frac{eB}{mc} = a\frac{eB}{mc}$$

First muon spin rotation experiment



PHYSICAL REVIEW VOLUME 118, NUMBER 1 APRIL 1, 1960

Accurate Determination of the μ^+ Magnetic Moment*

R. L. GARWIN,† D. P. HUTCHINSON, S. PENMAN,‡ AND G. SHAPIRO§
Columbia University, New York, New York

(Received August 4, 1959)

Note added in proof.—Experiments which have recently been reported to us [J. Lathrop, et al. and A. Bearden et al., Phys. Rev. Letters (to be published)] indicate a mass value of $M_\mu = 206.76_{-0.02}^{+0.03} M_e$. This yields a value of $\boxed{g_\mu = 2(1.00113_{-0.00012}^{+0.00016})}$. Although the assigned errors are now slightly greater than above, it is to be noted that the new result represents a direct measurement, rather than a lower limit. The agreement

$$a = \frac{\alpha}{2\pi} = 0.001161$$

Subsequent (g-2) experiments measured the difference frequency, ω_a, between the spin and momentum precession

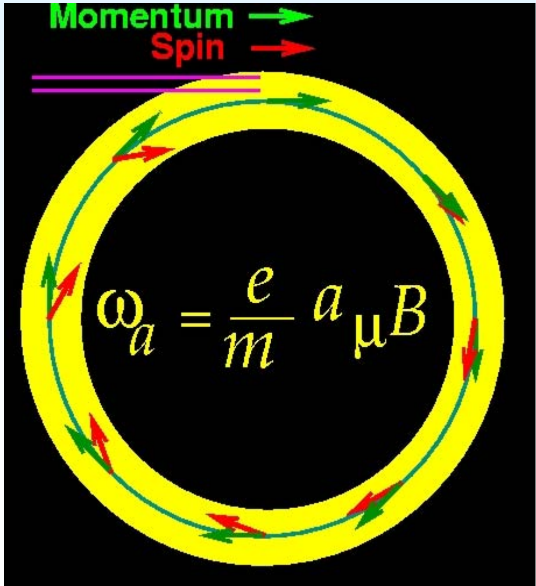
With an electric quadrupole field for vertical focusing

$$\vec{\omega}_a = -\frac{e}{m}\left[a_\mu \vec{B} - \cancelto{0}{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}}\right]$$

$$B \Rightarrow \langle B \rangle_{\mu-\mathrm{dist}}$$

$$\gamma_{\mathrm{magic}} = 29.3$$

$$p_{\mathrm{magic}} = 3.09\ \mathrm{GeV/c}$$



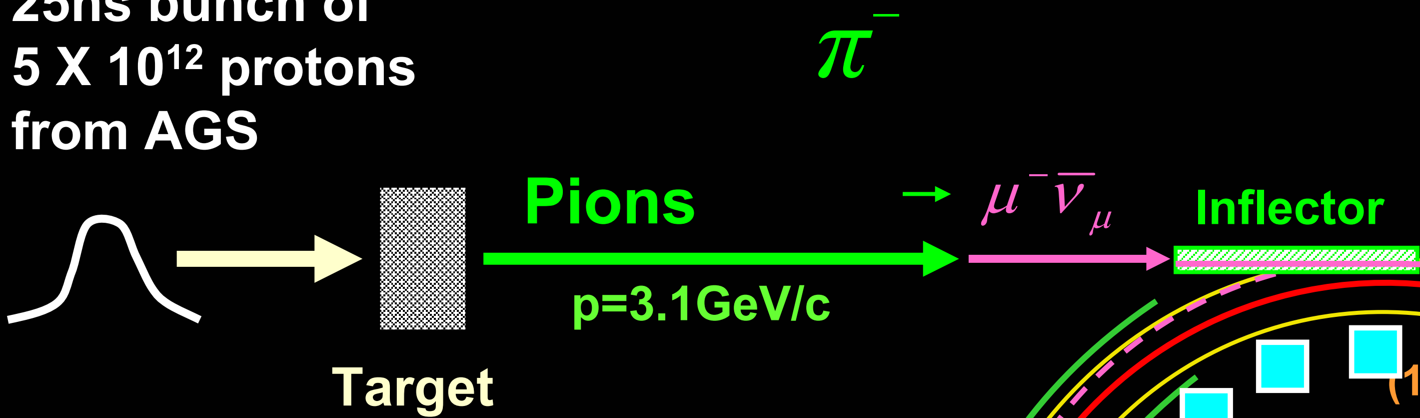
Experimental Technique

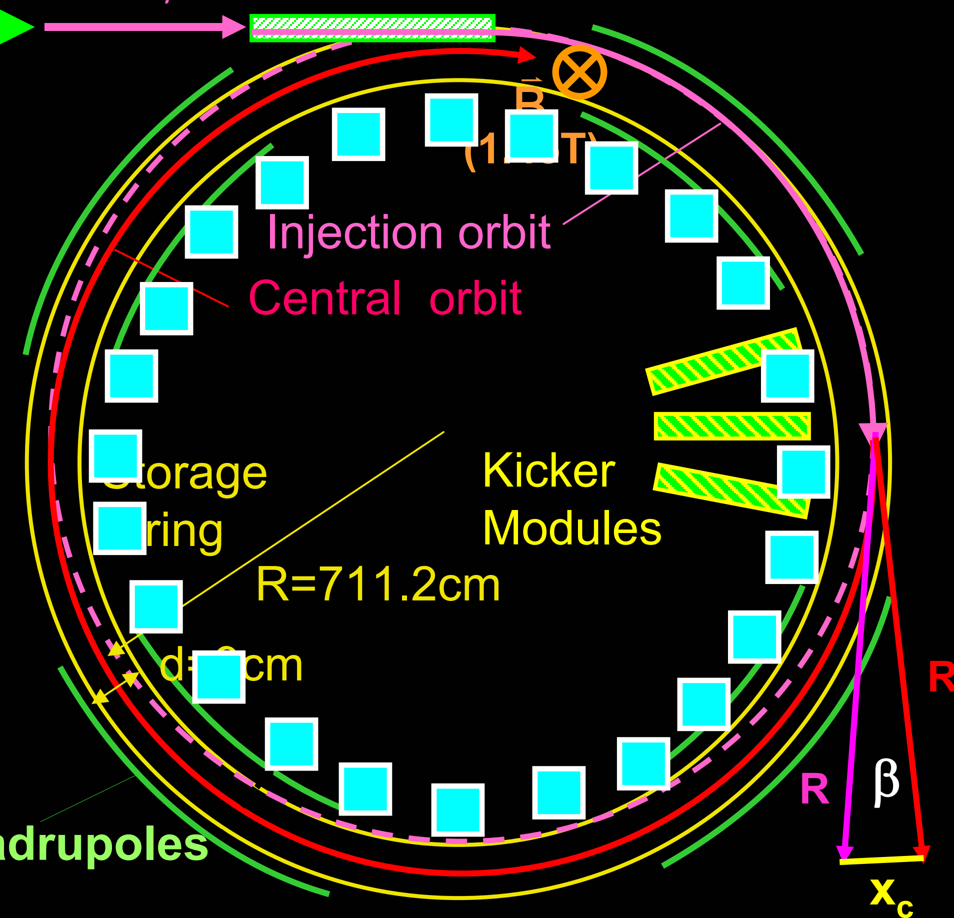
25ns bunch of
5 X 10^{12} protons
from AGS

$x_c \approx 77$ mm

$\beta \approx 10$ mrad

$B \cdot dl \approx 0.1$ Tm



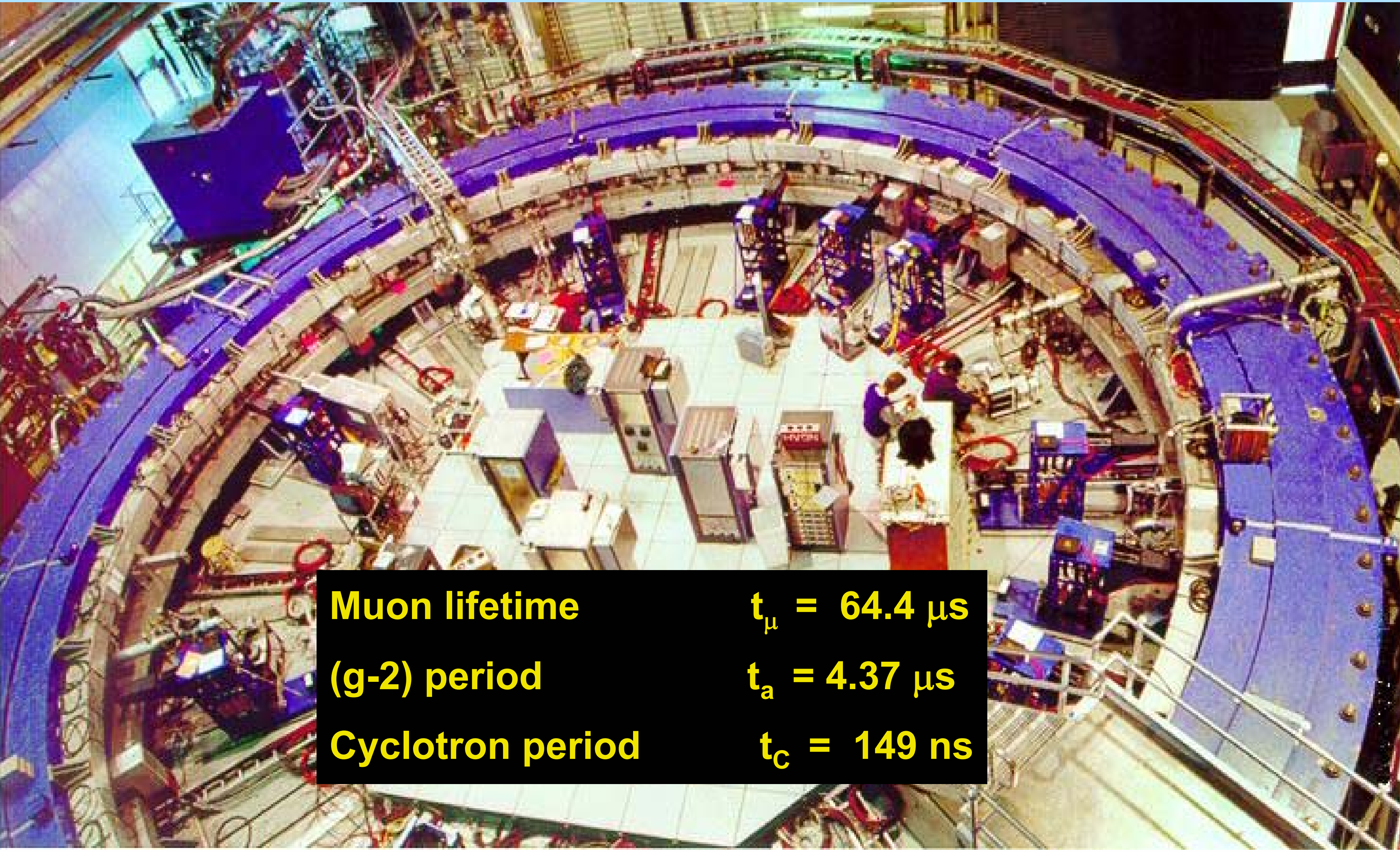


- Muon polarization
- Muon storage ring
- injection & kicking
- focus with Electric Quadrupoles
- 24 electron calorimeters

