MeV Dark Matter in the 3+1+1 Model

Jinrui Huang
Collaboration with Ann E. Nelson
arXiv: 1306.6079

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Los Alamos
NATIONAL LABORATORY
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Outline

- Motivation
- Current Status of Light Sterile Neutrinos
  - Neutrino Experiments
  - Cosmology
- Dark 3+1+1 Model
  - BBN and Supernovae Bounds
- Other implications
  - Dark Matter
  - Explanation of INTEGRAL 511 keV Gamma Line
  - Fermion Mass Hierarchy and Mixings: U(1)’ Family Symmetry
  - Baryon Number Asymmetry: Dirac leptogenesis
- Conclusion

Motivation

- LSND + MiniBooNE, reactor antineutrino anomaly, Gallium anomaly indicate one light (~eV) sterile neutrino
- 3+1 model disfavored by global fit with null results disappearance neutrino exps.
- The 3+1+1 model constrained by cosmology, collider exps., but viable in MeV region
- Natural MeV dark matter candidate in the 3+1+1 model to explain INTEGRAL
- With additional U(1) family symmetry, all fermion mass hierarchy and mixings can be explained
- Bonus: Dirac leptogenesis to explain Baryon Number Asymmetry

Hints of Sterile Neutrino

- LSND, MiniBooNE, 3.8 σ of ~eV sterile neutrino

  A. Aguilar-Arevalo et al. [LSND Collaboration], PRD 64, 112007 (2001)

  A. Aguilar-Arevalo et al. [MiniBooNE Collabroation], PRL 98, 231801 (2007); PRL 102, 101802 (2009); PRL 105, 181801 (2010); arXiv:1303.2588

- Reactor Antineutrino Anomaly, 2 σ $|\Delta m^2_{\text{new}}|>1.5\ \text{eV}^2$ and $\sin^2(2\theta_{\text{new}})=0.14+/-0.08$

  Mention, G. et al. Phys. Rev. D 83 (2011) 073006

  Huber, Patrick Phys. Rev. C 84 (2011) 024617, Erratum-ibid. C85 (2012) 029901

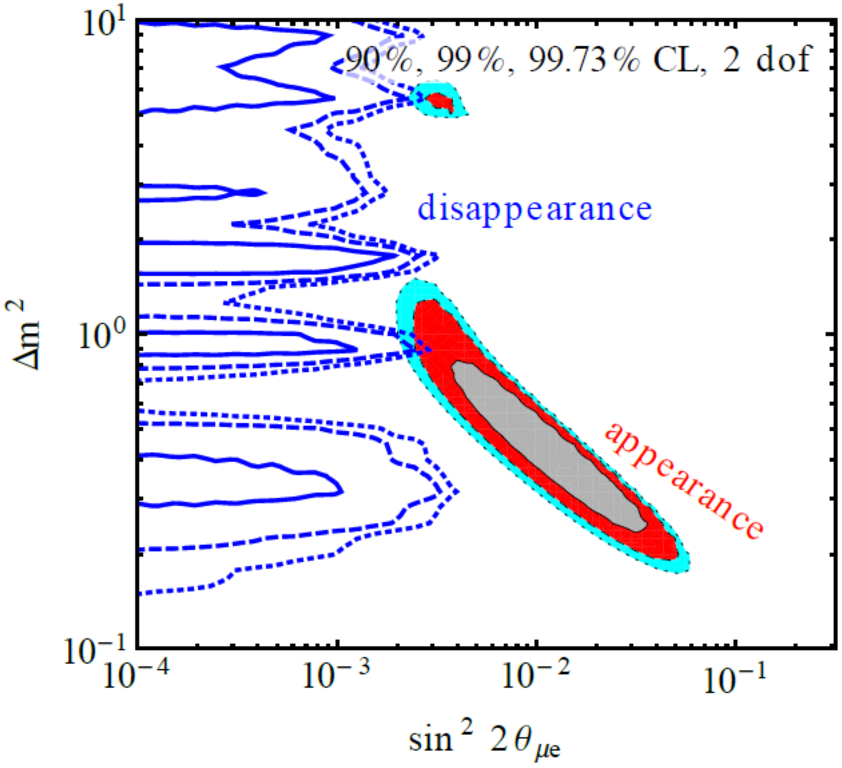
Sterile Neutrino Models

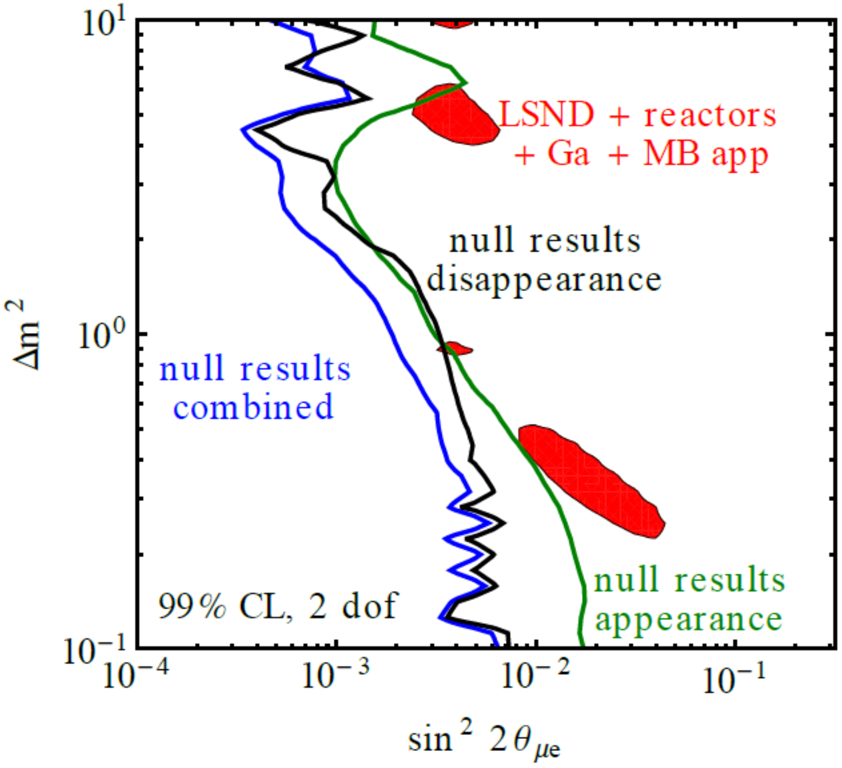
- **3+1 Model:** one additional flavor eV scale sterile neutrino
- **3+2 Model:** two additional flavors eV scale sterile neutrinos, $\Delta m_{41}^2 > 0$ and $\Delta m_{51}^2 > 0$
- **1+3+1 Model:** two additional flavors eV scale sterile neutrinos, $\Delta m_{41}^2 < 0$ or $\Delta m_{51}^2 < 0$
- **3+1+1 Model:** two additional flavors sterile neutrinos, one eV scale and one 33 eV~40 GeV scale
- **3+s (s>2) Model:** more than two additional flavors sterile neutrinos

3+1 Model In Tension

$$\sin^2 2\theta_{\mu e} = 4|U_{e4}|^2|U_{\mu 4}|^2$$

J. Kopp, et al, JHEP 1305 (2013) 050



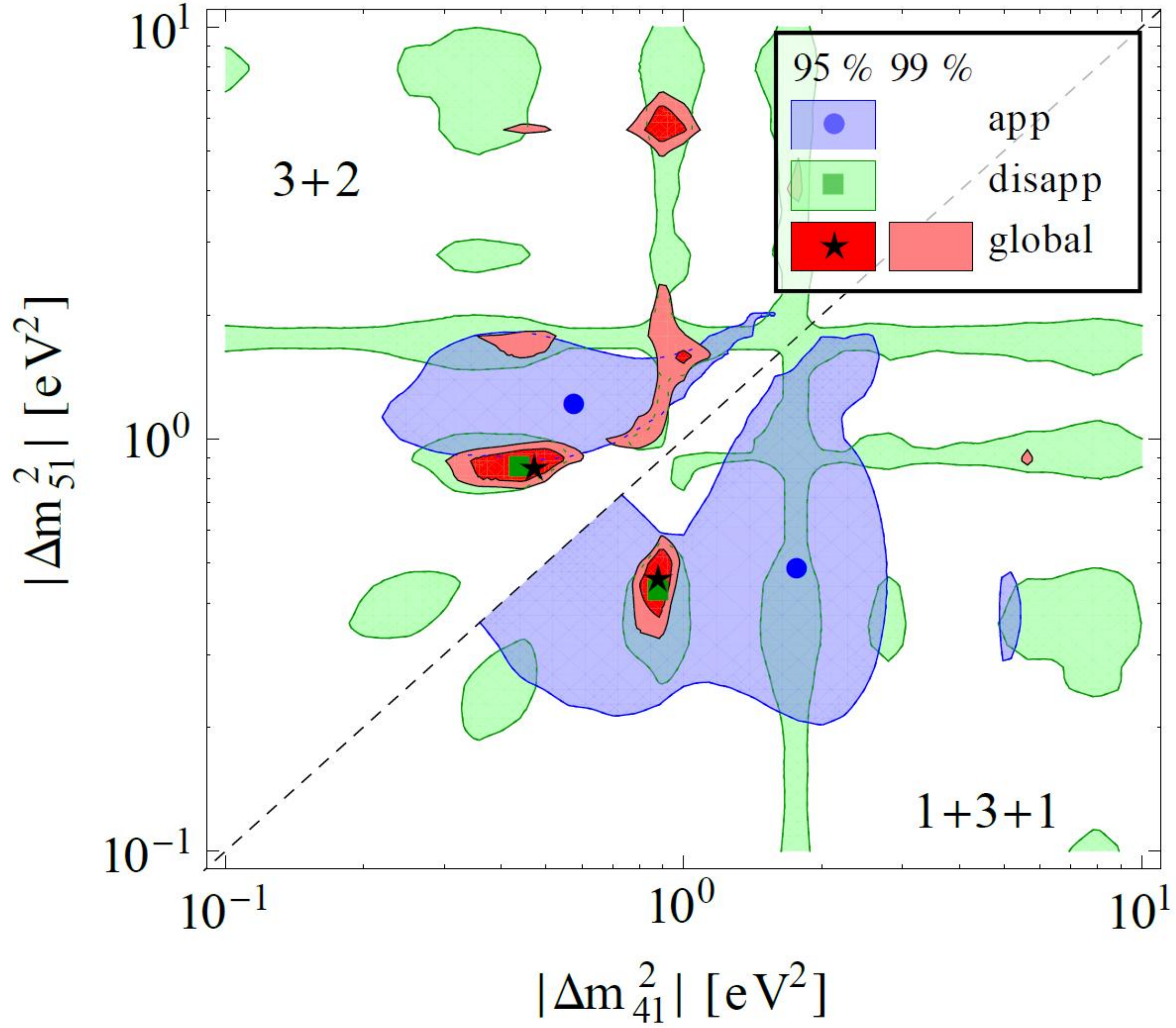


Appearance: LSND, MiniBooNE appearance, NOMAD, KARMEN, ICARUS, E776

Disappearance: atmospheric, solar, reactors, Gallium, CDHS, MINOS, MiniBooNE disappearance, KARMEN, LSND ν_e- ^{12}C scattering

