Precision Muon Lifetime Experiments

Steven Clayton
University of Illinois at Urbana-Champaign
Improved Measurement of the Positive-Muon Lifetime and Determination of the Fermi Constant

D. B. Chitwood, 1 T. I. Banks, 2 M. J. Barnes, 3 S. Battu, 4 R. M. Carey, 5 S. Cheekatmalla, 4 S. M. Clayton, 1 J. Crnkovic, 1 K. M. Crowe, 2 P. T. Debevec, 1 S. Dhamija, 4 W. Earle, 5 A. Gafarov, 5 K. Giovanetti, 6 T. P. Gorringe, 4 F. E. Gray, 1, 2 M. Hance, 5 D. W. Hertzog, 1 M. F. Hare, 5 P. Kammel, 1 B. Kiburg, 1 J. Kunkle, 1 B. Lauss, 2 I. Logashenko, 5 K. R. Lynch, 5 R. McNabb, 1 J. P. Miller, 5 F. Mulhauser, 1 C. J. G. Onderwater, 1, 7 C. S. Özben, 1 Q. Peng, 5 C. C. Polly, 1 S. Rath, 4 B. L. Roberts, 5 V. Tishchenko, 4 G. D. Wait, 3 J. Wasserman, 5 D. M. Webber, 1 P. Winter, 1 and P. A. Žolnierczuk 4

(MuLan Collaboration)

Measurement of the Muon Capture Rate in Hydrogen Gas and Determination of the Proton’s Pseudoscalar Coupling $g_P$

V. A. Andreev, 1 T. I. Banks, 2 T. A. Case, 2 D. B. Chitwood, 3 S. M. Clayton, 3 K. M. Crowe, 2 J. Deutsch, 4 J. Egger, 5 S. J. Freedman, 2 V. A. Ganzha, 1 T. Gorringe, 6 F. E. Gray, 2 D. W. Hertzog, 3 M. Hildebrandt, 5 P. Kammel, 1, 4 B. Kiburg, 3 S. Knaack, 3 P. A. Kravtsov, 1 A. G. Krivshchik, 1 B. Lauss, 2 K. L. Lynch, 7 E. M. Maev, 1 O. E. Maev, 1 F. Mulhauser, 3, 5 C. S. Özben, 3 C. Petitjean, 5 G. E. Petrov, 1 R. Prieels, 4 G. N. Schapkin, 1 G. G. Semenchuk, 1 M. A. Soroka, 1 V. Tishchenko, 6 A. A. Vasilyev, 1 A. A. Vorobyov, 1 M. E. Vznuzdaev, 1 and P. Winter 3

(MuCap Collaboration)

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Villigen, Switzerland

1.8 mA cyclotron, 590 MeV protons
MuLan: Muon Lifetime Analysis

\[ \mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \]
Muon Lifetime

- Fundamental electro-weak couplings

\[ G_F \quad \alpha \quad M_Z \]

9 ppm  0.0007 ppm  23 ppm

- \( G_F \)
  Implicit to all EW precision physics

\[ \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} (1 + \Delta r(m_t, m_H, \ldots)) \]

Uniquely defined by muon decay

\[ \frac{1}{\tau_{\mu^+}} = \frac{G_F^2 m_\mu^5}{192\pi^3} (1 + q) \]

P. Kammel
The Standard Model Fermi extraction is no longer theory limited.

$$\frac{\delta G_F}{G_F} = \frac{1}{2} \sqrt{\left(\frac{\delta \tau}{\tau}\right)^2 + \left(5 \frac{\delta m_\mu}{m_\mu}\right)^2 + \left(\frac{\delta \Delta q}{\Delta q}\right)^2}$$

<table>
<thead>
<tr>
<th>Year</th>
<th>Uncertainty</th>
<th>18 ppm</th>
<th>0.09 ppm</th>
<th>30 ppm</th>
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<td>Mid 90s:</td>
<td>17 ppm</td>
<td>18 ppm</td>
<td>0.09 ppm</td>
<td>30 ppm</td>
</tr>
<tr>
<td>1999:</td>
<td>9 ppm</td>
<td>18 ppm</td>
<td>0.09 ppm</td>
<td>&lt; 0.3 ppm</td>
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<tr>
<td>2007:</td>
<td>5 ppm</td>
<td>9.6 ppm</td>
<td>0.09 ppm</td>
<td>&lt; 0.3 ppm</td>
</tr>
</tbody>
</table>

Uncertainty on the muon lifetime error now limits the uncertainty on $G_F$.