$$\vec{\omega}_a = -\frac{e}{m} a_\mu \vec{B}$$

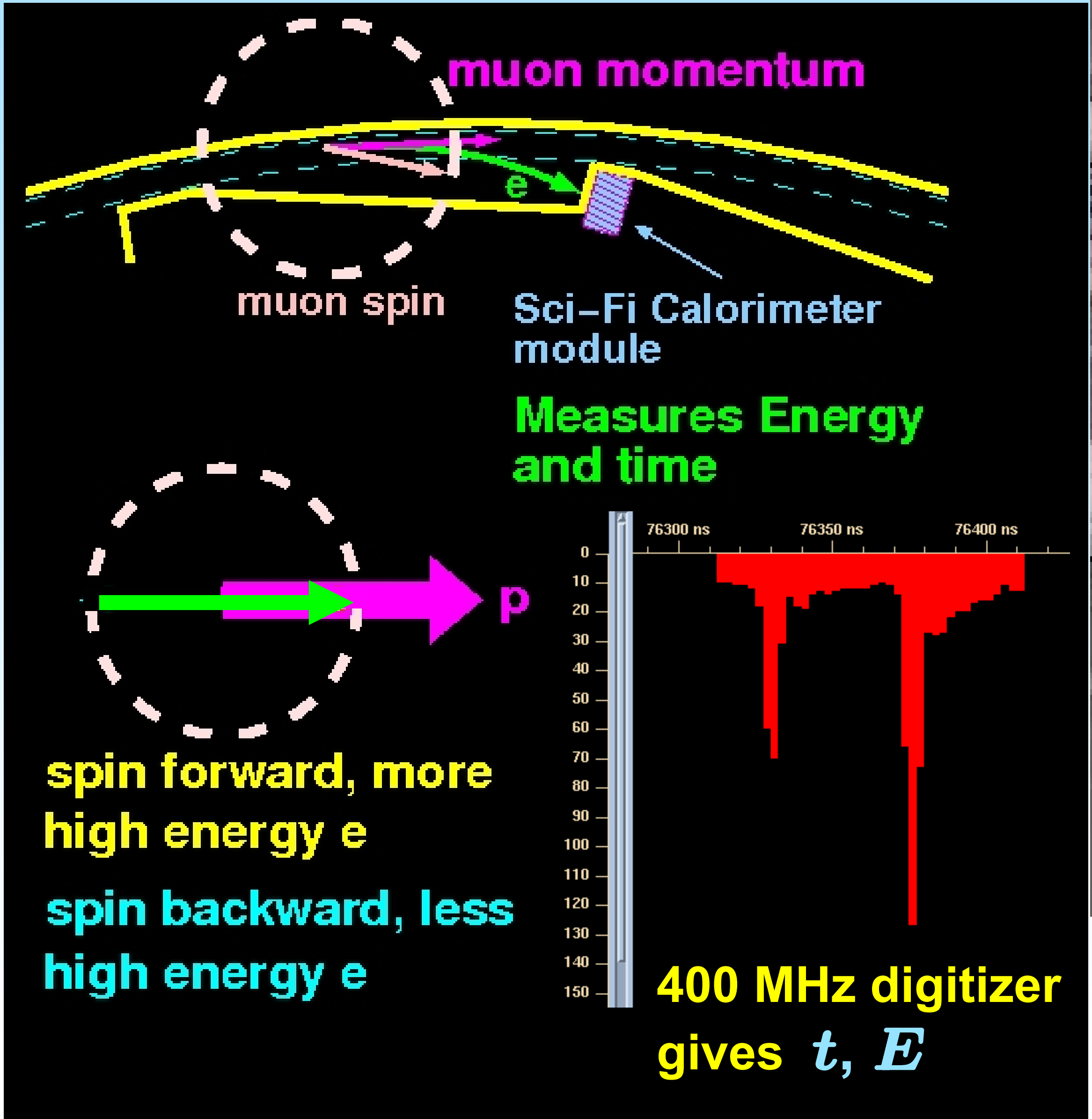
(thanks to Q. Peng)

muon (g-2) storage ring



Muon lifetime	t_μ = 64.4 μs
(g-2) period	t_a = 4.37 μs
Cyclotron period	t_C = 149 ns

To measure ω_a, we used Pb-scintillating fiber calorimeters.

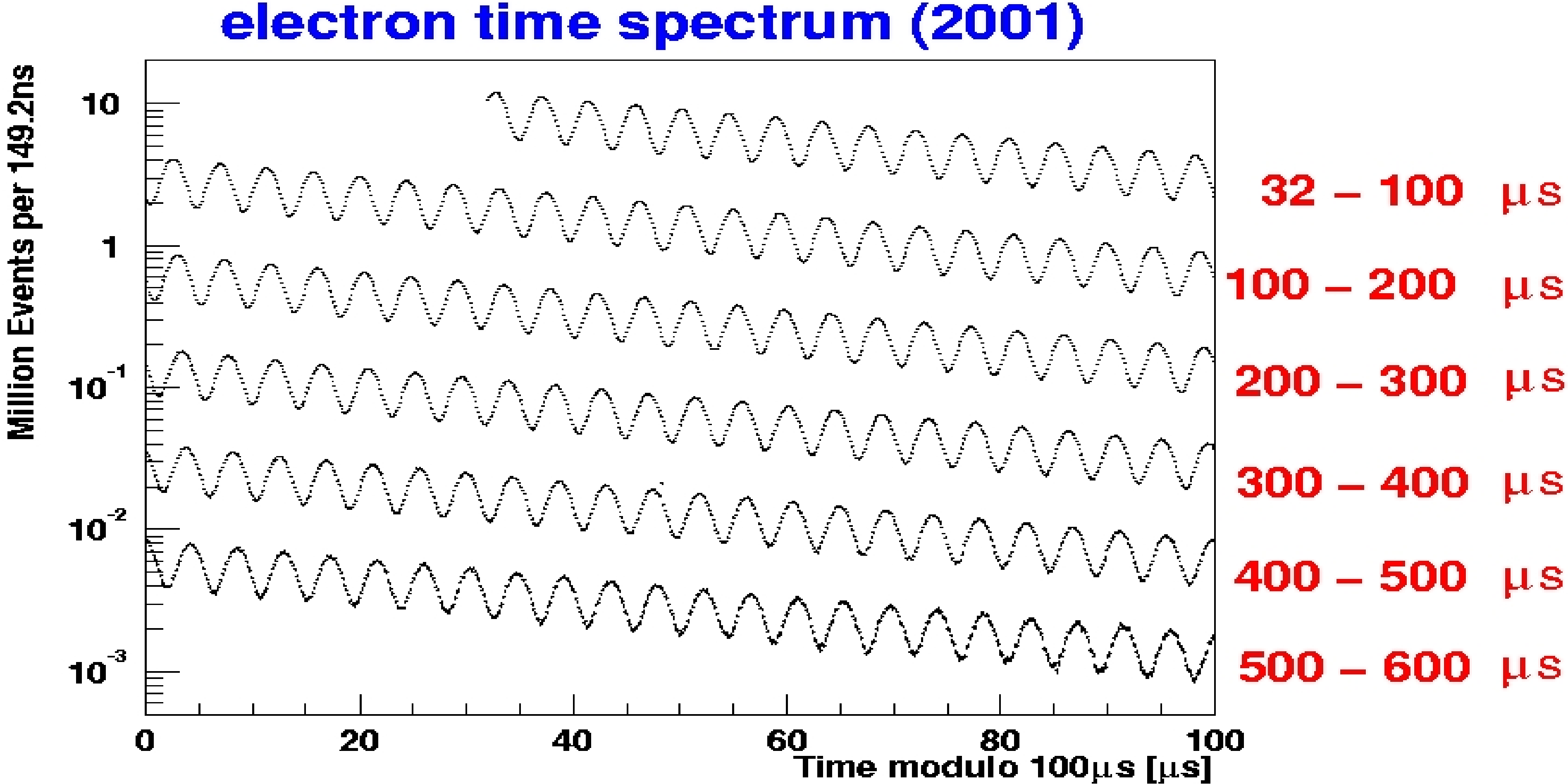


Count number of e^- with $E_e \geq 1.8$ GeV

We count high-energy electrons as a function of time.

$$4 \times 10^9 \quad e,\ E_{e^-} \geq 1.8\ \text{GeV}$$

$$f(t) \simeq N_0 e^{-\lambda t}[1 + A\cos\omega_a t + \phi)]$$



The ± 1 ppm uniformity in the average field is obtained with special shimming tools.