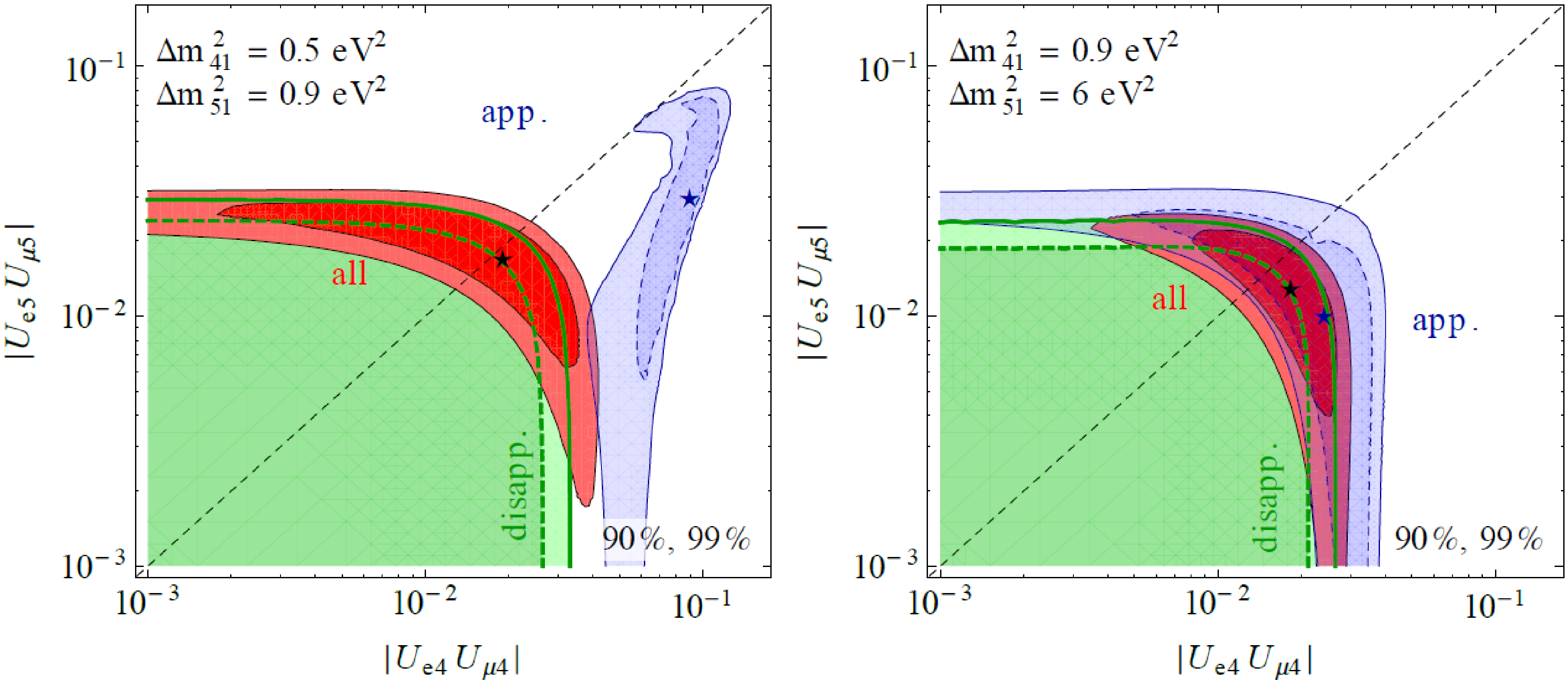
Masses in 3+2 and 1+3+1 Models

J. Kopp, et al, JHEP 1305 (2013) 050



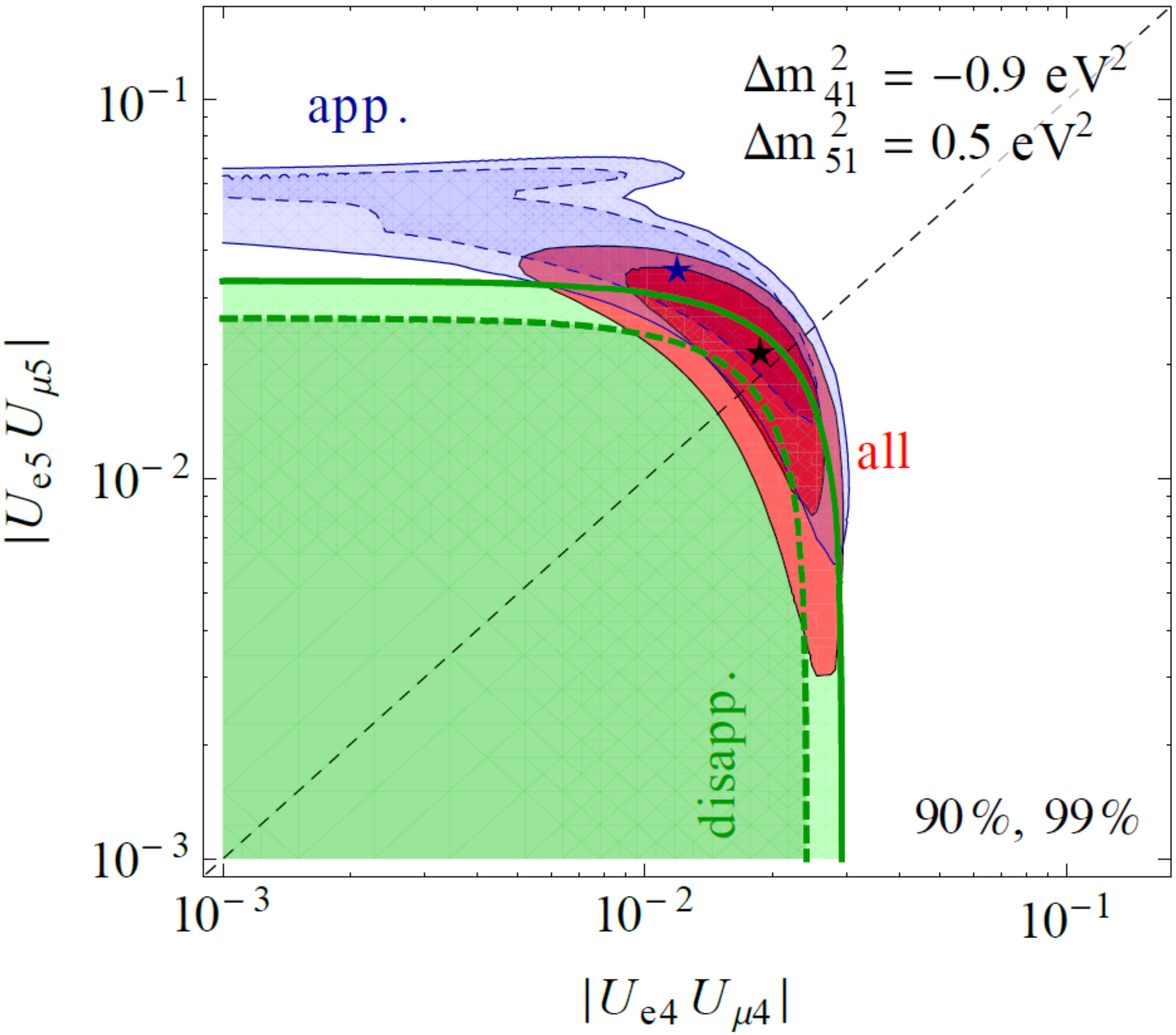
Mixings in 3+2 Model

J. Kopp, et al, JHEP 1305 (2013) 050



Mixings in 1+3+1 Model

J. Kopp, et al, JHEP 1305 (2013) 050



Oscillation Probability in 3+1+1 Model

A. E. Nelson, PRD 84 (2011) 053001
E.Kuflik, S. D. McDermott, K.M.Zurek PRD 86 (2012) 033015

Appearance Probability:

$$P_{\nu_\mu(\bar{\nu}_\mu)\to\nu_e(\bar{\nu}_e)} = \sin^2 2\theta_{\mu e}\sin^2\left(x_{41}\pm\beta\right)+\kappa$$

$$\sin^2 2\theta_{\mu e} \;=\; 4\left|U_{\mu 4}\right|^2\left|U_{e4}\right|^2 r \qquad x_{ij} \;=\; \Delta m^2_{ij}L/4E \;=\; 1.27\frac{(m_i^2-m_j^2)L/E}{\mathrm{eV}^2\,\mathrm{m}/\,\mathrm{MeV}}$$

$$r \;\equiv\; \left|U^*_{\mu 4}U_{e4}+U^*_{\mu 5}U_{e5}\right| / \left|U^*_{\mu 4}U_{e4}\right|$$

$$\beta \;\equiv\; \tfrac{1}{2}\tan^{-1}\left(\frac{\sin\phi|U_{e5}||U_{\mu 5}|}{|U_{e4}||U_{\mu 4}|+\cos\phi|U_{e5}||U_{\mu 5}|}\right) \qquad \phi\equiv\arg\left(\frac{U_{e5}U^*_{\mu 5}}{U_{e4}U^*_{\mu 4}}\right)$$

$$\kappa \;=\; \left|U_{\mu 4}\right|^2\left|U_{e4}\right|^2\left\{(1-r)^2+a\left[(1-r)^2+4r\sin^2\beta\right]\right\}$$

Disappearance Probability:

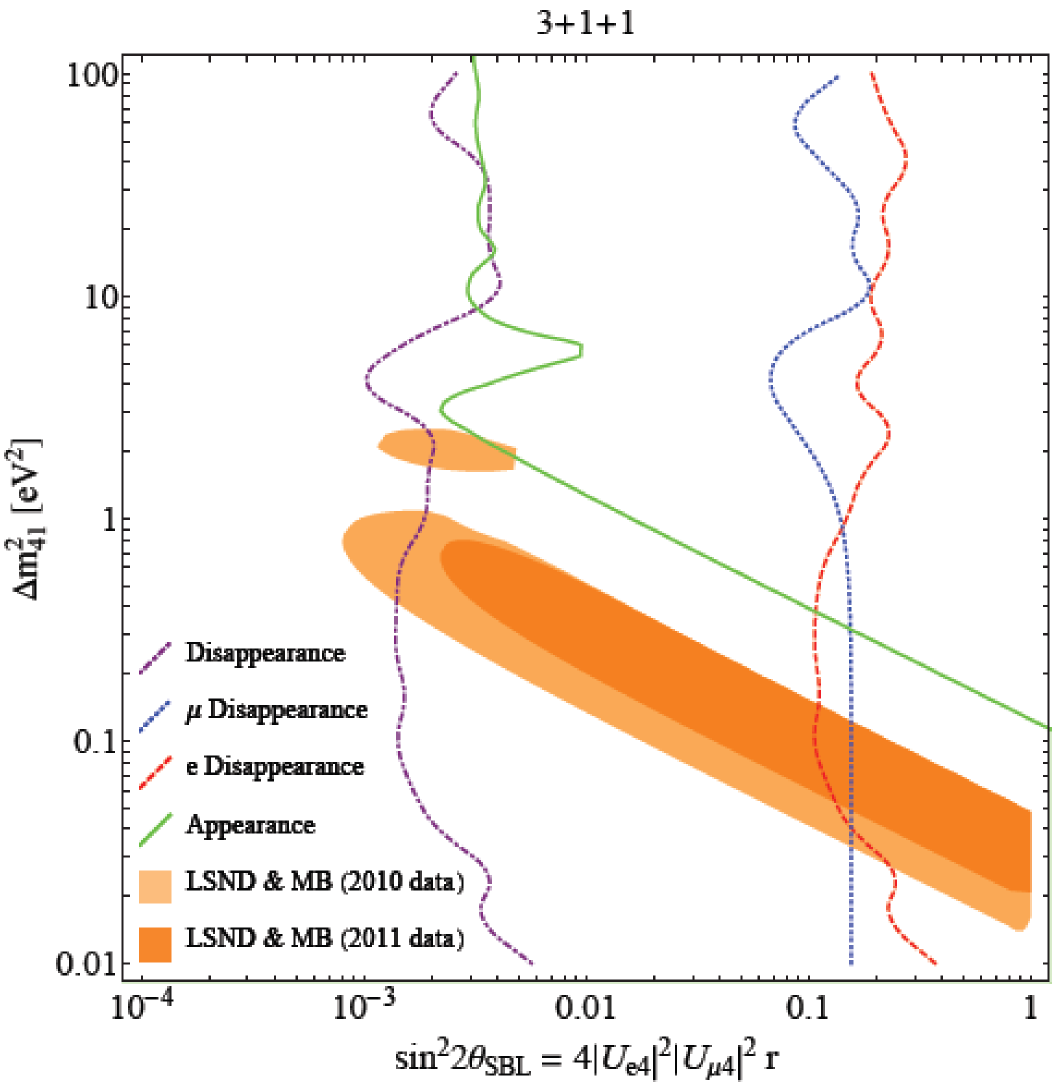
$$1-P_{\nu_\alpha\to\nu_\alpha} = \sin^2 2\theta_{\alpha 4}\sin^2 x_{41}+2|U_{\alpha 5}|^2\left(1-\frac{a+1}{2}|U_{\alpha 5}|^2\right)$$

$$\sin^2 2\theta_{\alpha 4} \;=\; 4|U_{\alpha 4}|^2(1-|U_{\alpha 4}|^2-|U_{\alpha 5}|^2)$$

a: phase space parameter associated with ν_5 production
β: CP odd parameter associated with CP violation

Fitting in 3+1+1 Model

E.Kuflik, S. D. McDermott, K.M.Zurek PRD 86 (2012) 033015



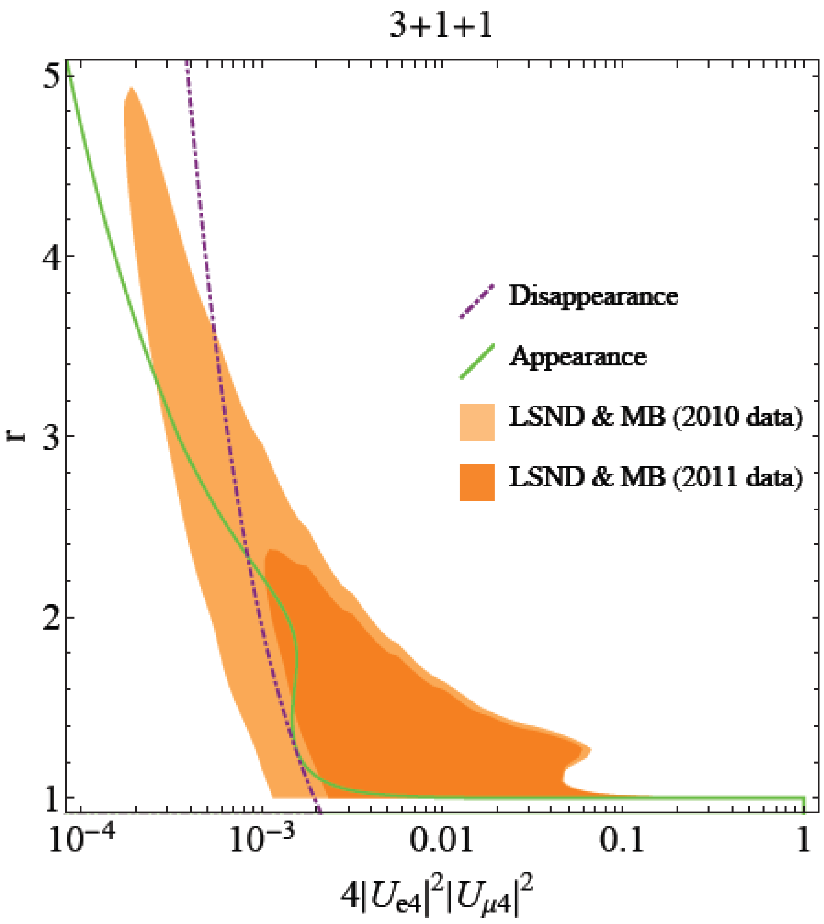
Appearance: KARMEN, E776, NOMAD, CCFR, NuTeV;