3 exp. efforts: MuLan, FAST, RIKEN-RAL
μSR rotation results in an oscillation of the measurement probability for a given detector.

\[ \omega = g_\mu \frac{eB}{2m_\mu c} \]

\[ g_\mu \approx 2 \]

\[ N(t) = N_0 \exp\left(-\frac{t}{\tau}\right) \left[ 1 + aP \cos(\omega t + \phi) \right] \]

This oscillation is easily detected

This oscillation is not easily detected and systematic errors may arise
The experimental concept in one animation ...

Fast-switching electric kicker on Fill Period

Number (log scale)

-12.5 kV

12.5 kV

Real data

Electron energy

Relative Intensity

53 MeV

Accumulation Period

Measurement Period

Time Relative to Kicker [ns]

Counts

Kicker On

Background

100% polarized muons at ~4 MeV

Rapidly precessed here
Create a time-structured “surface” muon beam with flux of roughly $10^7 \mu^+ \text{ Hz} @ 28 \text{ MeV/c}, (\sim 4 \text{ MeV}).$
In beam, backed off
2004 Physics Results:
~10 ppm statistical uncertainty

The 7-parameter fit function includes:
- The muon lifetime,
- A flat background, and
- An independently validated electronics oscillation (with low correlation to the lifetime)

\[ \chi^2 / \text{dof} = 453 / 484 \]

The analyzers are blind to the clock frequency
The fit residuals show no structure...

... and fit start and stop time scans are consistent with statistical fluctuation.
More fit consistency

... and a host of other variables argue for consistency of the global fit.
Systematics

“Early-to-late” changes

- Instrumental shifts
  - Gain or threshold
  - Time response
  - Kicker and accidentals

- Effective acceptance
  - Residual polarization or precession
    - target: Arnokrome III (AK-3) internal ~4000 G
    - symmetric detector
    - stray muons studied

- Pileup leads to missed events

### MuLan

<table>
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<tr>
<th>Source</th>
<th>Size (ppm)</th>
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<tbody>
<tr>
<td>Extinction stability</td>
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<tr>
<td>Errant muon stops</td>
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<tr>
<td>Dead time correction</td>
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<tr>
<td>Gain stability</td>
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<tr>
<td>MTDC response</td>
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<tr>
<td>Repeated events (+1 ppm shift)</td>
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<tr>
<td>Multiple hit timing shifts</td>
<td>0.8</td>
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<tr>
<td>Queuing loss</td>
<td>0.7</td>
</tr>
<tr>
<td>Total</td>
<td>5.2</td>
</tr>
</tbody>
</table>

500 MHz WFD

B

P. Kammel
MuLan result from the 2004 Data is in excellent agreement with the world average

\[ \tau_\mu = 2197.013(21)(11) \text{ ns (11.0 ppm)} \]

MuLan goal: 1 ppm uncertainty on \( \tau_\mu \) (0.5 ppm on \( G_F \))

\[ G_F = 1.166371(5) \times 10^{-5} \text{ GeV}^{-2} \quad (4.1 \text{ ppm}) \]

First physics from MuCap

\[ \mu + p \rightarrow n + \nu_\mu \]

capture rate \( \Lambda_S \)