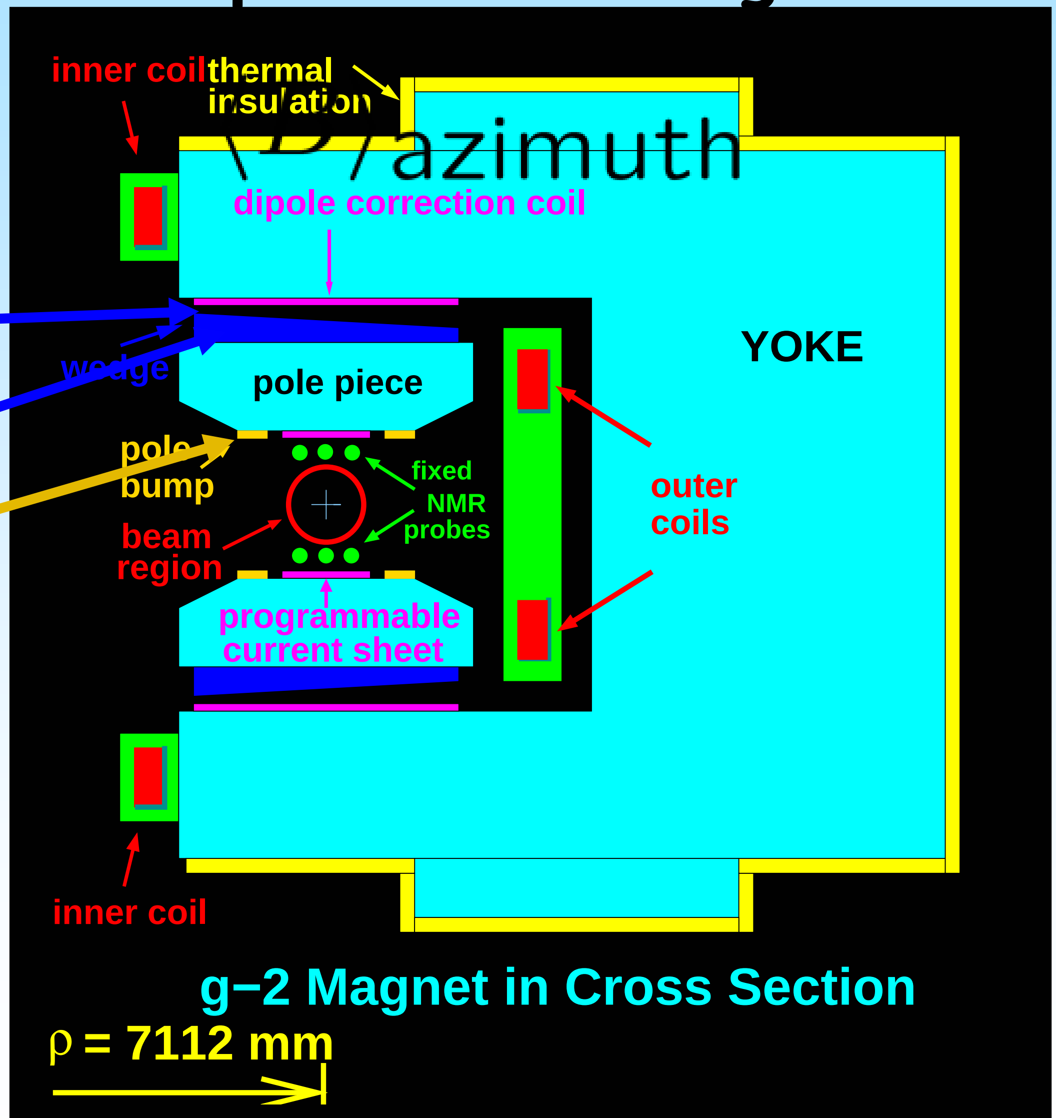
We can shim the

dipole,

quadrupole

sextupole

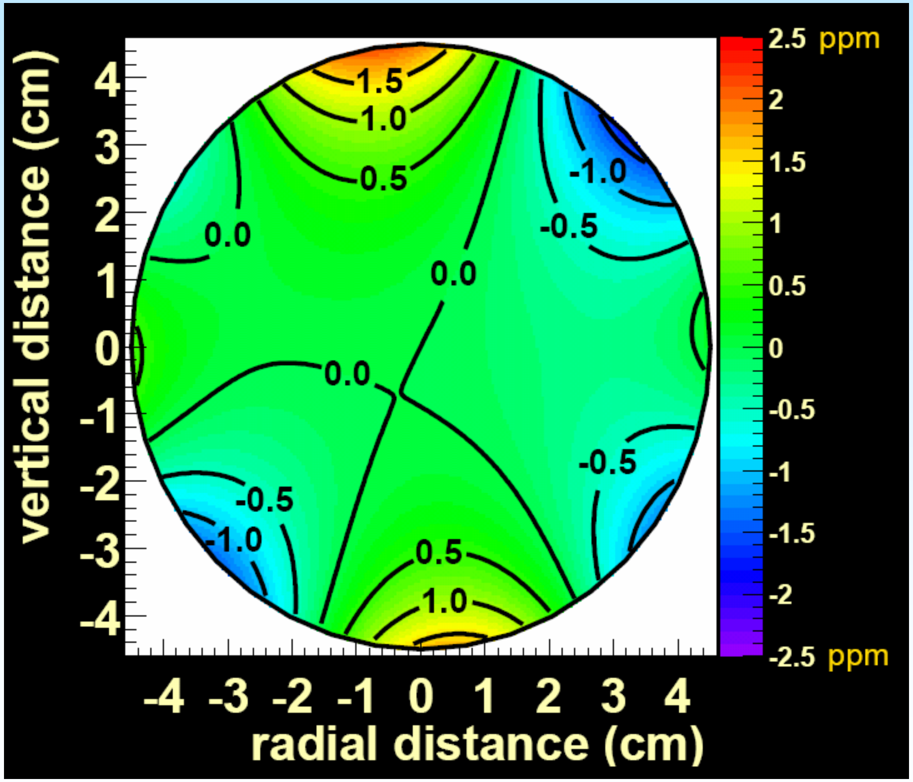
independently



0.5 ppm contours

The ± 1 ppm uniformity in the average field is obtained with special shimming tools.

$$\langle B \rangle_{\text{azimuth}}$$

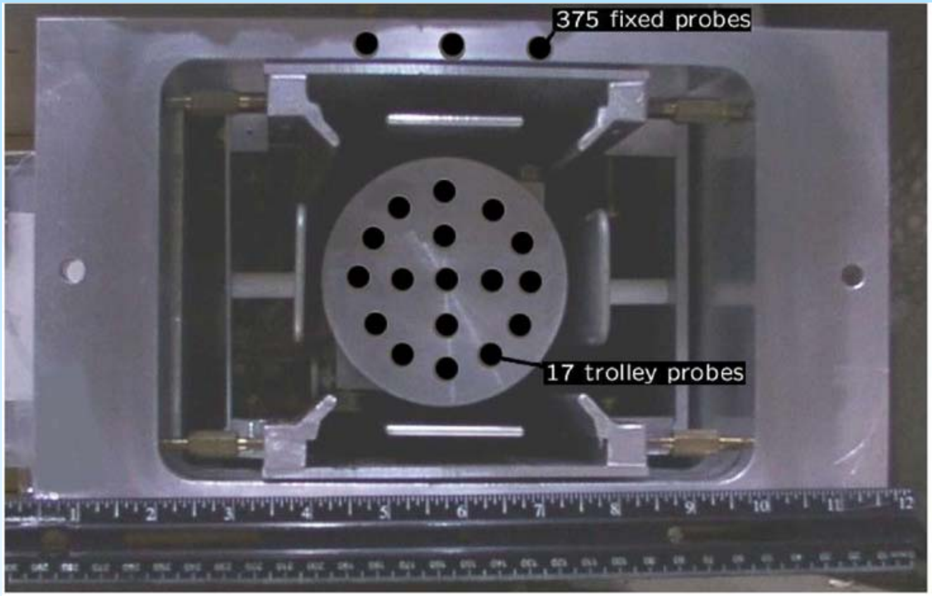


0.5 ppm contours

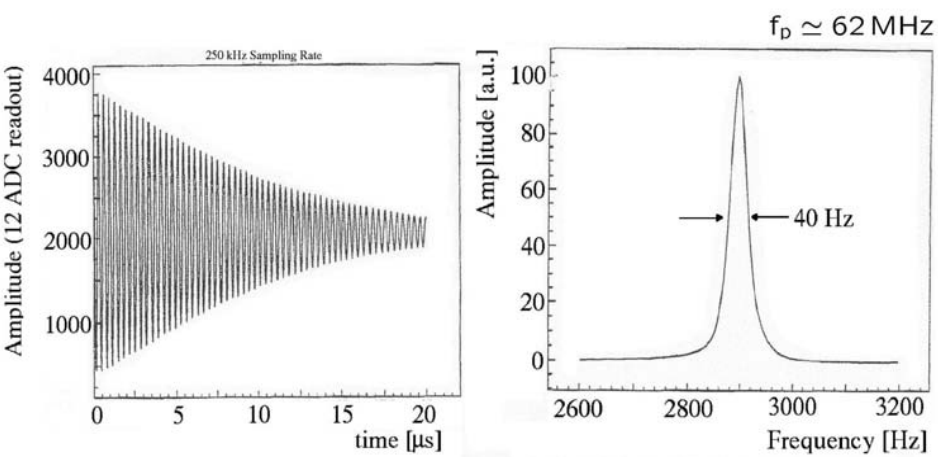
$$\sigma_{\text{syst}} \text{ on } \langle B \rangle_{\mu-\text{dist}} = \pm 0.03 \text{ ppm}$$

BOSTON UNIVERSITY

The magnetic field is measured and controlled using pulsed NMR and the free-induction decay.



Free induction decay signals:



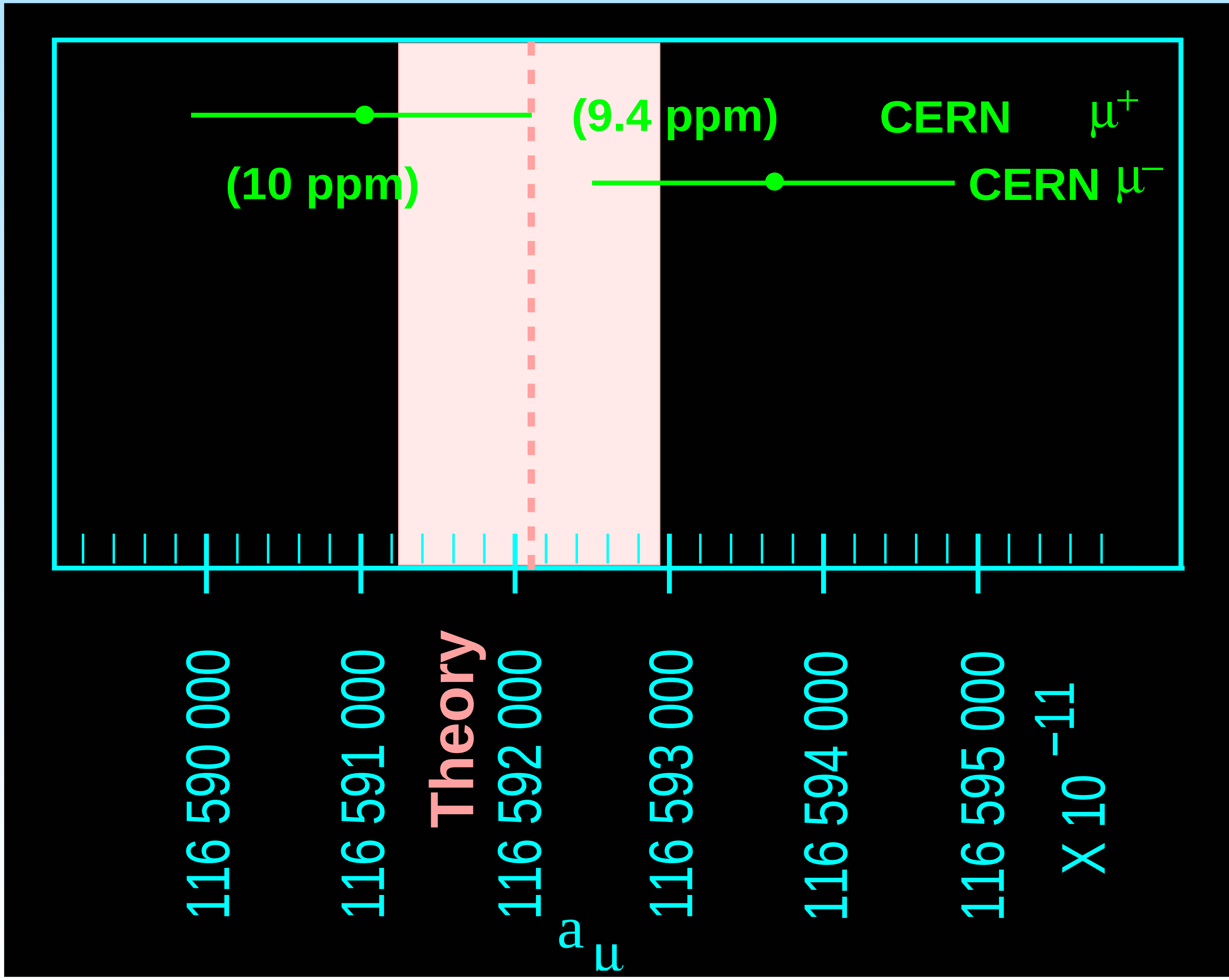
- Calibration to a spherical water sample that ties the field to the Larmor frequency of the free proton ω_p.
- So we measure ω_a and ω_p

$$a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_\mu}{\mu_p} - \frac{\omega_a}{\omega_p}}$$