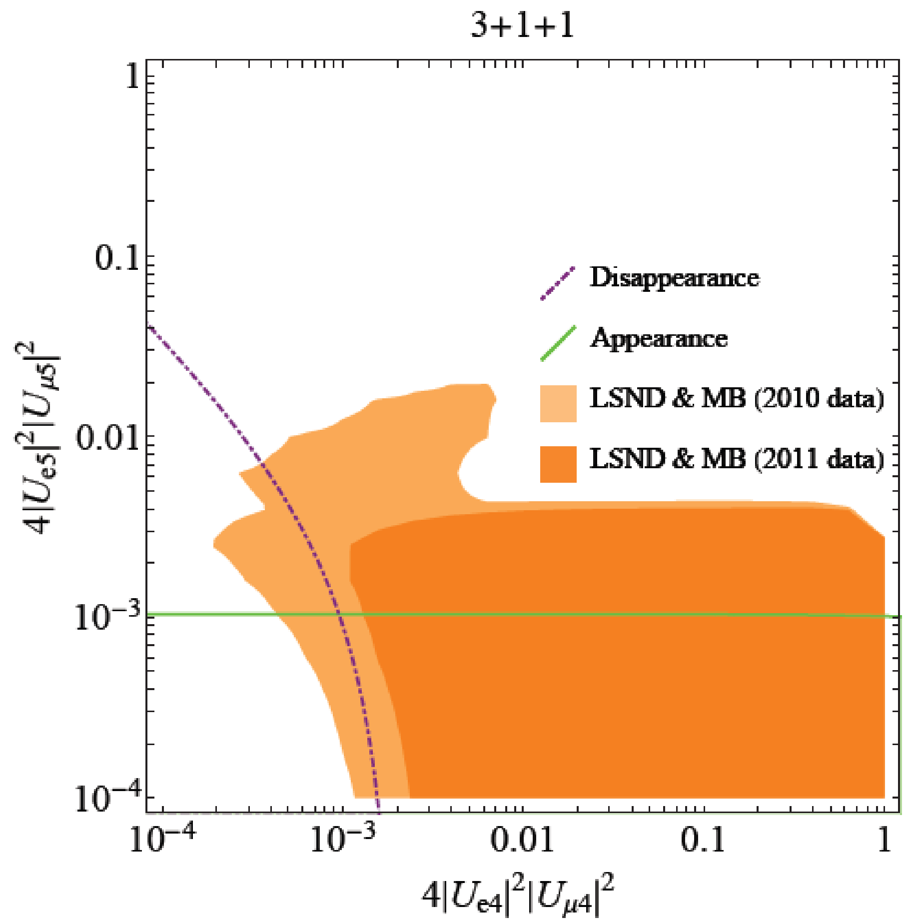
ν_e disappearance: SBL reactor experiments with new reactor flux predictions; ratio of flux observed in the Bugey 40 m and 15 m detectors;

ν_μ disappearance: CDHS, CCFR, atmospheric at Super-Kamiokande.

Fitting in 3+1+1 Model-Cont.

E.Kuflik, S. D. McDermott, K.M.Zurek PRD 86 (2012) 033015





Summary: Sterile Neutrino Models

- 3+1 Model: tension between appearance and disappearance neutrino exps.
- 3+2 Model: allowed, eV scale, O(0.1) mixings
- 1+3+1 Model: allowed, eV scale, O(0.1)mixings
- 3+1+1 Model: allowed, one eV scale, one heavier, O(0.1) mixings
- 3+s (s>2) Model: allowed, no qualitatively difference compared to two sterile neutrino scenarios M. Maltoni, T. Schwetz, PRD (2007) 093005

Dark Radiation

- BBN: **Y. I. Izotov, T. X. Thuan, Astrophys. J. 710, L67 (2010)**
  - $N_{eff} = 3.68^{+0.80}_{-0.70}$ (2 σ); $N_{eff} = 3.80^{+0.80}_{-0.70}$ (2 σ)
- Nine-year WMAP: **C. L. Bennett et al. [WMAP Collaboration], arXiv:1212.5225; arXiv:1212.5226**
  - $N_{eff} > 1.7$ (2 σ); $N_{eff}=3.84+/-0.40$ (1 σ) (WMAP + ACT+SPT + BAO + H_0)
- High resolution ground-base CMB experiments:
  - Atacama Cosmology Telescope (ACT)
    - $N_{eff}=2.79+/-0.56$ (2 σ) **J. L. Sievers et al., arXiv:1301.0824**
  - South Pole Telescope (SPT)
    - $N_{eff}=3.71+/-0.35$ (1.9 σ) (SPT+WMAP7+H_0+BAO) **S. Zhou, arXiv:1212.6267**

PLANCK

P. A. R. Ade et al. [Planck Collaboration], arXiv:1303.5076

- Combine Planck, nine-year WMAP, Baryon Acoustic Oscillations (BAO), ACT, SPT:

$$N_{eff} = 3.30^{+0.54}_{-0.51} \ (2\ \sigma)$$

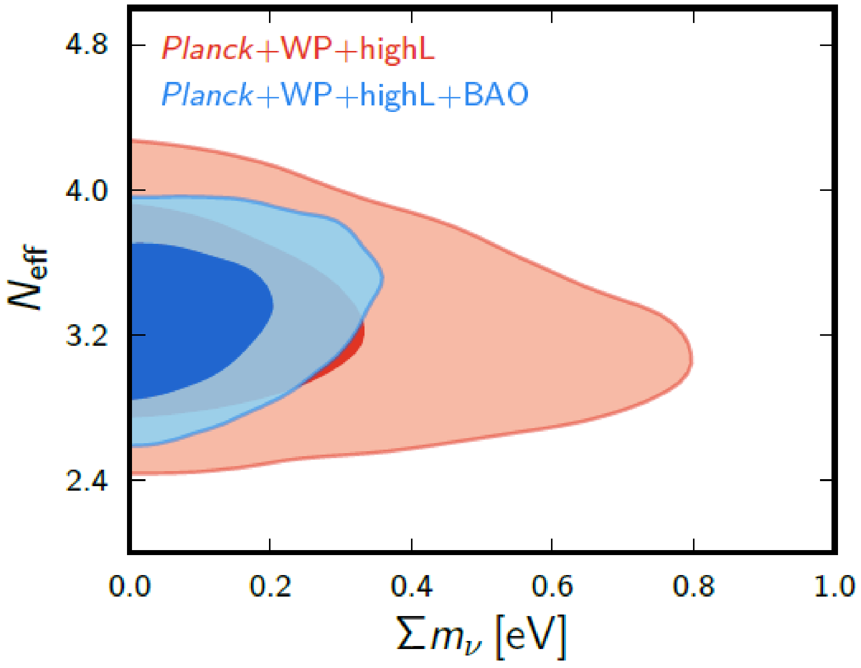
- Include additional direct measurements of the Hubble Constant $H_{0:}$

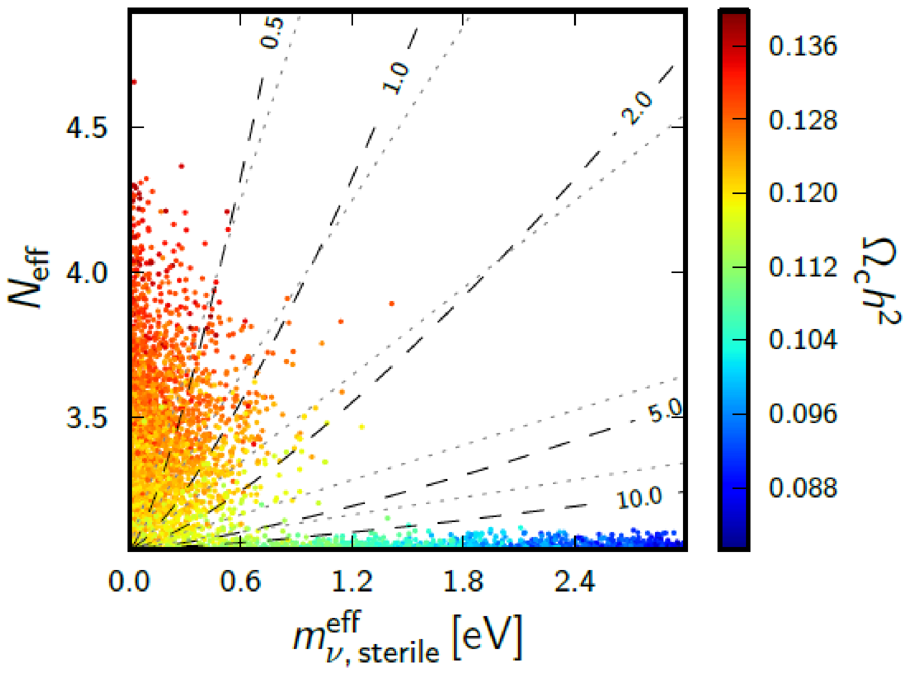
$$N_{eff} = 3.52^{+0.48}_{-0.45} \ (2\ \sigma)$$

Planck Result on Sterile Neutrino

Planck Collaboration, arXiv:1303.5076

There is still room for one sub-eV sterile neutrino (~0.6 eV).





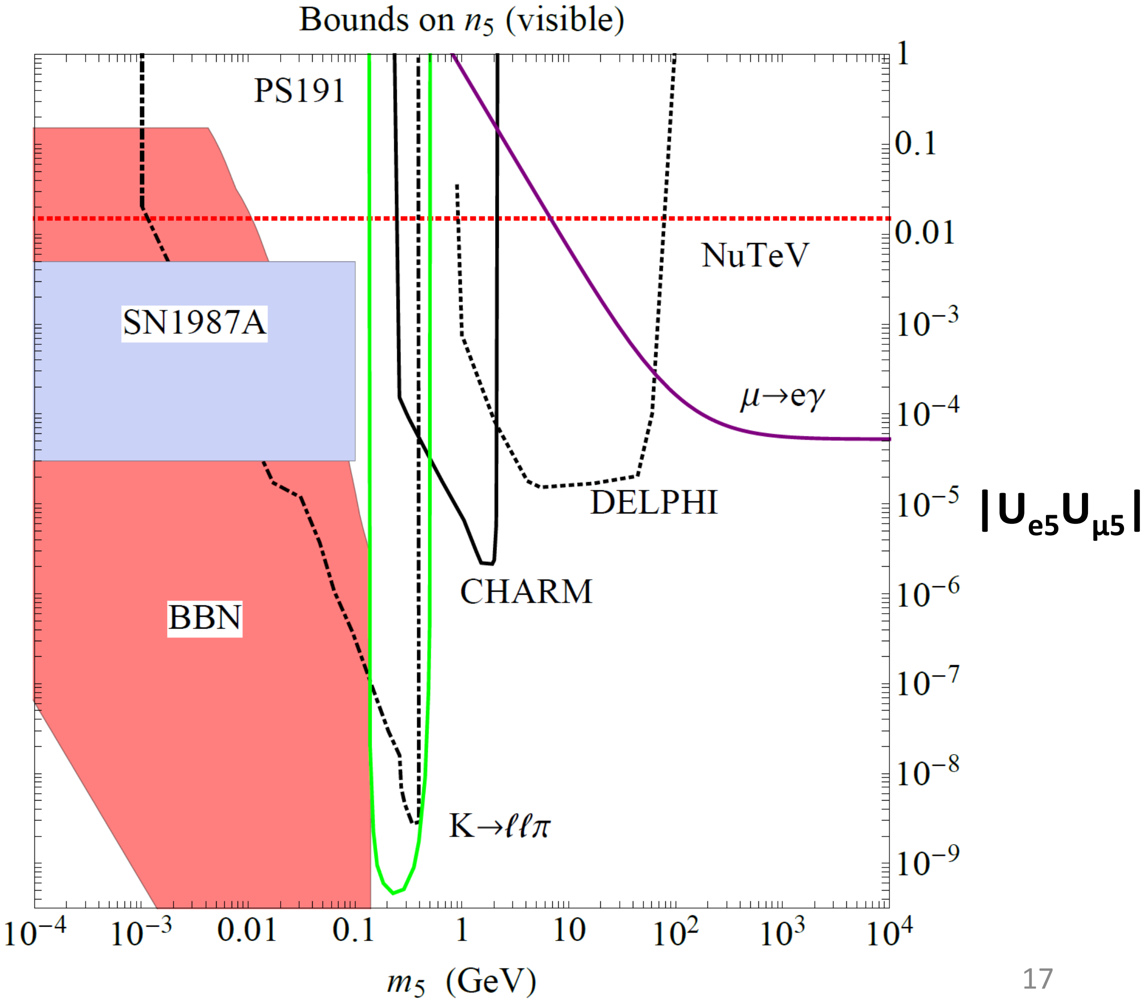
More Constraints on 3+1+1 Model

E.Kuflik, S. D. McDermott, K.M.Zurek PRD 86 (2012) 033015

Hierarchical: ν_{s1}~ eV, ν_{s2}~[33eV, 10GeV]

Mixing: θ ~ O(0.1) for LSN MiniBooNE

Ruled out by exps.