Proton’s pseudoscalar form factor \( g_P \)

nucleon level

\[ \mu \rightarrow W \rightarrow \nu \]
Nucleon Form Factors

\[ M = \frac{G_F}{\sqrt{2}} V_{ud} \langle \nu_\mu | \gamma^\alpha (1 - \gamma_5) | \mu \rangle \langle n | V^\alpha - A^\alpha | p \rangle_u \]

\[ V^\alpha = g_v(q^2) \gamma^\alpha + i g_m(q^2) \sigma^{\alpha\beta} \frac{q_\beta}{2M_N} + g_s(F^2) \frac{q^\alpha}{m_\mu} \]

\[ A^\alpha = g_a(q^2) \gamma^\alpha \gamma_5 + i g_t(q^2) \sigma^{\alpha\beta} \frac{q_\beta}{2M_N} \gamma_5 + g_p(q^2) \frac{q^\alpha}{m_\mu} \gamma_5 \]

CVC

EM FF's

n-decay

G-parity

quark level

nucleon level
Axialvector Form Factor $g_A$

**Neutron Decay Experiments**

- **$\beta$ Asymmetry**
  - PDG 2006
- **Lifetime**
  - Severijns et al. (2006) RMP

**Axial radius**

- $\nu+N$ scattering
  - $M_A = (1.026 \pm 0.021) \text{ GeV}$
  - $\sqrt{\langle r_A^2 \rangle} = (0.666 \pm 0.014) \text{ fm}$

consistent with $\pi$ electroproduction (with ChPT correction)

Introduces 0.45% uncertainty to $\Lambda_S$ (theory)

\[
g_a(q^2) = g_a(0) \left(1 + \frac{1}{6} \langle r_A^2 \rangle q^2 \right)
\]

\[
g_a(0) = 1.2695 \pm 0.0029
\]

\[
g_a(-0.88 m_{\mu}^2) = 1.247 \pm 0.004
\]
Pseudoscalar Form Factor $g_p$

$g_p$ determined by chiral symmetry of QCD:

$$g_p(q^2) = \frac{2m_\mu g_{\pi NN}(q^2)F_\pi}{m_\pi^2 - q^2}$$

$g_p = (8.74 \pm 0.23)$

PCAC pole term

ChPT leading order

- solid QCD prediction via ChPT (2-3% level)
- basic test of QCD symmetries

Recent reviews:

V. Bernard et al., Nucl. Part. Phys. 28 (2002), R1
Gives an expression in terms of form factors $g_V$, $g_M$, $g_A$, $g_P$.

W.f.s are solutions to the Dirac equation.

\[ e^{-iE_{\mu}t}\psi_{\mu}(\vec{x}) = e^{-iE_{\mu}t}\phi_{\mu}(\vec{x}) \begin{pmatrix} \chi_{\mu} \\ 0 \end{pmatrix}, \quad \phi_{\mu}(\vec{x}) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \]

Non-relativistic expansion to order $v_{\text{nucleon}}/c$:
- effective Hamiltonian in terms of “Primikoff factors” and Pauli matrices.
- particle states in terms of 2-spinors ($\chi$).
- results in an explicit expression for the transition rate $W$:

\[ W = \frac{C_\mu^2}{2\pi^2 a_0^3} \frac{E_\nu^2}{1 + E_\nu/\sqrt{m_n^2 + E_\nu^2}} G_V^2 (1 + 3\eta) \left( 1 - \frac{\langle \vec{\sigma} \cdot \vec{\sigma}_A \rangle \xi}{1 + 3\eta} \right) \]

total $\mu p$ spin dependence

$$\Lambda_S = W_{F=0} = 690.0 \text{ s}^{-1},$$  
$\mu p(\uparrow \downarrow)$ singlet

$$\Lambda_T = W_{F=1} = 11.3 \text{ s}^{-1},$$  
$\mu p(\uparrow \uparrow)$ triplet
Sensitivity of $\Lambda_S$ to Form Factors

$$\frac{\delta \Lambda_S}{\Lambda_S} = 2 \frac{\delta V_{ud}}{V_{ud}} + 0.466 \frac{\delta g_v}{g_v} + 0.151 \frac{\delta g_m}{g_m} + 1.567 \frac{\delta g_a}{g_a} - 0.179 \frac{\delta g_p}{g_p}$$

Contributes 0.45% uncertainty to $\Lambda_S$ (theory)

Examples:

2.4% ↔ 13.6%
1.0% ↔ 6.1%
0.5% ↔ 3.8%

$g_p$ vs. $\Lambda_S$

slope = -0.065 s (w/rad. corr.)