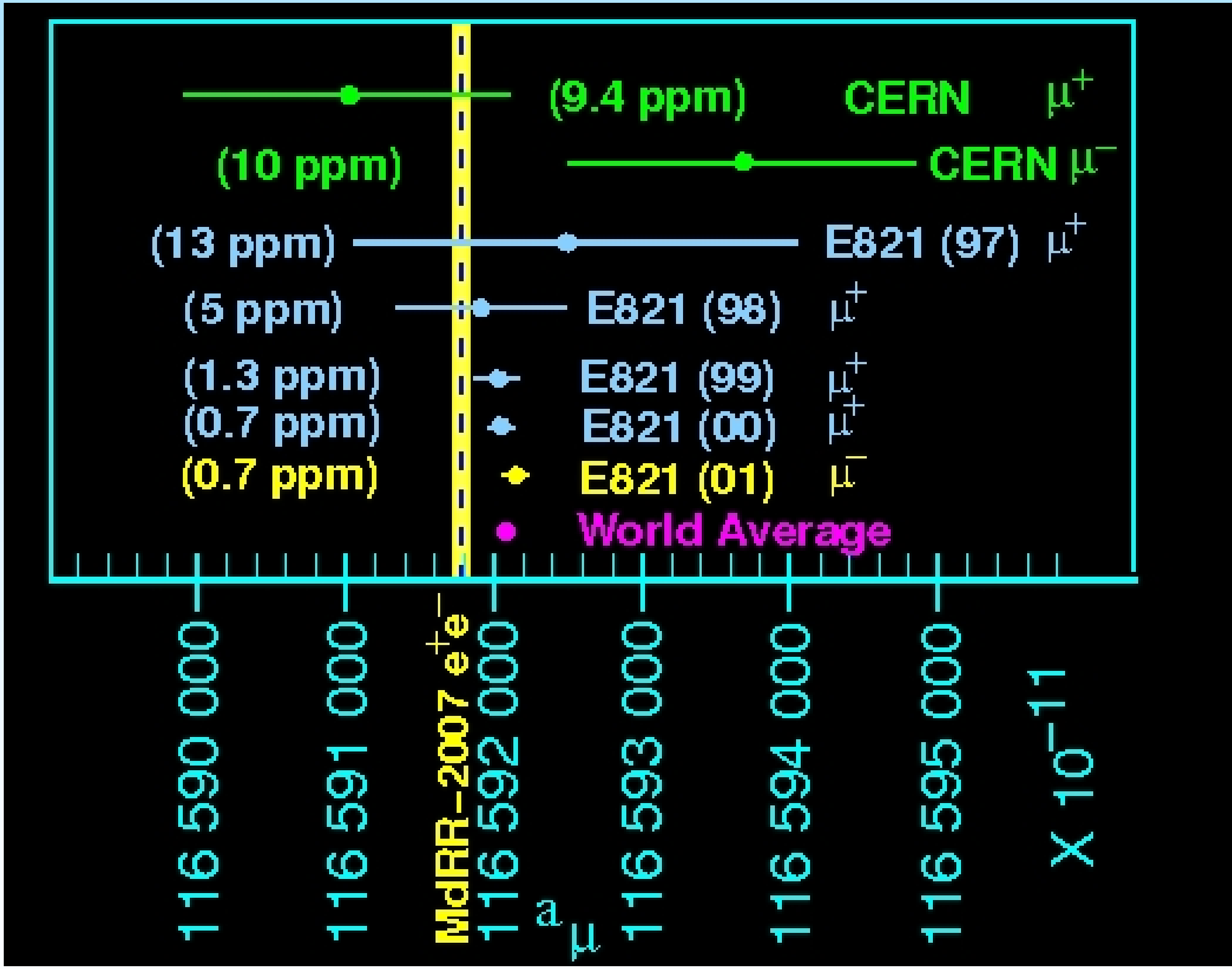
When we started in 1983, theory and experiment were known to about 10 ppm.

Theory uncertainty was ~ 9 ppm

Experimental uncertainty was 7.3 ppm



E821 achieved 0.5 ppm and the e^+e^- based theory is also at the 0.6 ppm level. Difference is 3.7 σ



MdRR=Miller, de Rafael, Roberts, Rep. Prog. Phys. **70** (2007) 795

$$\Delta a_\mu^{(\mathrm{EdR08})} = (29.7 \pm 7.9) \times 10^{-10}$$

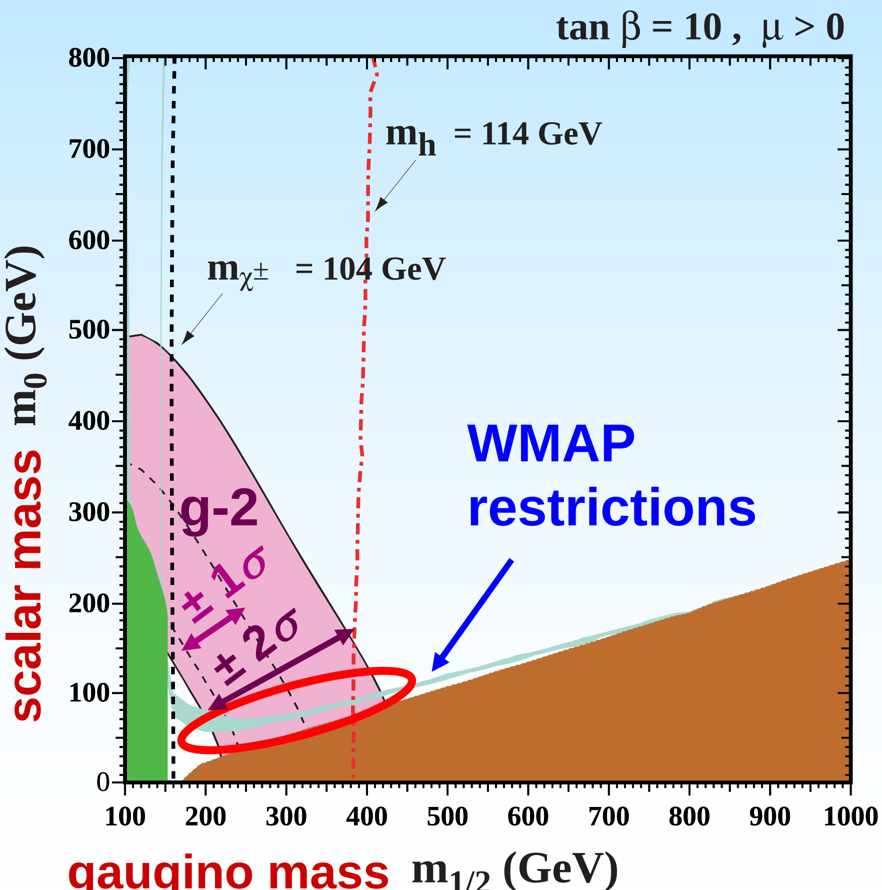
If the electroweak contribution is left out of the standard-model value, we get a 5.1 σ difference.

$$a_\mu^{EW} = 15.4(.1)(.2) \times 10^{-10}$$

$$\Delta(\text{no EW}) = 44.9(8.8) \times 10^{-10}$$

a_μ helps constrain new physics

In a constrained minimal supersymmetric model, $(g-2)_\mu$ provides an independent constraint on the SUSY LSP (lightest supersymmetric partner) being the dark matter candidate.

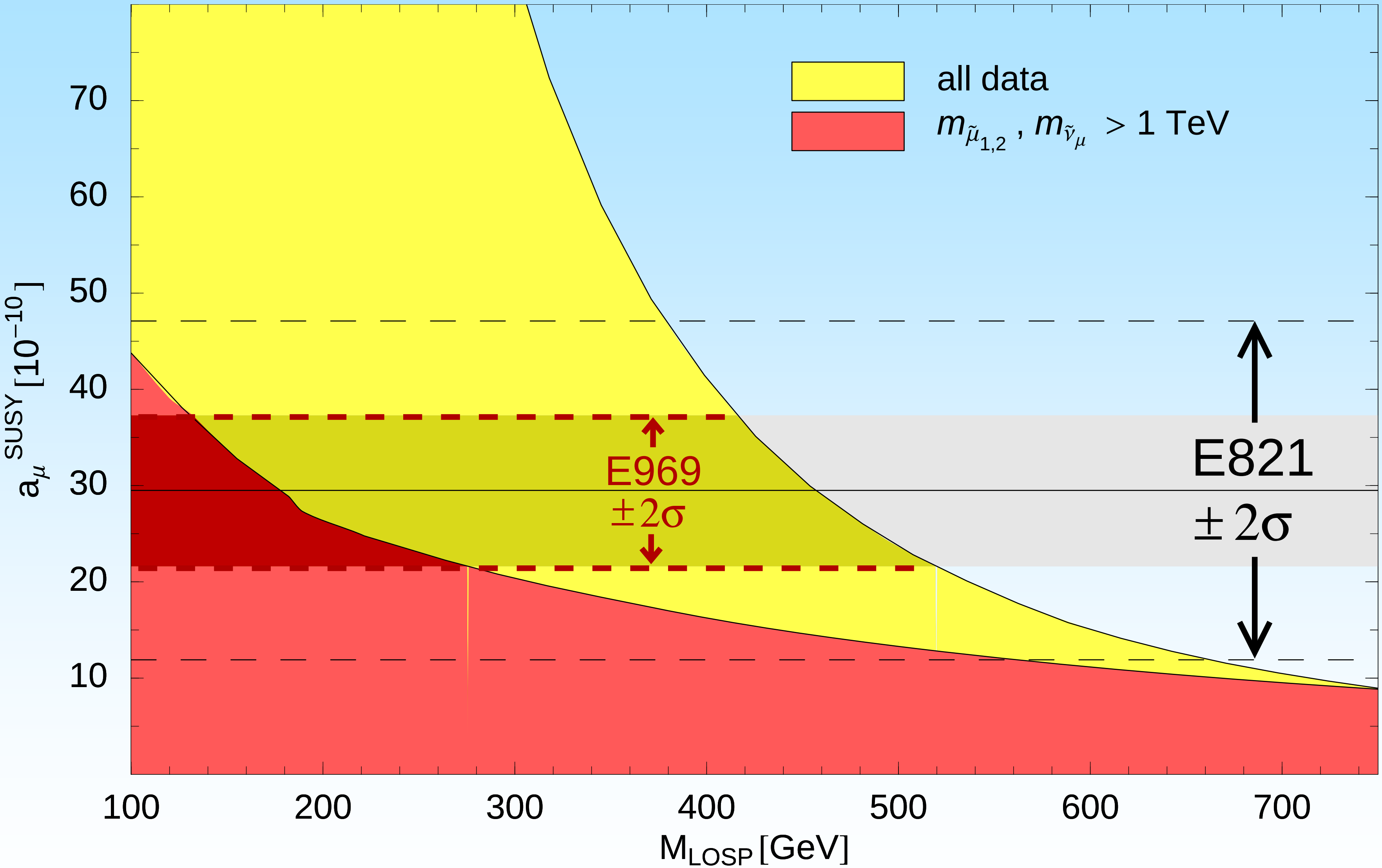


Historically muon (g-2) has played an important role in restricting models of new physics.

It provides constraints that are <u>independent and complementary</u> to high-energy experiments.