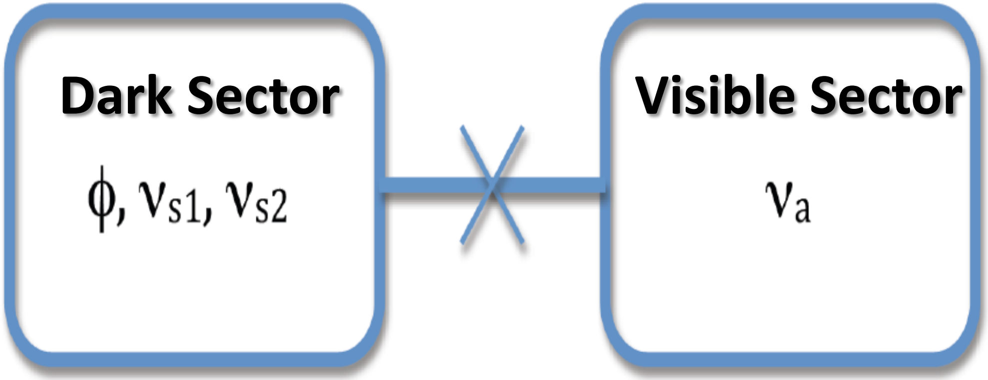


Dark 3+1+1 Model

JH, A E Nelson arXiv: 1306.6079

Introduce additional ϕ field and new interaction

$$\mathcal{L}_\phi = \lambda\phi\nu_{s1}\nu_{s2} + h.c.$$



ν_{s1}: subeV~eV sterile neutrino
ν_{s2}: [1,34]MeV sterile neutrino
ϕ: [1,7]MeV scalar
ν_a: active neutrino

X: Mixing between active and sterile neutrinos;
Particles in Dark Sector scatter among themselves.

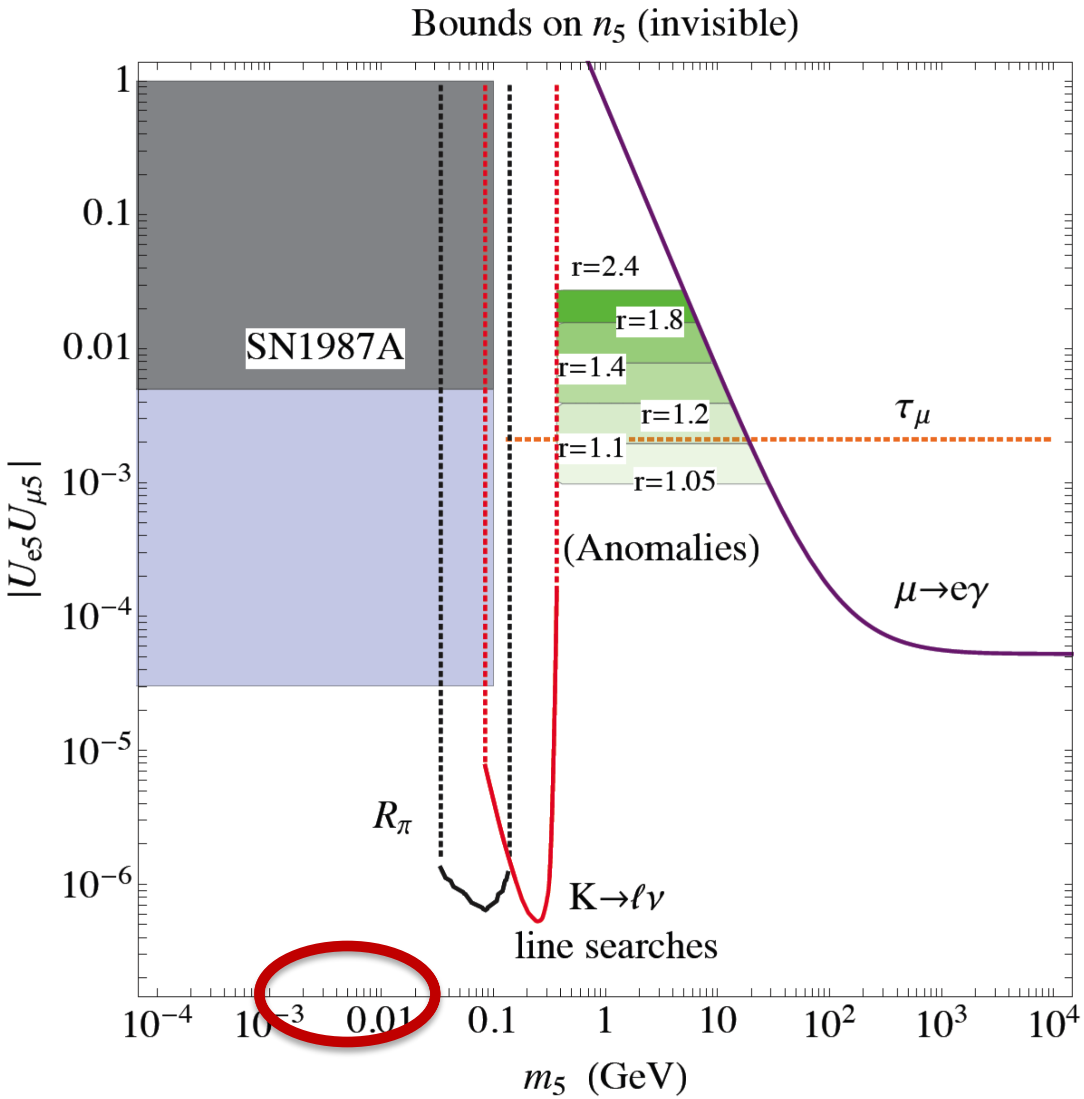
Lead to invisible decay $\nu_5 \rightarrow \nu_4\,\phi$, $\Gamma^{inv}_{\nu 5} \approx \lambda^2 m_5/(16\pi)$

Constraints on Dark 3+1+1Model

E. Kuflik, S. D. McDermott, K.M.Zurek PRD86 (2012) 033015

SN1987A, BBN controlled region can be open up

MeV ϕ field: dark matter candidate; explain 511keV INTEGRAL



BBN Constraint on Dark 3+1+1 Model

A. D. Dolgov, S. H. Hansen, G. Raelt D. V. Semikoz, Nucl. Phys. B 590, 562 (2000);
A. D. Dolgov, F. L. Villante, Nucl. Phys. B 679, 261 (2004);
O. Ruchayskiy, A. Ivashko, JCAP 1210, 014 (2012)

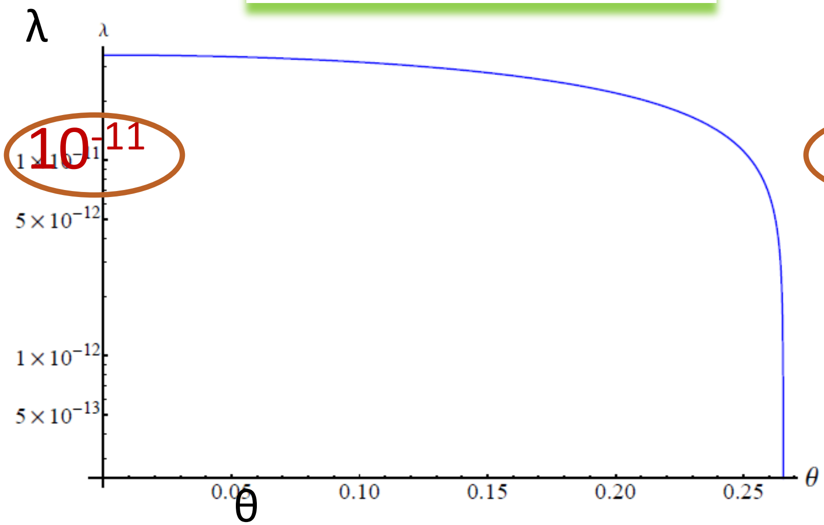
$\nu_5 \rightarrow \nu_a e^+ e^-$ decay modifies the Helium abundance:

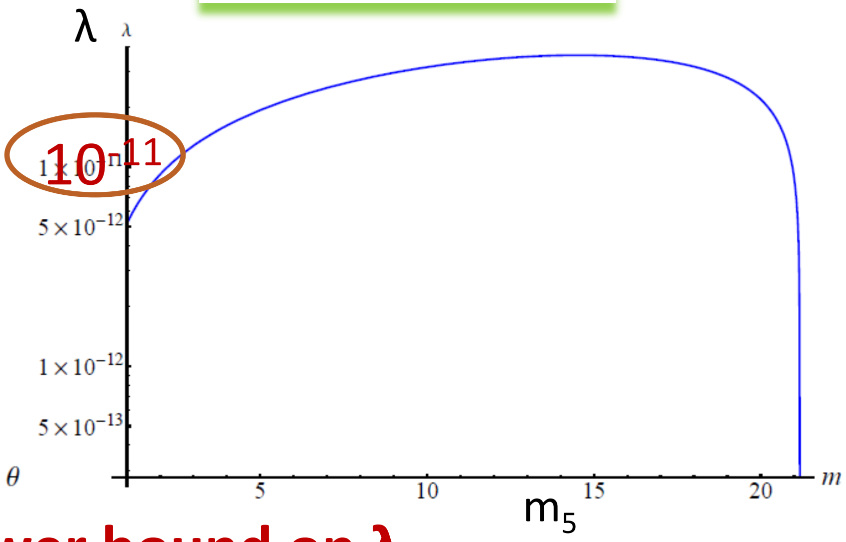
short lifetime avoid the bound:

$$\frac{\hbar}{\Gamma^{\rm inv}_{\nu_5} + \Gamma^{\rm ee}_{\nu_5}} < t_1 \left(\frac{m_5}{MeV}\right)^{\beta} + t_2$$

$$\Gamma^{\rm ee}_{\nu_5} \simeq \frac{\left(\frac{m_5}{10{\rm MeV}}\right)^5 s^2_{2\theta_{e5}} \hbar}{0.7}$$

$$\Gamma^{\rm inv}_{\nu_5} \simeq \frac{1}{16\pi} \lambda^2 m_5$$

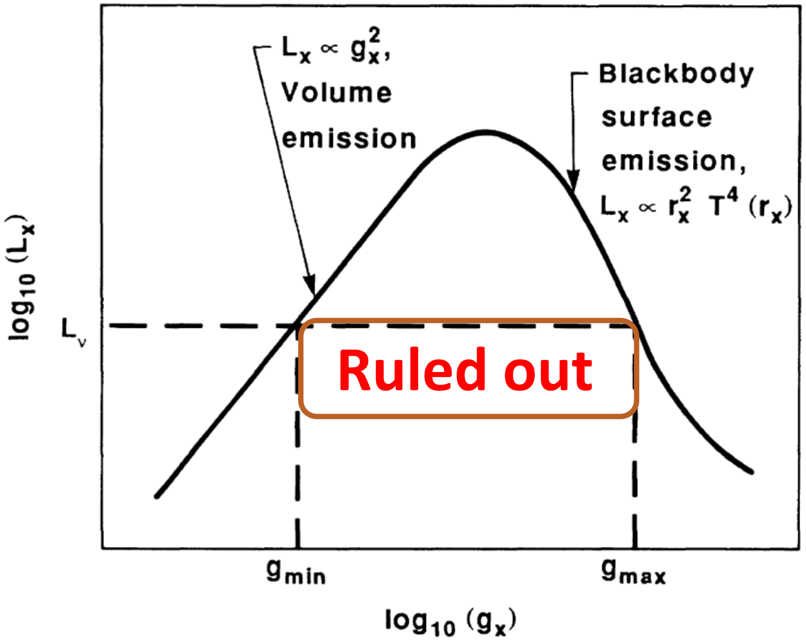




Weak lower bound on λ

SN1987a Constraint-I

G. Raelt, D. Seckel, PRL 60, 1793
K. Kainulainen, J. Maalampi, J. T. Peltoniemi,
Nucl. Phys. B 358, 435



In general, require new particle:
1) well trapped inside the supernovae: interaction length $l_{\nu 5} \le 1.5$ m;
Or 2) escape the supernovae: interaction length $l_{\nu 5} \ge 4.6\text{X}10^4$ km

ν_5 ν_4 ϕ: well trapped inside the supernovae core

$$\Gamma_{\nu s\ \nu a} \ge \Gamma_{\nu a N} \Rightarrow n_\nu \frac{(\lambda^\nu)^4}{<E>^2} \gtrsim n_N \frac{1}{2} \sin^2 2\theta_m G_F^2 <E>^2 |_{(\sin^2 2\theta_m = 2\times 10^{-2})}$$

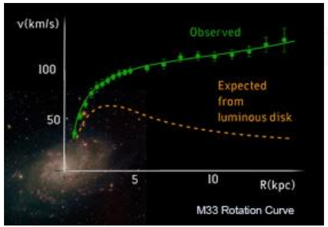
SN1987a Constraint-II

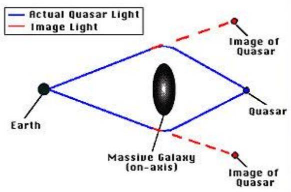
Interaction between dark sector/visible sector modifies active neutrino mean free path, needs weaker than EW interaction.