$\Lambda_S$ (s$^{-1}$)

\[\mu^- \text{ Stopping in Hydrogen}\]

- Quickly forms a \(\mu p\) atom, transitions to ground state, transitions to singlet hyperfine state.
  
  \[
  \text{Bohr radius } a \approx a_0 \frac{m_e}{m_\mu} \approx a_0/200
  \]

- Most of the time, the \(\mu\) decays:
  
  \[
  \mu^- \rightarrow \nu_\mu + e^- + \nu_e \quad \text{rate } \lambda_0 \approx 1/\tau_{\mu^+} \quad \text{BR} \approx 0.999
  \]

- Occasionally, it \textit{nuclear} captures on the proton:
  
  \[
  \mu^- + p \rightarrow \nu_\mu + n \quad \text{rate } \Lambda_S \quad \text{BR} \approx 10^{-3}
  \]

  \[
  \mu^- + p \rightarrow \nu_\mu + n + \gamma \quad \text{BR} \approx 10^{-8}, \ E > 60 \text{ MeV}
  \]

\textbf{The goal of } \mu \text{Cap is } \Lambda_S \text{ to 1\% precision:}

\[
\Lambda_S = \lambda - 1/\tau_{\mu^+}
\]

Complications: molecular formation/transitions, transfer to impurity atoms, …
Muon Atomic/Molecular State in Experiment must be known to connect with theory.

\[ \Lambda_s \approx 700 \text{ s}^{-1} \]

\[ \Lambda_{om} \approx \frac{3}{4} \Lambda_s \]

\[ \Lambda_{pm} \approx \frac{1}{4} \Lambda_s \]

\[ \phi = 1 \text{ (Liquid)} \]

\[ \phi = 0.01 \text{ (~10 bar gas)} \]
Previous Data on $g_p$

No common region of overlap between both expts. and theory

$g_p$ basic and experimentally least known weak nucleon form factor
Muon atomic transitions set stringent purity requirements.

\[ \Lambda_T \mu p(F=1) \]
\[ \Lambda_s \mu p(F=0) \]
\[ \Lambda_{pp} \phi \mu p(J=1) \text{ Ortho} \]
\[ \Lambda_{om} \mu p(J=0) \text{ Para} \]
\[ \Lambda_{pd} \mu d \]
\[ \Lambda_d \mu Z \]
\[ \Lambda_{pZ} c_Z \]
\[ \Lambda_{Z} \sim \Lambda_{s} Z^4 \]
\[ n + \nu_\mu + (Z-1)^* \]

H\textsubscript{2} must be pure isotopically and chemically: c\textsubscript{d} < 1 ppm, c\textsubscript{Z} < 10 ppb
μd Diffusion into Z > 1 Materials

μd scattering in $H_2$

displacement (from $\mu^-$ stop position) at time of decay

- Ramsauer-Townsend minimum in the scattering cross section

- $\mu d$ can diffuse ~10 cm before muon decay, possibly into walls.

Monte Carlo
Adamczak and Gronowski
Experimental Challenges

1) Unambiguous interpretation requires low-density hydrogen target to reduce $\mu$-molecular formation.

2) $\text{H}_2$ must be pure chemically ($c_O, c_N < 10 \text{ ppb}$) and isotopically ($c_d < 1 \text{ ppm}$).

3) All neutral final state of muon capture is difficult to detect (would require absolute calibration of neutron detectors, accurate subtraction of backgrounds).
**μCap Method: Lifetime Technique**

μCap measures the lifetime of μ⁻ in 10 bar Hydrogen.

\[ \lambda_{μ⁻} \equiv \frac{1}{τ_{μ⁻}} \]

Repeat 10\(^{10}\) times for a 10 ppm precision lifetime measurement.