CMSSM calculation Following Ellis, Olive, Santoso, Spanos, provided by K. Olive

MSSM scan of M_{LOSP} vrs. a_μ^{SUSY}

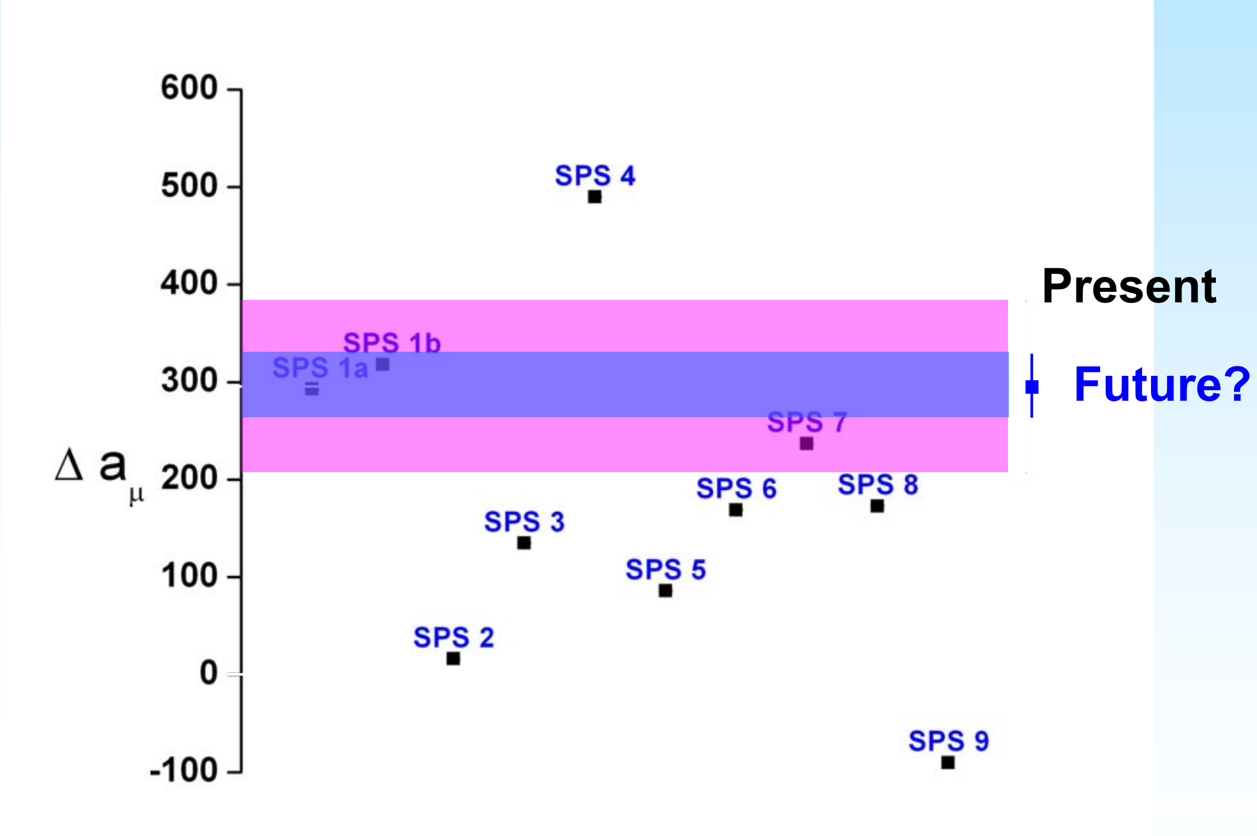


D. Stöckinger, J. Phys. G 34, R45 (2007)

The Snowmass Points and Slopes give reasonable benchmarks to test observables with model predictions

Muon g-2 is a powerful discriminator ...

no matter where the final value lands!

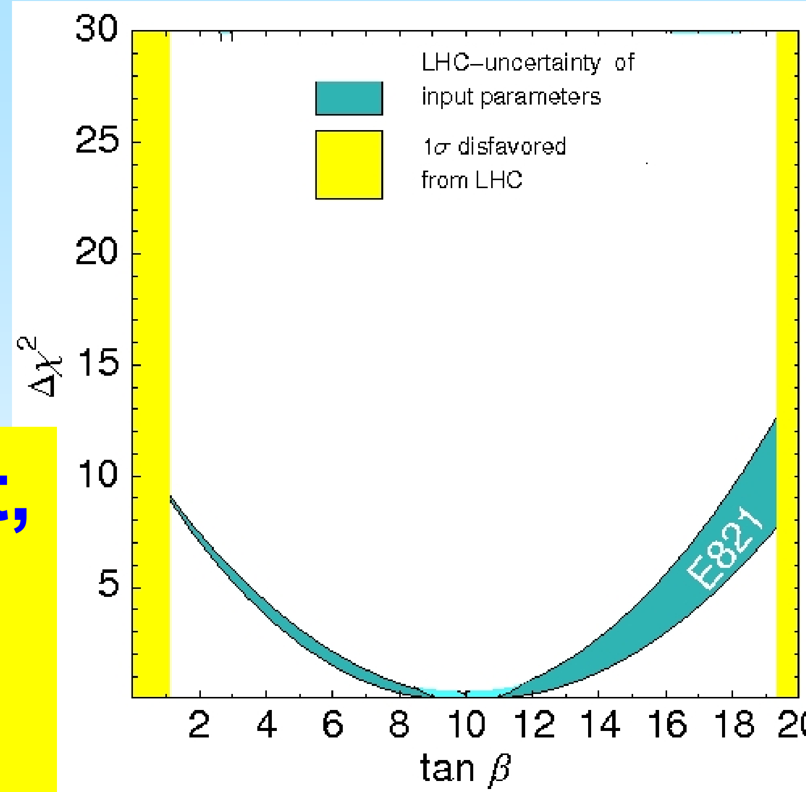


a_μ will help constrain the interpretation of LHC data, e.g. tan β and sgn μ parameter

MSSM reference point SPS1a

With these SUSY parameters, LHC gets tan β of 10.22 ± 9.1.

See: arXiv:0705.4617v1 [hep-ph]



Even with no improvement, a_μ will provide the best value for tan β, and show $\mu > 0$ to $> 3\ \sigma$

$$\Delta a_\mu^{(\text{today})} = (29.5 \pm 8.8) \times 10^{-10}$$

Improved experiment and theory for a_μ is important

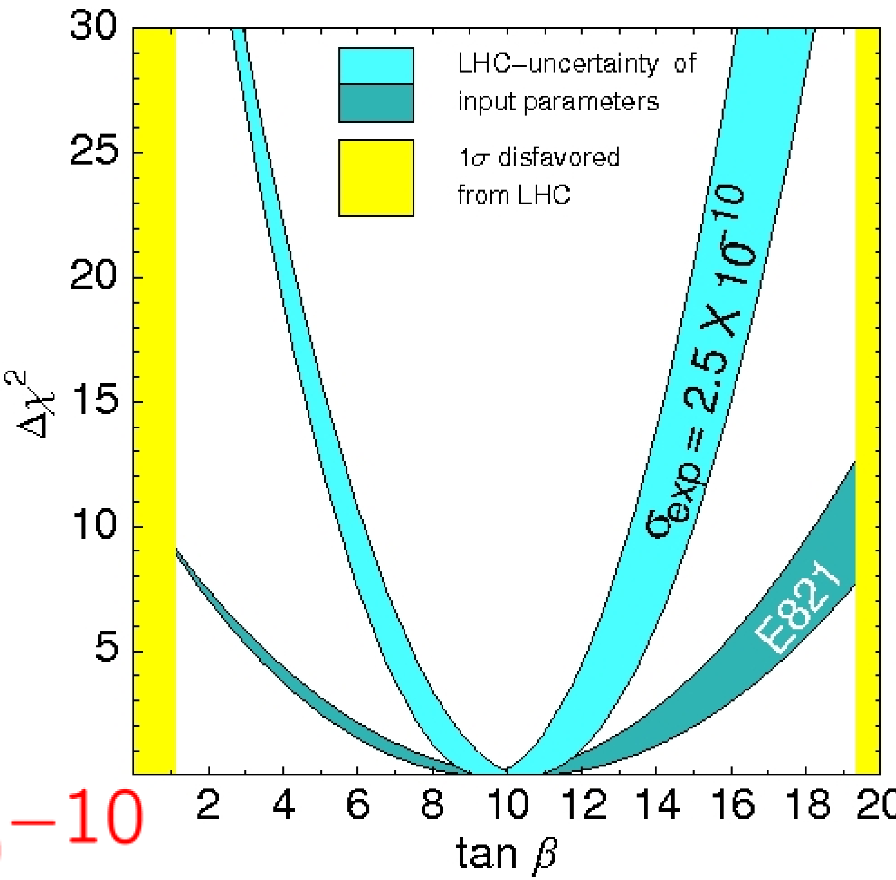
MSSM reference point SPS1a

With these SUSY parameters, LHC gets tan β of 10.22 ± 9.1.

See: arXiv:0705.4617v1 [hep-ph]

$\mu > 0$ by $> 6\,\sigma$

tan β to < 20%



$$\sigma^{E821} \quad (6.3 \to 2.5) \times 10^{-10}$$

$$\sigma^{SM} \quad (6.1 \to 3.0) \times 10^{-10}$$

$$\Delta a_\mu^{\text{(future)}} = (29.5 \pm 3.9) \times 10^{-10}$$

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The search for a Muon Electric Dipole Moment

Electric Dipole Moment: ~~P T~~

$$\mathcal{H} = -\vec{\mu}\cdot\vec{B} - \vec{d}\cdot\vec{E}$$

$\vec{\mu},\ \vec{d} \parallel$ to $\vec{\sigma}$

	$\vec{E}$	$\vec{B}$	$\vec{\mu}$ or $\vec{d}$
P	$-$	$+$	$+$
C	$-$	$-$	$-$
T	$+$	$-$	$-$

Transformation Properties

If CPT is valid, an EDM would imply non-standard model ~~*CP*~~.

Purcell and Ramsey: EDM would violate Parity
Proposed to search for an EDM of the neutron

"raises directly the question of parity."

On the Possibility of Electric Dipole Moments for Elementary Particles and Nuclei

E. M. PURCELL AND N. F. RAMSEY
Department of Physics, Harvard University, Cambridge, Massachusetts
April 27, 1950

IT is generally assumed on the basis of some suggestive theoretical symmetry arguments[1] that nuclei and elementary particles can have no electric dipole moments. It is the purpose of this note to point out that although these theoretical arguments are valid when applied to molecular and atomic moments whose electromagnetic origin is well understood, their extension to nuclei and elementary particles rests on assumptions not yet tested.

One form of the argument against the possibility of an electric dipole moment of a nucleon or similar particle is that the dipole's orientation must be completely specified by the orientation of the angular momentum which, however, is an axial vector specifying a direction of circulation, not a direction of displacement as would be required to obtain an electric dipole moment from electrical charges. On the other hand, if the nucleon should spend part of its time asymmetrically dissociated into opposite magnetic poles of the type that Dirac[2] has shown to be theoretically possible, a circulation of these magnetic poles could give rise to an electric dipole moment. To forestall a possible objection we may remark that this electric dipole would be a polar vector, being the product of the angular momentum (an axial vector) and the magnetic pole strength, which is a pseudoscalar in conformity with the usual convention that electric charge is a simple scalar.

The argument against electric dipoles, in another form, raises directly the question of parity. A nucleon with an electric dipole moment would show an asymmetry between left- and right-handed coordinate systems; in one system the dipole moment

The authors wish to thank Mr. Smith for suggesting an important correction to our original calculation on the neutron.