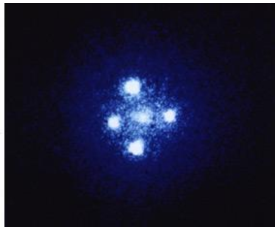
$$\Gamma_{\phi\ \nu a} \leq \Gamma_{EW} \quad \Rightarrow \quad \left(\frac{1}{2}\sin^2 2\theta_m\right)^2 n_\nu \frac{(\lambda^\nu)^4}{< E >^2} \leq n_N G_F^2 < E >^2$$

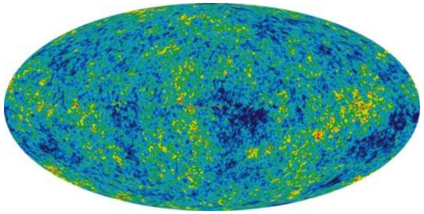
$$\boxed{2 \times 10^{-4} \lesssim \lambda^\nu \lesssim 0.5 \times 10^{-2}}$$

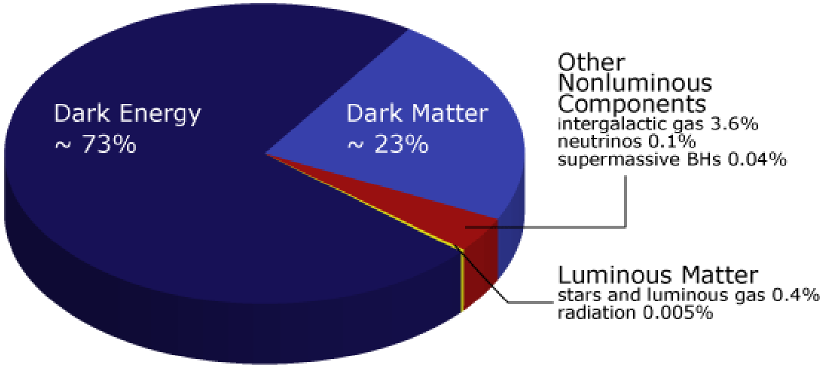
Dark Matter











Evidences of DM:

- Motion of Galaxy
- Structure simulations
- Temperature fluctuations in CMB
- Gravitational lensing
- Cooling clusters

Properties of DM:

- Not Luminous
- Long Lived
- Not Hot
- Not dissipational

Neutrinophilic MeV DM

- No confirmed signals of DM yet;
- Current DM direct detections not sensitive to MeV region;
- Detections assume DM interacting with quarks;
- Light MeV DM or neutrinophilic DM:
  - NOT covered in the current DM detection experiments.

Constraints on Neutrinophilic MeV DM from Supernovae

P. Fayet, D. Hooper , G. Sigl, PRL 96, 211302 (2006)

- cross section of DM annihilating into active neutrinos

~10^4 **stronger** than the weak interaction

→ interaction length of the active neutrino too short; ruled out by Supernovae SN1987a

[1, 30] MeV DM region ruled out.

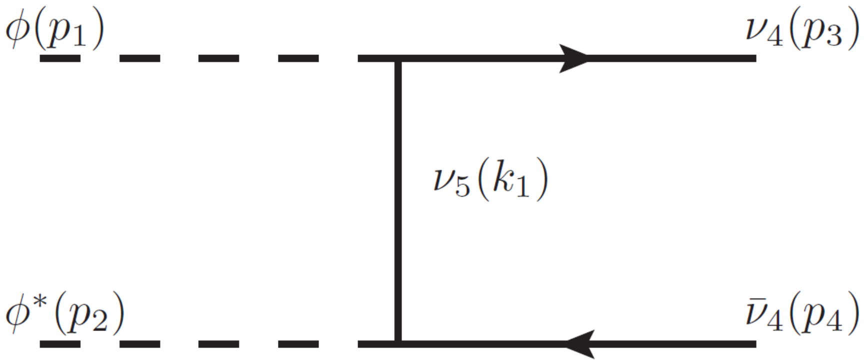
- In dark 3+1+1 Model, DM annihilates into sterile neutrino pairs dominantly,

→ cross section of the DM annihilation into active neutrino pairs **suppressed** by the mixing angles 10^{-4}

Avoid the supernovae bound.

DM Relic Abundance

DM annihilates into sterile neutrino pair



$$\sigma_{\rm ann} v_{\rm rel} = \frac{\lambda^4}{4\pi} \frac{m_5^2}{(m_\phi^2 + m_5^2)^2} \simeq 0.3 \left(\frac{\lambda}{10^{-3}}\right)^4 \left(\frac{10{\rm MeV}}{m_5}\right)^2 {\rm pb}$$

For right relic abundance, $\sigma_{\rm ann} v_{\rm rel} \simeq 0.2 \times \frac{x_F}{\sqrt{g_*}} \left(\frac{\Omega_{dm} h^2}{0.11}\right)^{-1} pb$

→ $\boxed{\lambda \sim 3 \times 10^{-4} - 2 \times 10^{-3}}$

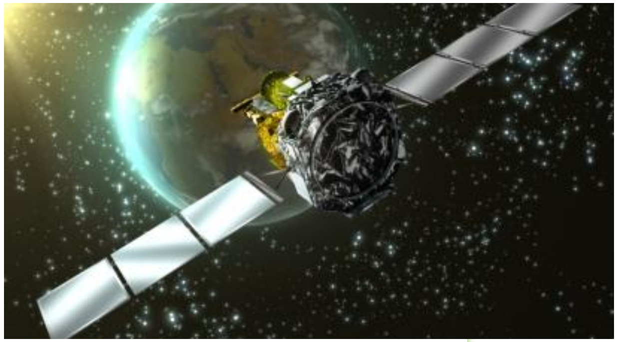
511 keV INTEGRAL Gamma-Ray Line

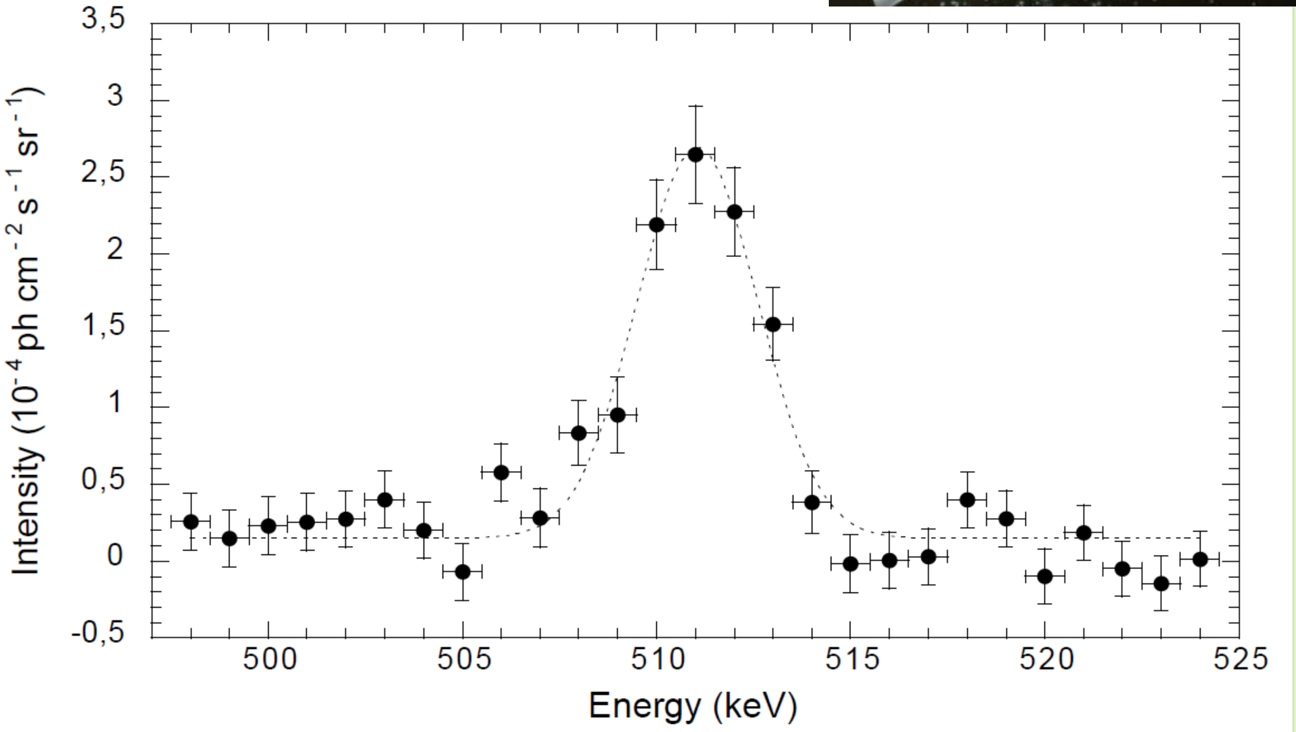
P. Jean et al, Astron.Astrophys.407:L55,2003

Flux: $9.9^{+4.7}_{-2.1} \times 10^{-4}$ ph cm^{-2} s^{-1}

Centroid: $511.06^{+0.17}_{-0.19}$

Width: $2.95^{+0.45}_{-0.51}$ keV (FWHM)





MeV DM and 511 keV INTEGRAL

- DM annihilation into electron pair:

C. Boehm, P. Fayet, Nucl. Phys. B 683, 219 (2004);
C. Boehm, et al, PRL 92, 101301 (2004)

- DM decay into electron pair:

C. Picciotto, M. Pospelov, PLB 605, 15 (2005)
M. Pospelov, A. Ritz, PLB 651, 208 (2007)

- Excited DM with MeV mass splitting:

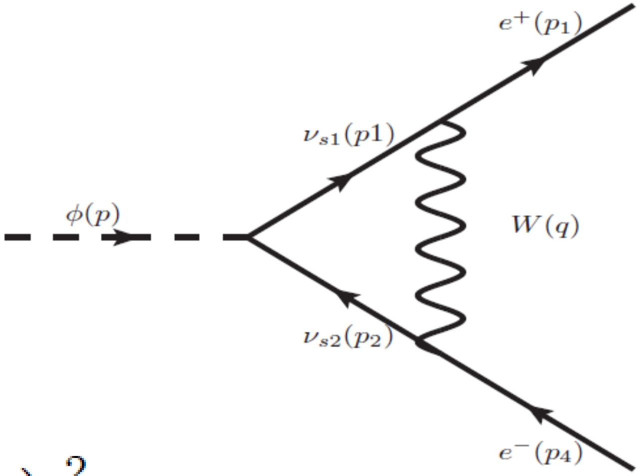
M. Pospelov, A. Ritz, PLB 651, 208 (2007);
D. P. Finkbeiner, N.Weiner, PRD 76, 083519 (2007);
N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer, N. Weiner, PRD79 (2009) 015014;
J. M. Cline, A. R. Frey, F. Chen, PRD 83, 083511 (2011);
Y. Bai, M. Su, Y. Zhao, JHEP 1302, 097 (2013)

INTEGRAL Constraints on DM Models

- To fit the continuum photon energy spectrum of 511 keV INTEGRAL gamma line:
  - $m_{DM} < 30$ MeV N. Prantzos et al., arXiv:1009.4620
- Avoid Internal Bremsstrahlung with positron production violating the COMPTEL and EGRET diffuse gamma-ray observation:
  - $m_{DM} < 20$ MeV J. F. Beacom, N. F. Bell, G. Bertone, PRL 94, 171301 (2005)
- DM annihilating into electron pair through new interaction can modify the fine structure constant:
  - $m_{DM} < 7$ MeV C. Boehm, Y. Ascasibar, PRD 70, 115013
- Be consistent with the diffuse Galactic gamma-ray data
  - Positron injection energy < 3MeV J. F. Beacom, H. Yuksel, PRL 97, 071102 (2006)

Explain INTEGRAL in Dark 3+1+1 Model

DM decay into electron pair



$$\Gamma_{\phi\to e^+e^-} \simeq 10^{-50}\left(\frac{\lambda}{10^{-3}}\cdot\frac{|U_{s15}|}{10^{-7}}\cdot\frac{m_5}{10\text{MeV}}\right)^2\left(\frac{m_\phi}{1\text{MeV}}\right)$$ **(GeV)**

$|U^*_{e5}U_{e5}|^2 \simeq 10^{-4}$ $|U_{s25}|^2 \simeq 1$ **have been applied**

Explain INTEGRAL: $\tau_\phi \sim 10^{18}$ years

Explain INTEGRAL in Dark 3+1+1 Model-Cont.

DM decay into neutrino pair

$$\Gamma_{\nu\nu} \simeq 10^{-50} \left(\frac{|U_{s1i}U^*_{s2i}|}{10^{-20}} \right)^2 \left(\frac{\lambda}{10^{-3}} \right)^2 \left(\frac{m_\phi}{\mathrm{MeV}} \right) \quad \textbf{(GeV)}$$

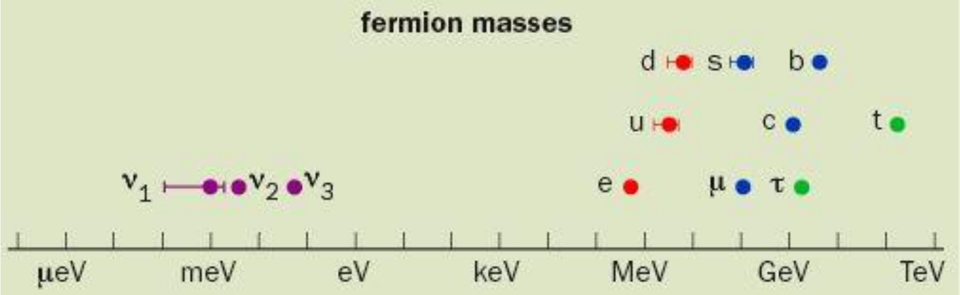
$$(i = 1, 2, 3, 4)$$

Highly suppressed to explain INTEGRAL

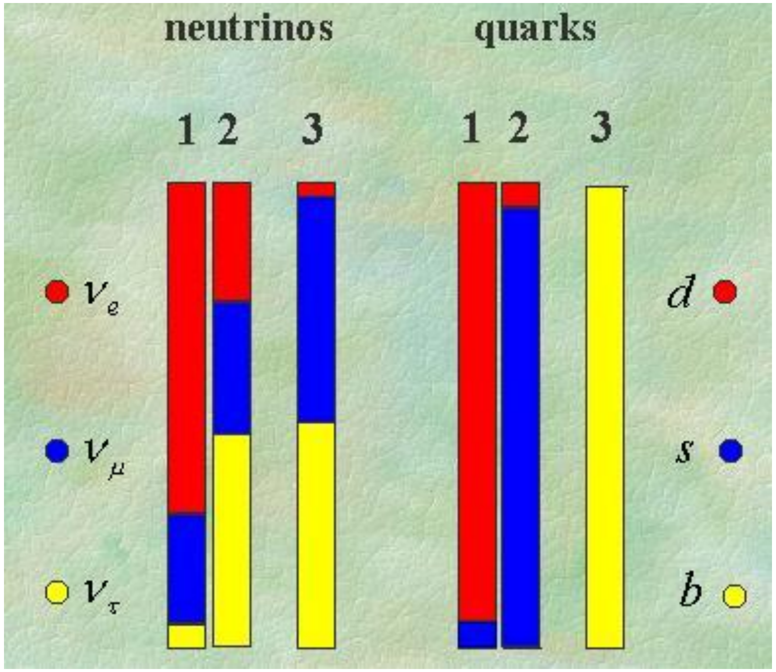
$|U_{s1i}U^*_{s2i}| < 10^{-20}$ **for** $\lambda \simeq 10^{-3}$ **and** $m_\phi \sim \mathrm{MeV}$

Fermion Masses, Mixings

P. Ramond, R. G. Roberts, G. G. Ross, Nucl. Phys. B406 (1993) 19



$m_b : m_s : m_d \cong 1 : \lambda^2 : \lambda^4$
$m_\tau : m_\mu : m_e \cong 1 : \lambda^2 : \lambda^4$
$m_t : m_c : m_u \cong 1 : \lambda^4 : \lambda^8$
Cabbibo angle: $\lambda \cong 0.22$



$m^2{}_2 - m^2{}_1 = 7.58\times10^{-5}\ \mathrm{eV}^2$
$|m^2{}_3 - m^2{}_1| = 2.35\times10^{-3}\ \mathrm{eV}^2$
m_4 ~ sub-eV
m_5 ~ MeV

Quark sector: CKM matrix
small mixing;

Lepton sector: PMNS matrix
large mixing.

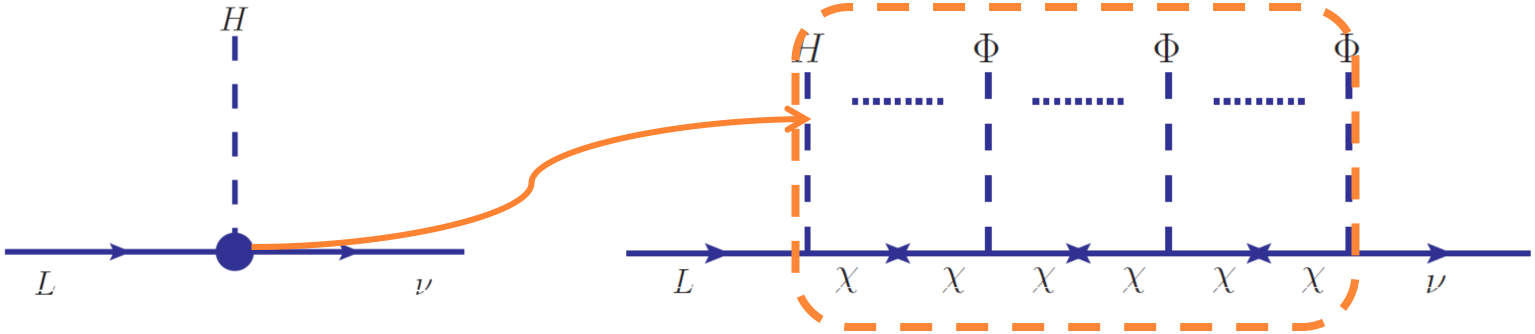
Models to Explain Fermion Masses, Mixings

- Family Symmetries
  - Continuous Symmetry
    - U(1), SU(2), SU(3) etc
  - Discrete Symmetry
    - A_4, T’, A_5, S_4 etc
- To explain small neutrino masses,
  - popular approach: seesaw mechanism
- In Dark 3+1+1 Model, all fermions are Dirac Particles,
  - no canonical seesaw mechanism,
  - one natural approach: higher dimensional operators with U(1)’ symmetry

Froggatt-Nielsen Mechanism

D. D. Froggatt, H. B. Nielsen, Nucl. Phys. B147, 277 (1979)

Higher dimension operator



η: flavon field;

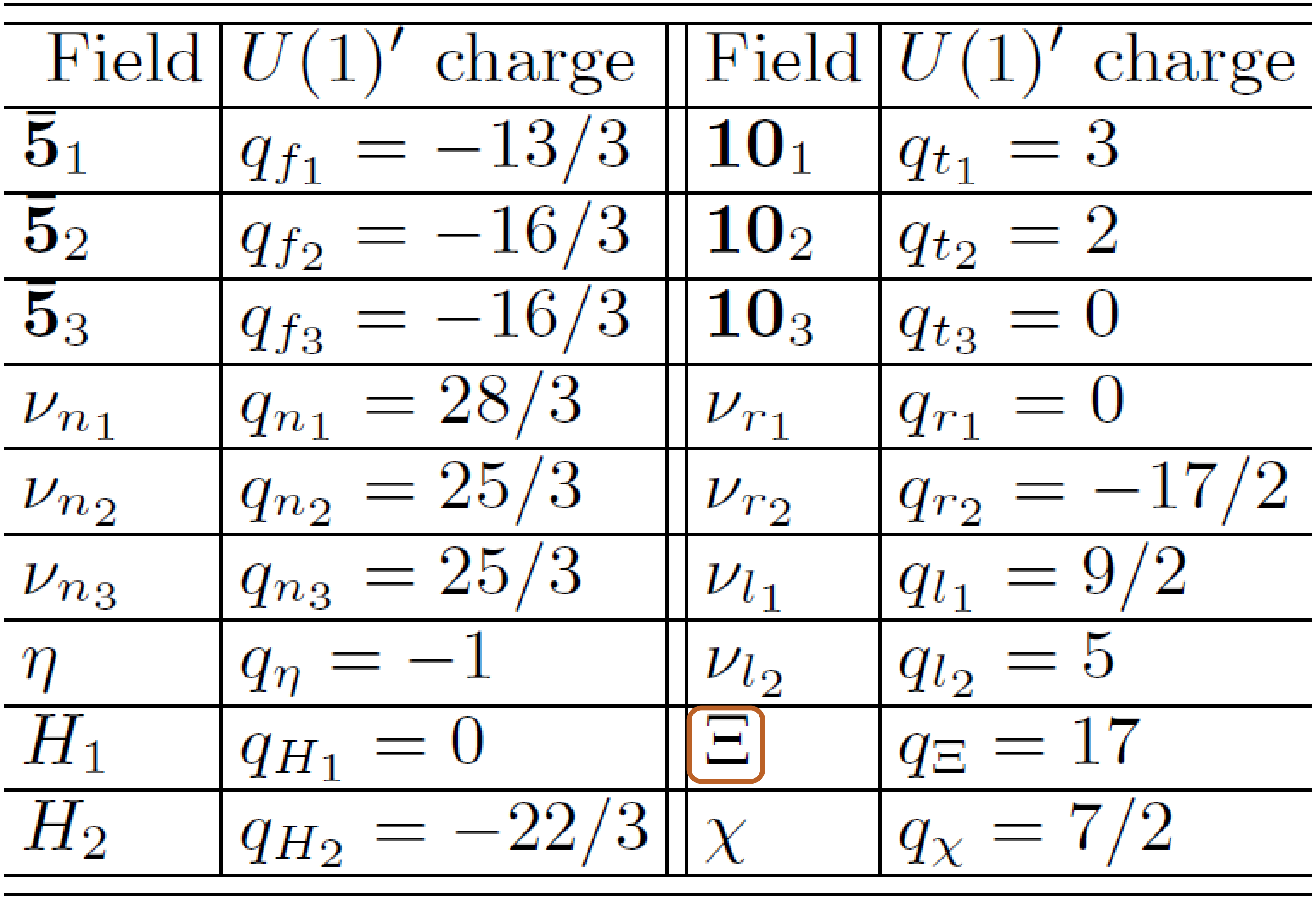
χ: vector like fermions

Integer number of η fields inserted!

$$Y_\nu \bar{L}\, \nu H_u \qquad Y_{ij} \sim \left(y_{ij} \frac{\eta}{\Lambda} \right)^{|q_i+q_j+q_H|} \sim \left(y_{ij} \lambda \right)^{|q_i+q_j+q_H|} \qquad \lambda = \frac{\langle \eta \rangle}{\Lambda} \cong 0.22$$

U(1)′ charges → suppression power → Y_{eff} pattern, μ_{eff}

U(1)' Charges



Field	$U(1)'$ charge	Field	$U(1)'$ charge
$\bar{\mathbf{5}}_1$	$q_{f_1} = -13/3$	$\mathbf{10}_1$	$q_{t_1} = 3$
$\bar{\mathbf{5}}_2$	$q_{f_2} = -16/3$	$\mathbf{10}_2$	$q_{t_2} = 2$
$\bar{\mathbf{5}}_3$	$q_{f_3} = -16/3$	$\mathbf{10}_3$	$q_{t_3} = 0$
ν_{n1}	$q_{n_1} = 28/3$	ν_{r1}	$q_{r_1} = 0$
ν_{n_2}	$q_{n_2} = 25/3$	ν_{r_2}	$q_{r_2} = -17/2$
ν_{n_3}	$q_{n_3} = 25/3$	ν_{l_1}	$q_{l_1} = 9/2$
η	$q_\eta = -1$	ν_{l_2}	$q_{l_2} = 5$
H_1	$q_{H_1} = 0$	$\boxed{\Xi}$	$q_\Xi = 17$
H_2	$q_{H_2} = -22/3$	χ	$q_\chi = 7/2$

Mass Matrices

Up-type Quark

$$M^u \sim \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon^0 \end{pmatrix} \langle H_1 \rangle$$

Down-type Quark

$$M^d \simeq \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^5 \\ \epsilon^5 & \epsilon^4 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \end{pmatrix} \langle H_2 \rangle$$

Charged Lepton $M^e \simeq (M^d)^T$

SU(5) embed

Active Neutrino

$$M^{an} \simeq \begin{pmatrix} \epsilon^{22} & \epsilon^{21} & \epsilon^{21} \\ \epsilon^{21} & \epsilon^{20} & \epsilon^{20} \\ \epsilon^{21} & \epsilon^{20} & \epsilon^{20} \end{pmatrix} \langle H_1 \rangle$$