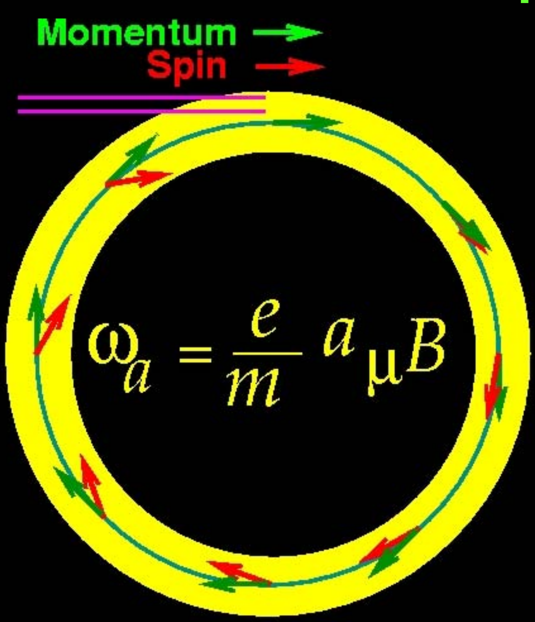
Phys. Rev. 78 (1950)

Spin Frequencies: μ in B field with MDM & EDM

$$\omega_C = \frac{eB}{mc\gamma} \qquad \omega_S = \frac{geB}{2mc} + (1-\gamma)\frac{eB}{\gamma mc}$$

$$\omega_a = \omega_S - \omega_C = \left(\frac{g-2}{2}\right)\frac{eB}{mc}$$

spin difference frequency = ω_s - ω_c



The EDM
spin to p
of plane.

The highest energy decay $e^{\pm}$ are along the muon spin direction

10^{-14} e cm

Spin Frequencies: μ in B field with MDM & EDM

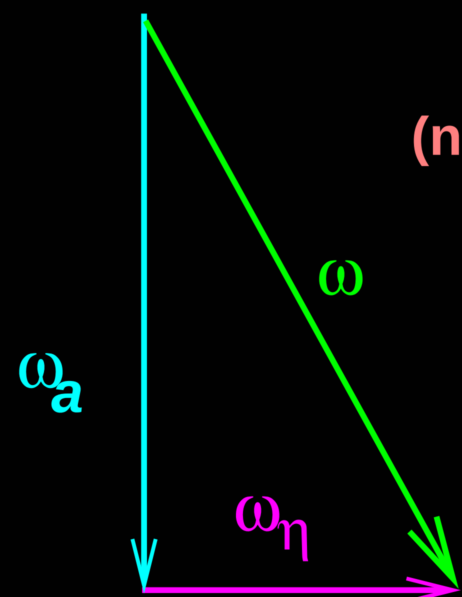
$$\vec{\omega} = -\frac{e}{m}\left[\underbrace{a_\mu \vec{B}}_{\omega_a} - \cancelto{0}{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}}\right] + \underbrace{\frac{e}{m}\left[\frac{\eta}{2}\left(\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}\right)\right]}_{\omega_\eta}$$

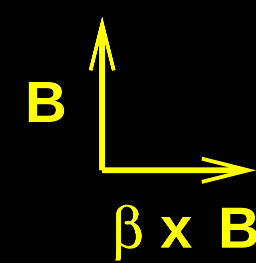
$$\gamma_{\text{magic}} = 29.3$$

The motional E - field, β X B, is (~GV/m).

(not to scale)

The EDM causes the spin to precess out of plane.





Spin Frequencies: μ in B field with MDM & EDM

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} - \cancelto{0}{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)} \frac{\vec{\beta} \times \vec{E}}{c}\right]$$

$\gamma_{\text{magic}} = 29.3$

$$+ \quad \omega_a \quad \frac{e}{m}\left[\frac{\eta}{2}\left(\frac{\vec{E}}{c} + \vec{\beta} \times \vec{B}\right)\right]$$

ω_η

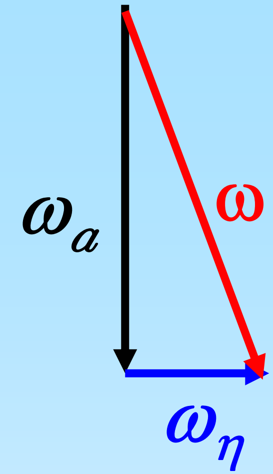
The motional E - field,
β X B, is (~GV/m).

$$d_\mu = \frac{\eta}{2}\left(\frac{e\hbar}{2mc}\right) \simeq \eta \times 4.7 \times 10^{-14} \ \ e \text{ cm}$$

and

$$a_\mu = \left(\frac{g-2}{2}\right)$$

Total frequency $$\omega = \sqrt{\omega_a^2 + \omega_\eta^2}$$



$$\vec{d} = \eta \frac{q}{2mc}\vec{s} \qquad \vec{\mu} = g\frac{q}{2m}\vec{s}$$

Plane of the spin precession tipped by the angle δ

$$\delta = \tan^{-1}\left(\frac{\eta\beta}{2a_\mu}\right)$$

$$\omega = \omega_a\sqrt{1+\tan^2\delta}$$

Number above (+) and below (-) the midplane will vary as:

$$N^{\pm}(t) \propto [1 \mp A_{EDM}\sin(\omega t + \phi) + A_\mu \cos(\omega t + \phi)]$$

since $\omega^2 = \omega_a{}^2 + \omega_\eta{}^2$, the $\Delta_{a\mu}$ could be an EDM

what value EDM would this correspond to?

$$|d_\mu| = 1.8(.5) \times 10^{-19}\ e\mathrm{cm}$$

obviously this would be exciting.

See: Feng, et al., Nucl. Phys. **B 613** (2001) 366

The CERN limits was: $< 1.05 \times 10^{-18}$ 95% CL

SM value $< 10^{-38}$

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E821 looked for this vertical oscillation in 3 ways

- 5-piece vertical hododscope in front of the calorimeters called an FSD
  - 14 detector stations
- Much finer x-y hododscope called a PSD
  - 5 detector stations
- Traceback straw tube array
  - 1 station
- No significant oscillation was found

$$d_\mu < 2 \times 10^{-19} \ 95\% \ \mathrm{CL}^*$$

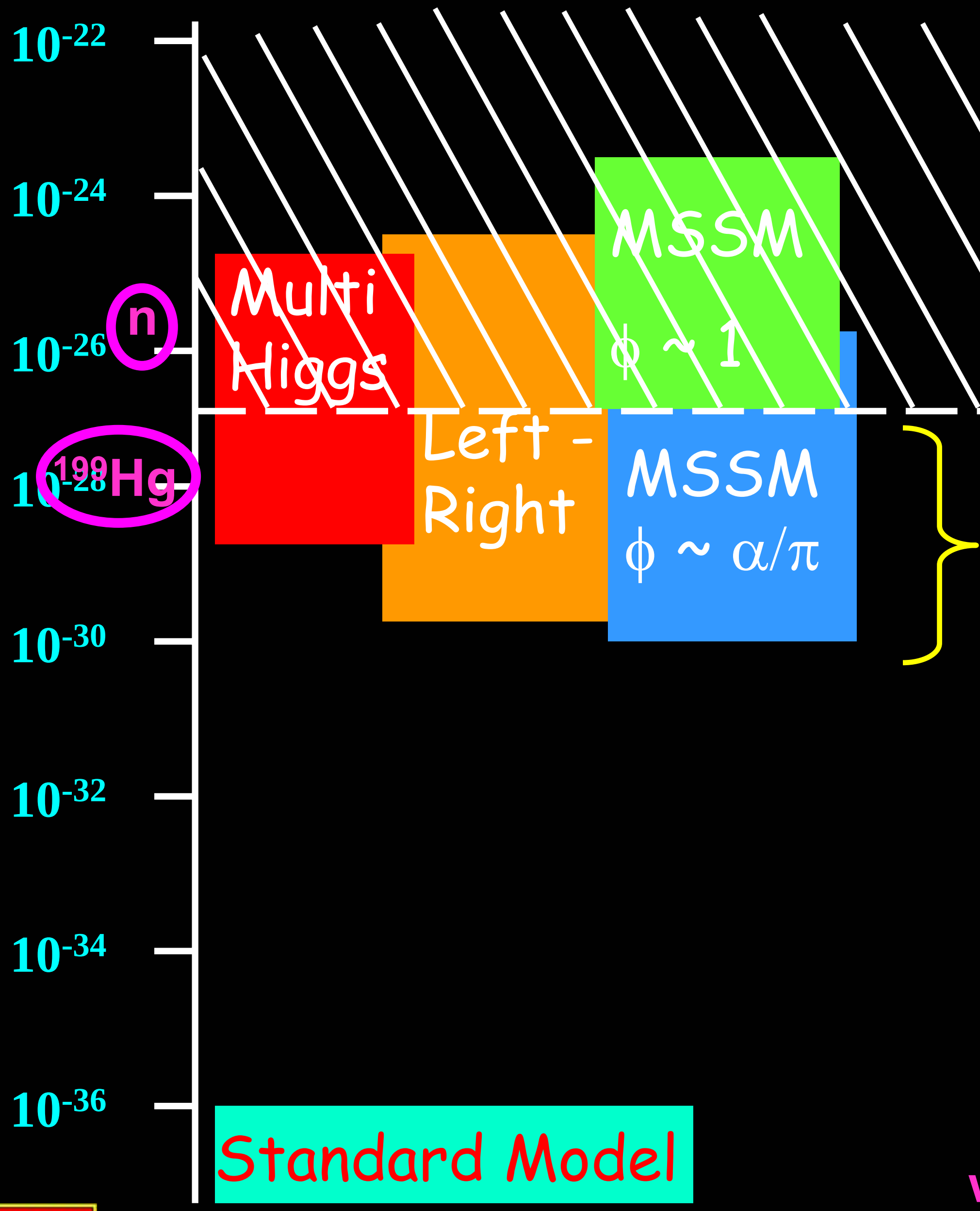
- The observed Δa_μ is not from an EDM at the 2.2 σ level

***Coming soon to a preprint server near you**

The present EDM limits are orders of magnitude from the standard-model value

Particle	*Present EDM limit* (e-cm)	*SM value* (e-cm)
n	2.9×10^{-26}	$10^{-32} - 10^{-31}$
e^-	$\sim 1.6 \times 10^{-27}$	$< 10^{-41}$
μ	2×10^{-19} * (E821)	$< 10^{-38}$
future μ exp	10^{-24} to 10^{-25}	

*final and will be submitted to PRD soon e EDM (e.cm)



Excluded region
(Tl atomic beam)
Commins (2002)
$d_e < 1.6 \times 10^{-27}$ e.cm

E. Hinds' e-EDM experiment
at Imperial College
with YbF molecules
is starting
to explore this region

with thanks to Ed Hinds

Dedicated EDM Experiment $\rightarrow 10^{-24} - 10^{-25}$

$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} - \cancelto{0}{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}}\right]$$

Use a radial E-field to turn off the ω_a precession

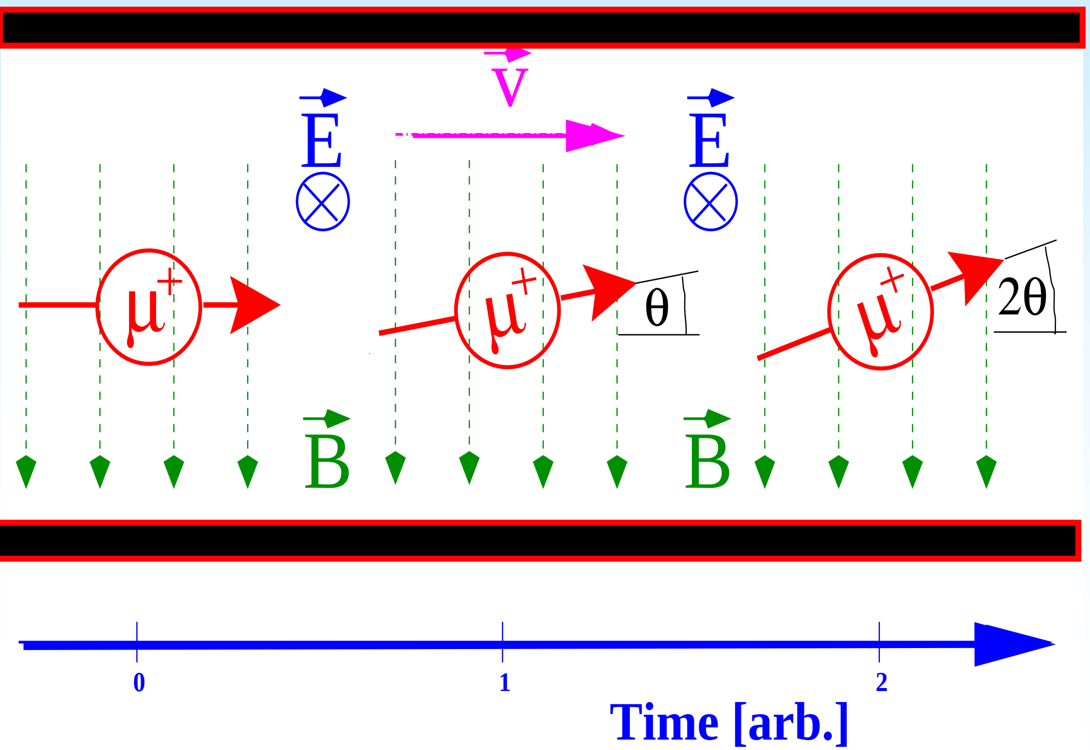
$$+ \quad \frac{e}{m}\left[\frac{\eta}{2}\left(\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}\right)\right]$$

With $\omega_a = 0$, the EDM causes the spin to steadily precess out of the plane.