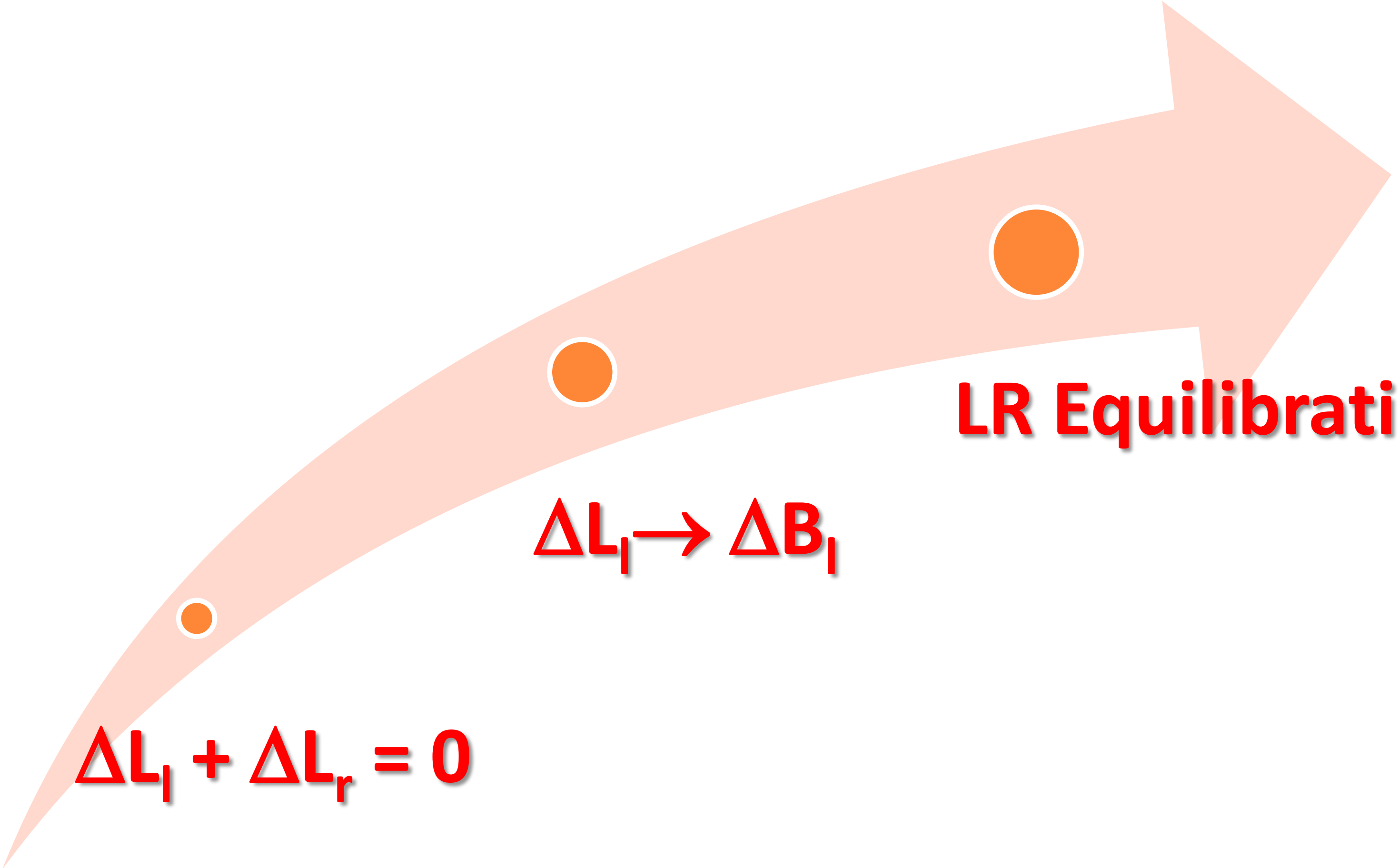
Sterile Neutrino

$$M^{lr} \simeq \begin{pmatrix} \epsilon^8 & 0 \\ 0 & 1 \end{pmatrix} \langle \chi \rangle$$

Dirac Leptogenesis

K. Dick, M. Linder, M. Ratz, D. Wright, PRL 84, 4039 (2000)

H. Murayama, A. Pierce, PRL 89, 271601 (2002)

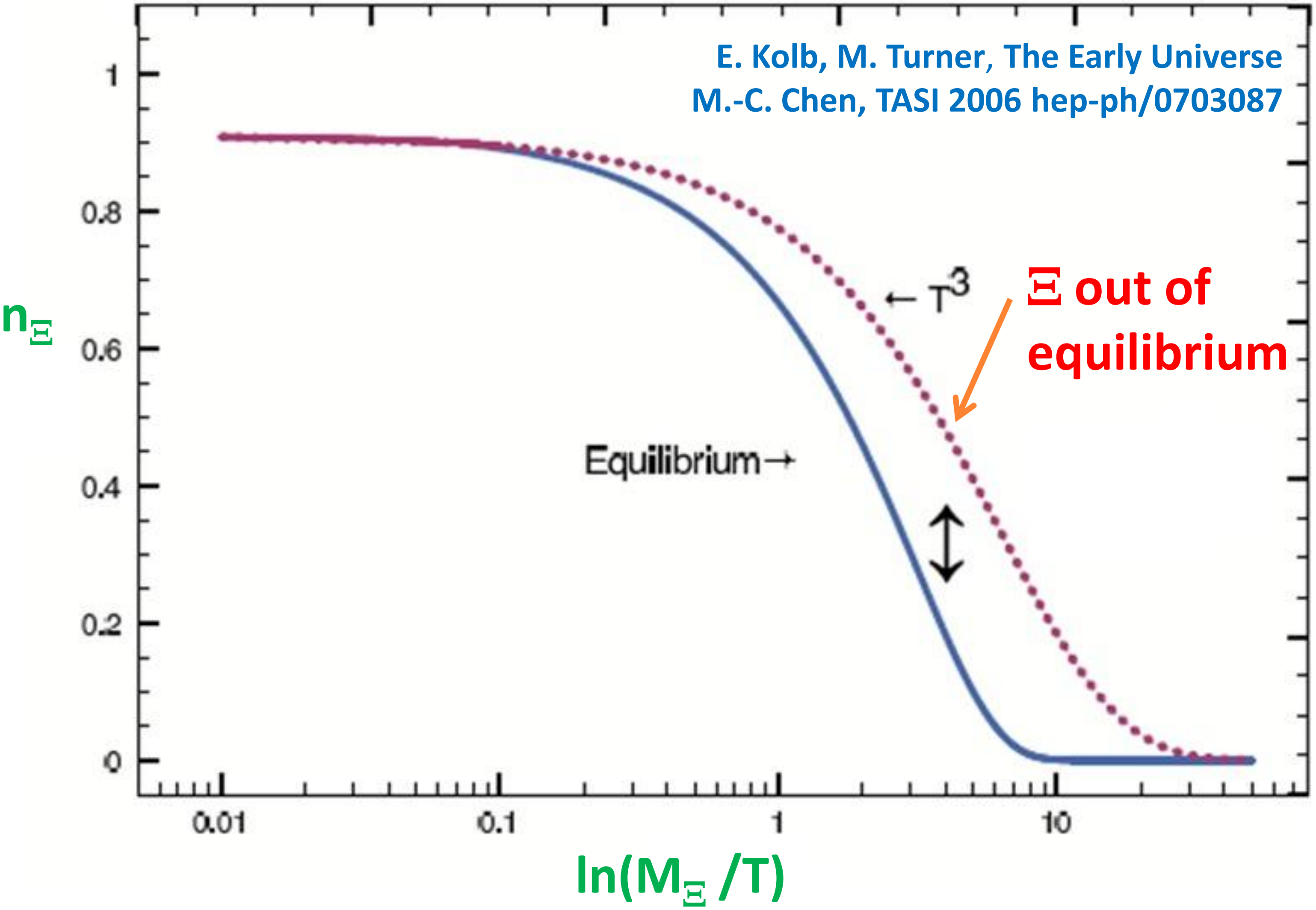


$\Delta L_l + \Delta L_r = 0$

$\Delta L_l \to \Delta B_l$

LR Equilibration

Ξ Freeze out



E. Kolb, M. Turner, The Early Universe
M.-C. Chen, TASI 2006 hep-ph/0703087

Ξ out of equilibrium

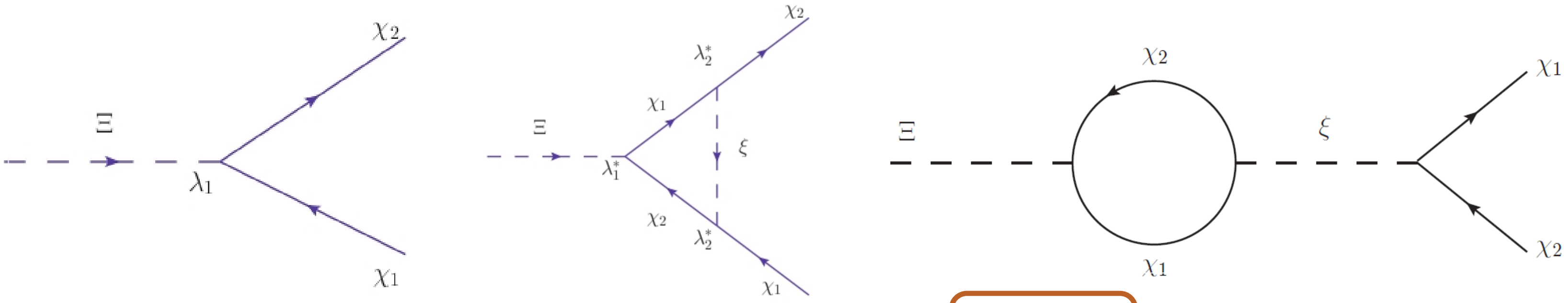
Baryon Number Asymmetry

M-C Chen, JH, W. Shepherd, JHEP 1211 (2012) 059

$$L \supset m_\Xi^2 |\Xi|^2 + m_\xi^2 |\xi|^2 + \lambda_1 \Xi \overline{\chi}_1 \chi_2 + \lambda_2 \xi \chi_1 \overline{\chi}_2$$

Once Ξ out of equilibrium, χ_1, χ_2 further decay to leptons

Interference → CP Asymmetry



CP Asymmetry:

$$\varepsilon_\Xi = \frac{8\,\mathrm{Im}[\lambda_1^2 \lambda_2^2]\,\boxed{\mathrm{Im}[I_{\Xi\xi}]}}{\Gamma_\Xi} \propto \lambda_2^2$$

B Asymmetry:

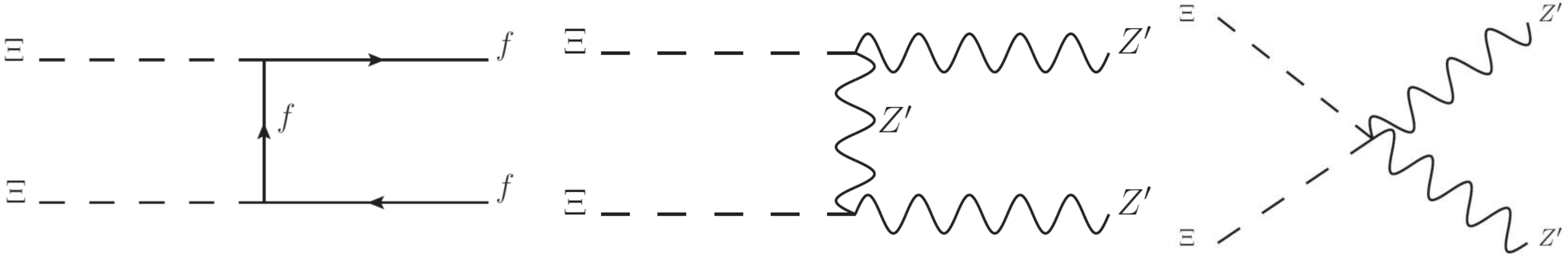
$$\eta_B \sim n_{\nu_R} \sim (\varepsilon_\Xi \boxed{\eta_\Xi} + \varepsilon_\xi \boxed{\eta_\xi})$$

Abundance

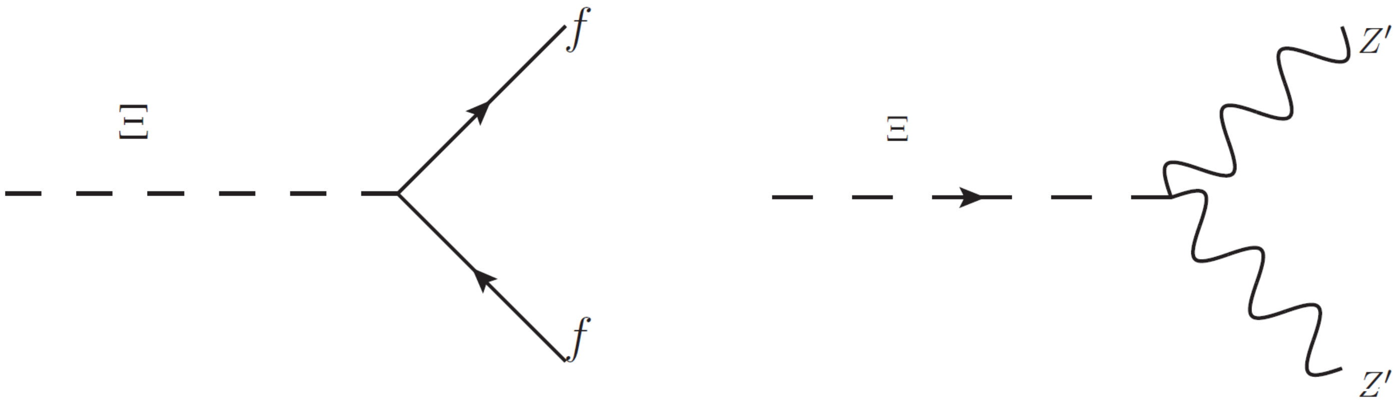
Wash Out Effects

M-C Chen, JH, W. Shepherd, JHEP 1211 (2012) 059

Annihilation:

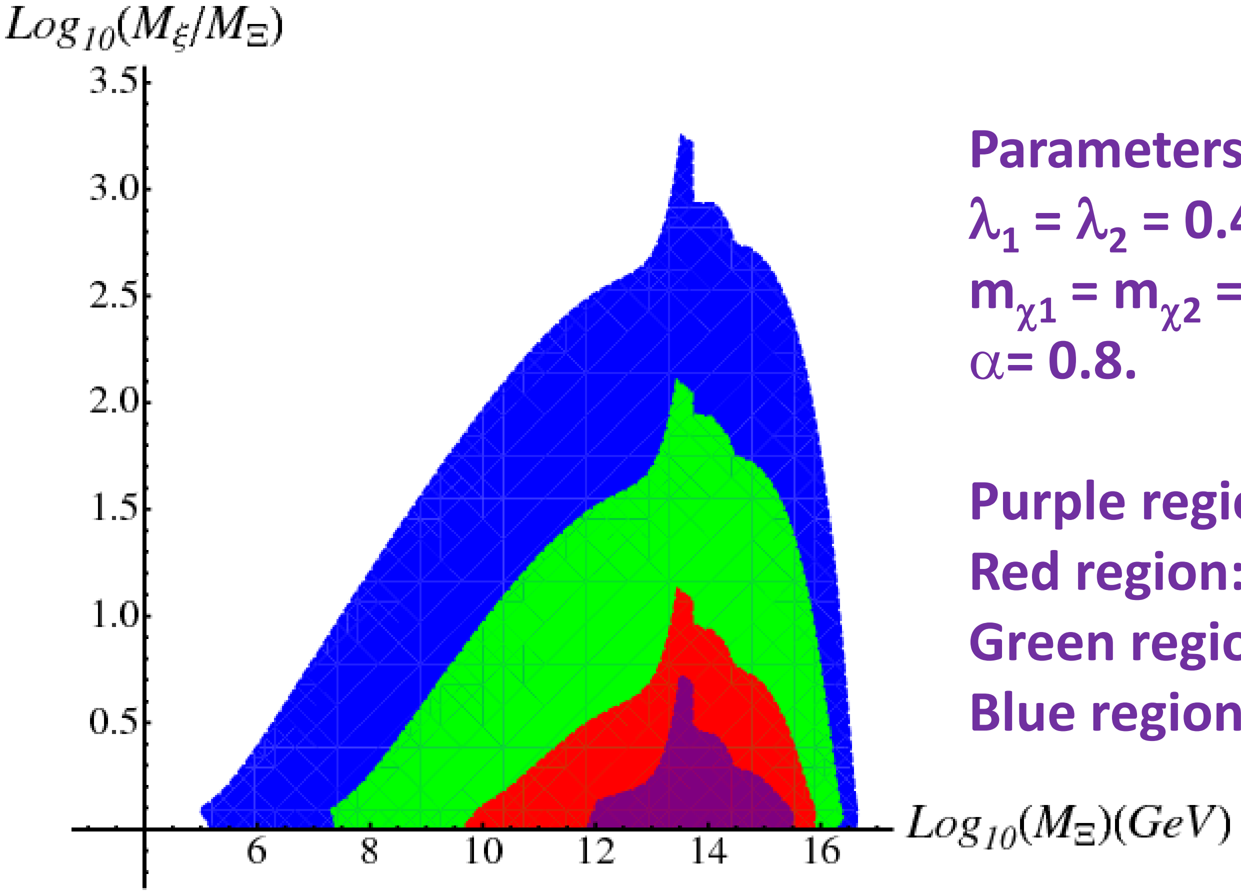


Inverse Decay:



Numerical Result

M-C Chen, JH, W. Shepherd, JHEP 1211 (2012) 059



Parameters:

$\lambda_1 = \lambda_2 = 0.4\, e^{-i\pi/8}$;

$m_{\chi 1} = m_{\chi 2} = 0.4 m_\Xi$;

$\alpha = 0.8$.

Purple region: $\eta_B \geq 6.19\times10^{-5}$

Red region: $\eta_B \geq 6.19\times10^{-6}$

Green region: $\eta_B \geq 6.19\times10^{-8}$

Blue region: $\eta_B \geq 6.19\times10^{-10}$

Conclusion

- The Dark 3+1+1 model can accommodate with the LSND, MiniBoone sterile neutrino exps. Anomaly
- It can satisfy various cosmology, collider, muon decay constraints etc.
- MeV scalar can be the dark matter candidate to explain the 511keV INTEGRAL gamma line
- With additional U(1)’ family symmetry, all fermion masses and mixings can be generated naturally
- Realistic Dirac Leptogenesis can be realized

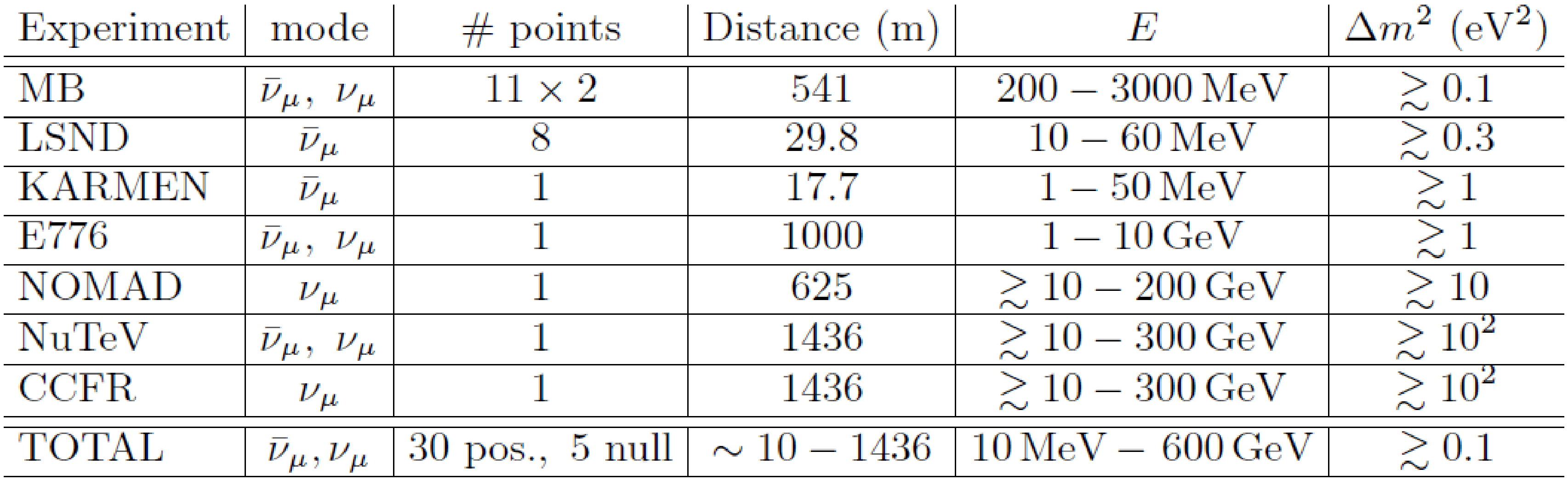
Thanks!

Back Up

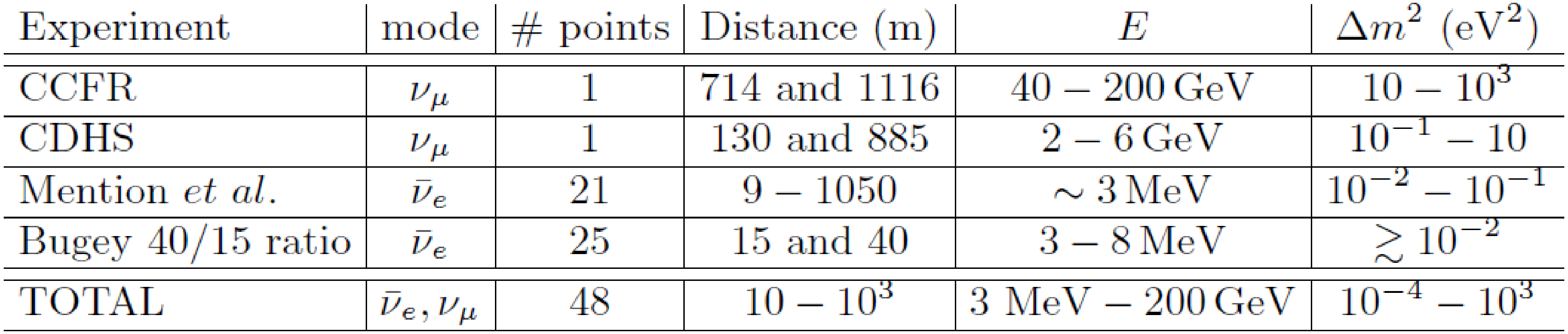
Parameter Counting

	A/P	LBL approx.	(A/P)	SBL approx.	(A/P)
3+2	9/5	$V_{35}V_{34}V_{25}O_{24}O_{23}O_{15}O_{14}V_{13}$	(8/4)	$V_{35}O_{34}V_{25}O_{24}O_{15}O_{14}$	(6/2)
3+1	6/3	$V_{34}O_{24}O_{23}O_{14}V_{13}$	(5/2)	$O_{34}O_{24}O_{14}$	(3/0)

Parameters of Neutrino Exps.



Experiment	mode	# points	Distance (m)	E	Δm^2 (eV^2)
MB	$\bar{\nu}_\mu,\ \nu_\mu$	11×2	541	$200 - 3000$ MeV	$\gtrsim 0.1$
LSND	$\bar{\nu}_\mu$	8	29.8	$10 - 60$ MeV	$\gtrsim 0.3$
KARMEN	$\bar{\nu}_\mu$	1	17.7	$1 - 50$ MeV	$\gtrsim 1$
E776	$\bar{\nu}_\mu,\ \nu_\mu$	1	1000	$1 - 10$ GeV	$\gtrsim 1$
NOMAD	ν_μ	1	625	$\gtrsim 10 - 200$ GeV	$\gtrsim 10$
NuTeV	$\bar{\nu}_\mu,\ \nu_\mu$	1	1436	$\gtrsim 10 - 300$ GeV	$\gtrsim 10^2$
CCFR	ν_μ	1	1436	$\gtrsim 10 - 300$ GeV	$\gtrsim 10^2$
TOTAL	$\bar{\nu}_\mu, \nu_\mu$	30 pos., 5 null	$\sim 10 - 1436$	10 MeV $-$ 600 GeV	$\gtrsim 0.1$



Experiment	mode	# points	Distance (m)	E	Δm^2 (eV^2)
CCFR	ν_μ	1	714 and 1116	$40 - 200$ GeV	$10 - 10^3$
CDHS	ν_μ	1	130 and 885	$2 - 6$ GeV	$10^{-1} - 10$
Mention *et al.*	$\bar{\nu}_e$	21	$9 - 1050$	~ 3 MeV	$10^{-2} - 10^{-1}$
Bugey 40/15 ratio	$\bar{\nu}_e$	25	15 and 40	$3 - 8$ MeV	$\gtrsim 10^{-2}$
TOTAL	$\bar{\nu}_e, \nu_\mu$	48	$10 - 10^3$	3 MeV $-$ 200 GeV	$10^{-4} - 10^3$

CKM Matrix

Definition:

$$L_{Yukawa} = Y^{u} Q\bar{u} H_{u} + Y^{d} Q\bar{d} H_{d}$$

$$M_{diag}^{f} = V_{L}^{f} Y^{f} V_{R}^{f+} (\upsilon / \sqrt{2})(f = u, d) \qquad V_{CKM} = V_{L}^{u} V_{L}^{d+}$$

Standard Parameterization:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}$ θ_{ij}: Mixing angle; δ_{13}: CP-violating phase

Experimental Results:

$$\sin\theta_{12} = 0.2255 \pm 0.0006; \quad \sin\theta_{23} = (4.115 \pm 0.045) \times 10^{-2};$$

$$\sin\theta_{13} = (3.61 \pm 0.12) \times 10^{-3}; \quad \delta_{13} = (69.9 \pm 3.0)^{\circ}$$

UTfit 2010

PMNS Matrix

Definition:

$$L_{Yukawa} = Y^e L\bar{e}H_d + Y^\upsilon L\bar{\nu}H_u + M_{RR}\nu\bar{\nu} + \frac{Y^{LL}}{\Lambda}LLH_uH_u$$

$$M^f_{diag} = V^f_L M^f_{eff} V^{f+}_R (f = \upsilon, e)$$

$$U_{PMNS} = V^{\upsilon+}_L V^e_L$$

Standard Parameterization:

$$U_{PMNS} = V \times diag(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}})$$

α_{21} α_{31}: Majorana CP-violating phase

Experimental Results:

G. Fogli et al., PRD84, 053007(2011)

$$\sin^2\theta_{12} = 0.306^{+0.018}_{-0.015}; \quad \sin^2\theta_{23} = 0.42^{+0.08}_{-0.03}; \quad \sin^2\theta_{13} = 0.021^{+0.007}_{-0.008}.$$

Large θ_{13}:

T2K, Double Chooz, Daya Bay, etc indicate non-zero θ_{13} ,

$$\sin^2 2\theta_{13} = 0.092 \pm 0.016(stat) \pm 0.005(syst)$$

Daya Bay @ 5.2σ