ω_η

"Frozen spin" technique to measure EDM

- Turn off the (g-2) precession with radial $\boldsymbol{E}$
- Up-Down detectors measure EDM asymmetry
- Look for an up-down asymmetry building up with time
- Side detectors measure (g-2) precession
  - To prove the spin is frozen



$$a = \frac{\text{up} - \text{down}}{\text{up} + \text{down}}$$

PSI suggestion:

A. Adelmann[1], K. Kirch[1], C.J.G. Onderwater[2], T. Schietinger[1], A. Streun[1]

Search for the muon electric dipole moment using a compact storage ring

A. Adelmann and K. Kirch*

Paul Scherrer Institut (PSI), CH-5232 Villigen PSI, Switzerland

hep-ex/0606034

(Dated: June 15, 2006)

The recently proposed *'New Method of Measuring Electric Dipole Moments in Storage Rings'* [1, 2, 3] could be used in an experiment using the existing muon beam μE1 at PSI. A high muon polarization and a rather low momentum of $p_\mu \sim 125$ MeV/c allow for an almost table-top storage ring and increase the intrinsic sensitivity and, thus, partially compensate for limitations due to lower event statistics. A measurement of the muon electric dipole moment with a sensitivity of better than $d_\mu \sim 5 \times 10^{-23}$ e·cm within one year of data taking appears feasible.

$B = 1$ T

$p_\mu = 125$ MeV/c

$\beta_\mu = 0.77, \quad \gamma_\mu = 1.57$

$P \approx 0.9$

$E = 0.64$ MV/m

$R = 0.35$ m

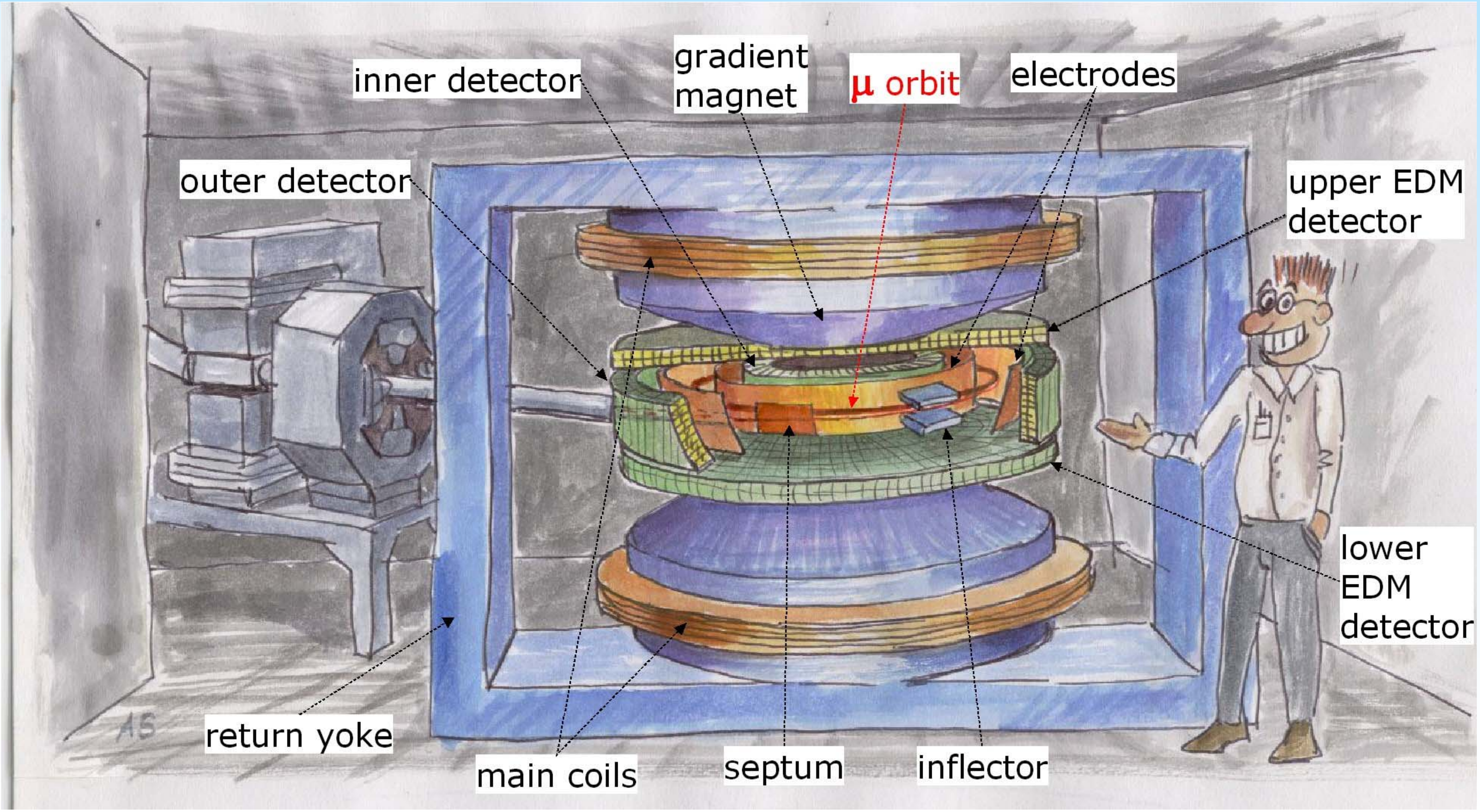
$$\sigma_{d_\mu} \simeq \frac{1.1 \times 10^{-16}}{\sqrt{N}}$$

In 1 year of running @ PSI

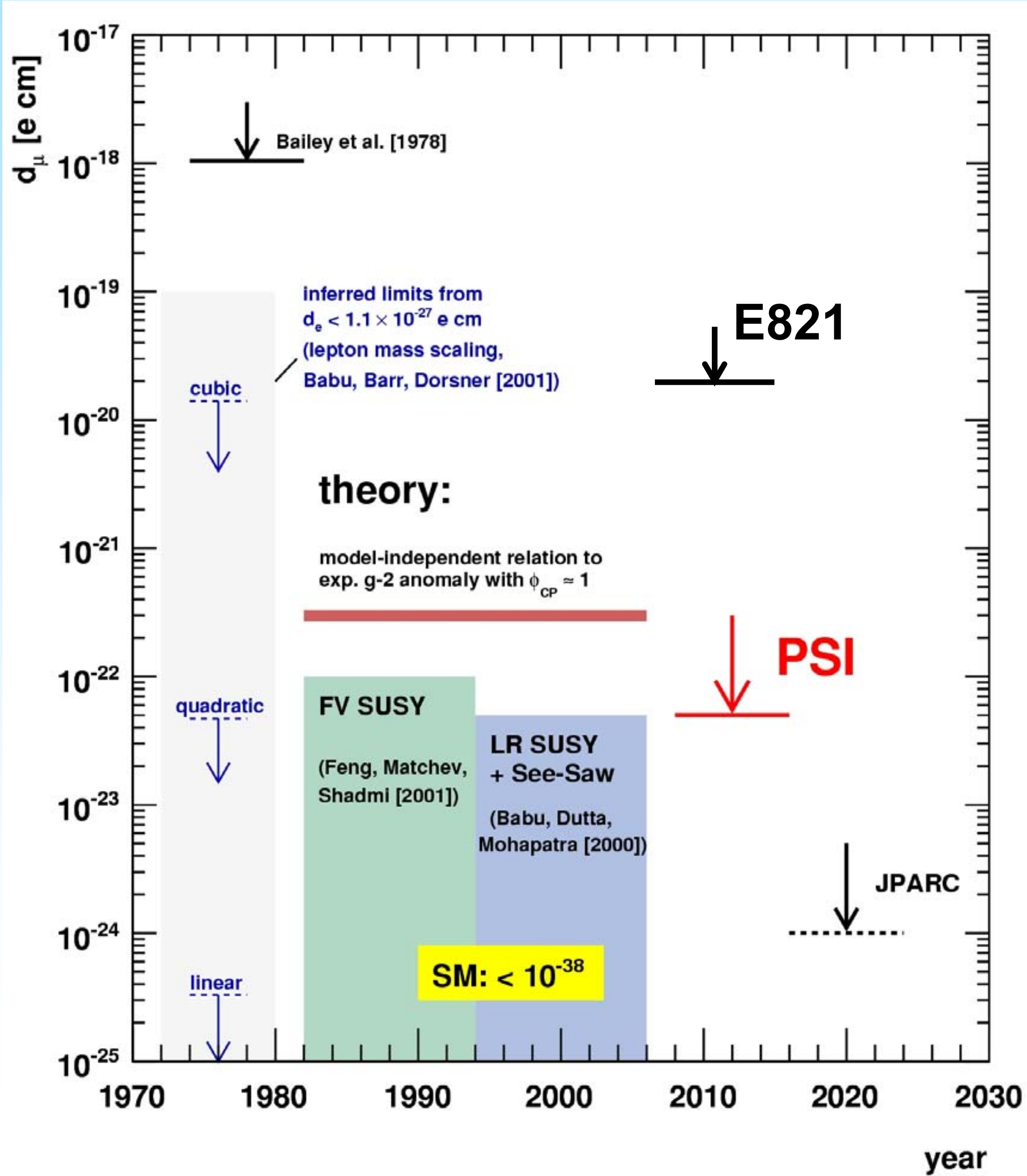
$$\sigma_{d_\mu} \simeq 5 \times 10^{-23} \mathrm{e \cdot cm}$$

The storage ring is modest in size

Injection studies look promising.



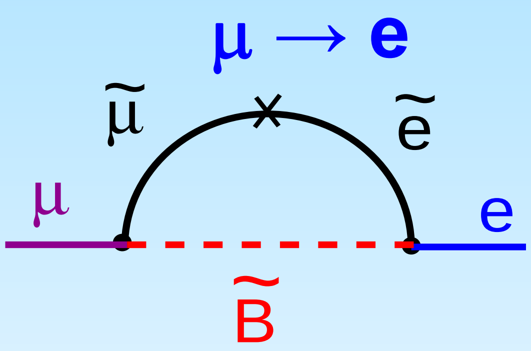
Muon EDM Limits: CERN3 and E821

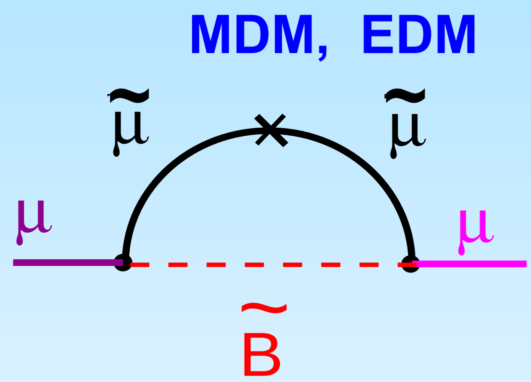


E821: G. Bennett, et al., (Muon g-2 collaboration) to be submitted to PRD 2008

Connection between MDM, EDM and the lepton flavor violating transition moment $\mu \to e$

SUSY $\Rightarrow$ slepton mixing





$$\begin{pmatrix} m^2_{\tilde{e}\tilde{e}} & \Delta m^2_{\tilde{e}\tilde{\mu}} & \Delta m^2_{\tilde{e}\tilde{\tau}} \\ \Delta m^2_{\tilde{\mu}\tilde{e}} & m^2_{\tilde{\mu}\tilde{\mu}} & \Delta m^2_{\tilde{\mu}\tilde{\tau}} \\ \Delta m^2_{\tilde{\tau}\tilde{e}} & \Delta m^2_{\tilde{\tau}\tilde{\mu}} & m^2_{\tilde{\tau}\tilde{\tau}} \end{pmatrix}$$

An Intermezzo:

The search for CPT and Lorentz violation in ω_a

PRL 100, 091602 (2008) PHYSICAL REVIEW LETTERS week ending 7 MARCH 2008

Search for Lorentz and *CPT* Violation Effects in Muon Spin Precession

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(Muon $g - 2$ Collaboration)

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(Received 29 September 2007; published 4 March 2008)

What we measure that could show CPT/Lorentz violation

$$\vec{\omega}_a = -\frac{e}{m}\left[a_\mu \vec{B} - \cancelto{0}{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}}\right]$$

$\omega_a = \omega_S - \omega_C$; where ω_C is unaffected by ~~CPT/Lorentz~~ to lowest order.

- BUT $\quad \omega_a = \omega_a(B) \Rightarrow \omega_a(\omega_p)$
- Instead we have to use $\quad \mathcal{R} = \dfrac{\omega_a}{\omega_p}$

CPT/Lorentz violation in the Lagrangian*

$$\mathcal{L}' = -a_\kappa \bar{\psi}\gamma^\kappa \psi - b_\kappa \bar{\psi}\gamma_5\gamma^\kappa \psi - \frac{1}{2}H_{\kappa\lambda}\bar{\psi}\sigma^{\kappa\lambda}\psi$$
$$+\frac{1}{2}ic_{\kappa\lambda}\bar{\psi}\gamma^\kappa \overleftrightarrow{D}^\lambda \psi + \frac{1}{2}id_{\kappa\lambda}\bar{\psi}\gamma_5\gamma^\kappa \overleftrightarrow{D}^\lambda \psi$$

- a_κ, b_κ are CPT odd, others CPT even
- All terms violate Lorentz invariance
- In lowest-order, a_μ is insensitive to violating terms

- Two tests of CPT/Lorentz violation:
  - Difference between $\mathcal{R}(\mu^+)$ and $\mathcal{R}(\mu^-)$
  - Sidereal time variation in ω_a

*Bluhm, Kostelecký, Lane, PRL 84,1098 (2000)

Difference between ω_a for μ^+ and μ^-

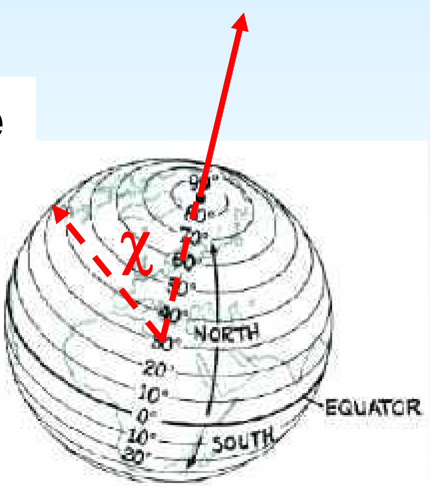
$$\mathcal{L}' = -a_\kappa \bar{\psi}\gamma^\kappa\psi - b_\kappa \bar{\psi}\gamma_5\gamma^\kappa\psi - \frac{1}{2}H_{\kappa\lambda}\bar{\psi}\sigma^{\kappa\lambda}\psi$$

$$+\frac{1}{2}ic_{\kappa\lambda}\bar{\psi}\gamma^\kappa \overleftrightarrow{D^\lambda}\,\psi + \frac{1}{2}id_{\kappa\lambda}\bar{\psi}\gamma_5\gamma^\kappa \overleftrightarrow{D^\lambda}\,\psi$$

$$\Delta\omega_a \equiv \langle\omega_a^{\mu^+}\rangle - \langle\omega_a^{\mu^-}\rangle = \frac{4b_Z}{\gamma}\cos\chi$$

To compare frequencies, in the experiment we must use

$$\mathcal{R} = \frac{\omega_a}{\omega_p}$$

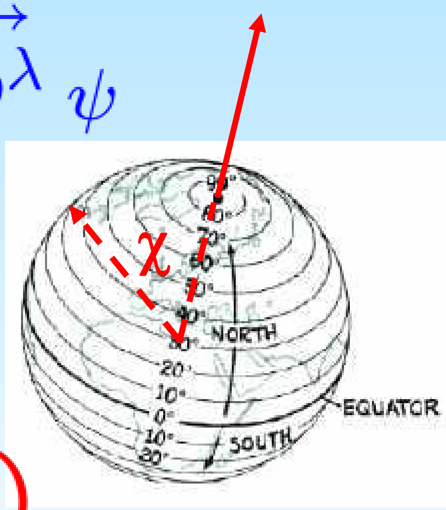


Separate studies show that any variation in ω_p is much less that our limits for ω_a. T_{sidereal} **= 86164.09 s**

T_{solar} **= 86400 s**

For two measurements with different colatitudes and ω_p:

$$\mathcal{L}' = -a_\kappa \bar{\psi}\gamma^\kappa\psi - b_\kappa\bar{\psi}\gamma_5\gamma^\kappa\psi - \frac{1}{2}H_{\kappa\lambda}\bar{\psi}\sigma^{\kappa\lambda}\psi + \frac{1}{2}ic_{\kappa\lambda}\bar{\psi}\gamma^\kappa \overleftrightarrow{D}^\lambda \psi + \frac{1}{2}id_{\kappa\lambda}\bar{\psi}\gamma_5\gamma^\kappa \overleftrightarrow{D}^\lambda \psi$$



$$\Delta\mathcal{R} = \frac{2b_Z}{\gamma}\left(\frac{\cos\chi_1}{\omega_{p1}} + \frac{\cos\chi_2}{\omega_{p2}}\right) + 2(m_\mu d_{Z0} + H_{XY})\left(\frac{\cos\chi_1}{\omega_{p1}} - \frac{\cos\chi_2}{\omega_{p2}}\right)$$

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For the difference, we find

$$\Delta\mathcal{R} = -(3.6 \pm 3.7) \times 10^{-9}$$

Bennett, et al., Phys. Rev. D73, 072003-1

$$b_Z = -(8.7 \pm 8.9) \times 10^{-24}\ \text{GeV}$$

$$r_{\Delta\omega_a} \equiv \frac{\Delta\omega_a}{m_\mu} = -(1.0 \pm 1.1) \times 10^{-23}$$

Approaches to search for an oscillation signal:

G. Bennett, et al., Phys. Rev. Lett. **100**, 091602 (2008)
(Thesis of Xiaobo Huang)

- Multi-parameter fit
  - good for all data
- Fourier Transform
  - only works on equally spaced data
- Lomb-Scargle test
  - designed for unequally spaced data

- All gave comparable results.
  - No significant oscillation

These limits translate into 95% CL limits on parameters

$$b_T^{\mu^+} = \sqrt{(\check{b}_X^{\mu^+})^2 + (\check{b}_Y^{\mu^+})^2} \leq 1.4 \times 10^{-24}\ \mathrm{GeV}$$

$$\check{b}_T^{\mu^-} = \sqrt{(\check{b}_X^{\mu^-})^2 + (\check{b}_Y^{\mu^-})^2} \leq 2.6 \times 10^{-24}\ \mathrm{GeV}$$

dividing by m_μ

$$r_{A_\Omega}^{\mu^+} \leq 2 \times 10^{-23} \qquad r_{A_\Omega}^{\mu^-} \leq 3.8 \times 10^{-23}$$

Muonium hyperfine structure $\quad r^{\mu^+} \leq 5 \times 10^{-22}$

electron in a penning trap $\quad r^{e} \leq 1.6 \times 10^{-21}$

note that

$$\frac{m_\mu}{M_P} = 8.7 \times 10^{-21}$$

Future Improvements in a_μ?

$\sigma_{\text{stat}} = \pm 0.46$ ppm $\sigma_{\text{syst}} = \pm 0.28$ ppm

- Theory (strong interaction part) will improve.
  - both lowest order, and light-by-light
- If money were no object, how well could the experiment be improved?
  - The limit of our technique is between ~0.1 and 0.06 ppm.

The error budget for a new experiment represents a continuation of improvements already made during E821

Systematic uncertainty (ppm)	1998	1999	2000	2001	E??? Goal
Magnetic field – ω_p	0.5	0.4	0.24	0.17	≤0.1
Anomalous precession – ω_a	0.8	0.3	0.31	0.21	≤0.1
Statistical uncertainty (ppm)	4.9	1.3	0.62	0.66	?
Total Uncertainty (ppm)	5.0	1.3	0.73	0.72	≈0.1

- Field improvements: better trolley calibrations, better tracking of the field with time, temperature stability of room, improvements in the hardware
- Precession improvements will involve new beam scraping scheme, lower thresholds, more complete digitization periods, better energy calibration

Possible Future Experiments ?

- Brookhaven
  - E969 aimed for 0.2 ppm overall error
  - No funding, most unlikely
- Fermilab
  - the $\mu \rightarrow e$ conversion experiment is top priority in the recent P5 recommendations.
  - g-2 is mentioned as important, but with the three sites mentioned as possibilities. We would aim for 0.1 ppm total error. It could be done at FNAL, and we have received significant interest there.
- J-PARC
  - Significant interest in moving the ring there.

    goal is ≤ 0.1 total error

Summary

- The measurement of e^- and $\mu^\pm$ magnetic dipole moments has been an important benchmark for the development of QED and the standard model of particle physics.
- The muon anomaly has been particularly valuable in restricting physics beyond the standard model, and will continue to do so in the LHC Era
- There appears to be a difference between a_μ and the standard-model prediction at the 3.4 (3.7) σ level.
- Much activity continues on the theoretical front.
- A new limit on the EDM is now available
- The experiment can certainly be improved...

  and we look forward to discussions with FNAL and J-PARC