Compare to μ⁺ lifetime:

\[ \lambda_{μ⁻} ≈ \lambda_{μ⁺} + Λ_S \]

\[ ⇒ Λ_S \text{ to } 1\% \text{ precision} \]
3D tracking w/o material in fiducial volume

Time Projection Chamber (TPC)

10 bar ultra-pure hydrogen, 1% LH₂
2.0 kV/cm drift field
>5 kV on 3.5 mm anode half gap
bakable glass/ceramic materials

µ Beam

µ Stop

Beam View

Side View

y

µ Stop

y

x

Z

X
3D tracking w/o material in fiducial volume

Time Projection Chamber (TPC)

10 bar ultra-pure hydrogen, 1% LH$_2$
2.0 kV/cm drift field
>5 kV on 3.5 mm anode half gap
bakable glass/ceramic materials

Observed muon stopping distribution
\( \mu \text{Cap Method: Clean } \mu \text{ Stop Definition} \)

Each muon is tracked in a time projection chamber.

Only muons stopped well-away from non-hydrogen are accepted.
μCap Detailed Diagram

- Tracking of Muon to Stop Position in Ultrapure H₂ Gas
- Tracking of Decay Electron
Commissioning and First Physics Data in 2004

(Target Pressure Vessel, Pulled Back)
Gas impurities \((Z > 1)\) are removed by a continuous \(H_2\) ultra-purification system (CHUPS).


Comissioned 2004

\(c_{N_2}, c_{O_2} \leq 0.01\) ppm
Isotopically Pure “Protium” Target

1) Generate H₂ from deuterium-depleted water (c₃ ~1 ppm)

2) On-site isotopic purifier 2006 (PNPI, CRDF)

\[ c_d < 0.006 \text{ ppm} \]
Analysis of MuCap data collected in 2004

- Led to first physics result published July 2007
- Based on $1.6 \times 10^9$ observed muon decay events
- Conditions:
  - Full muon tracking
  - Full electron tracking
  - CHUPS running ($c_Z \sim 10$ ppb)
  - DC muon beam $\sim 20$ kHz
  - No isotopic purification column ($c_d \sim 1$ ppm)
Impact Parameter Cuts
(also known as $\mu$-e vertex cuts)

The impact parameter $b$ is the distance of closest approach of the e-track to the $\mu$ stop position.
Lifetime Spectra

Electron Definition:
Scintillator Hodoscope
Full e-Detector
12 cm Impact Par. Cut

Fit of 12 cm Cut Spectrum
to \( y = 0.040 N \lambda e^{-\lambda t} + B \)

\[ \chi^2/\text{ndf} = 578.6/594 \]

\( N = 1.60 \times 10^9 \pm 4.1 \times 10^4 \)

\( \lambda = 0.455 \pm 1.2 \times 10^{-5} \mu\text{s}^{-1} \)

\( B = 2.15 \times 10^3 \pm 5.7 \times 10^0 \)

Normalized residuals ("pull")
Internal corrections to $\lambda_{\mu^-}$

<table>
<thead>
<tr>
<th>Source</th>
<th>Correction ($s^{-1}$)</th>
<th>Uncertainty ($s^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z &gt; 1$ impurities ($\Delta \lambda_Z$)</td>
<td>$-17.4$</td>
<td>$4.6$</td>
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<tr>
<td>Deuterium ($\Delta \lambda_d$)</td>
<td>$-12.1$</td>
<td>$1.8$</td>
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<tr>
<td>$\mu p$ Diffusion ($\Delta \lambda_k$)</td>
<td>$-3.1$</td>
<td>$0.1$</td>
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<tr>
<td>Unseen $\mu + p$ scatters ($\Delta \lambda_{sc}$)</td>
<td>$0.0$</td>
<td>$3.0$</td>
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<td>$\mu$ stop definition ($\Delta \lambda_{tr}$)</td>
<td>$0.0$</td>
<td>$2.0$</td>
</tr>
<tr>
<td>$\mu$ pileup veto inefficiency ($\Delta \lambda_{\kappa}$)</td>
<td>$0.0$</td>
<td>$3.0$</td>
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<tr>
<td>Analysis methods ($\Delta \lambda_{Ana}$)</td>
<td>$0.0$</td>
<td>$5.0$</td>
</tr>
<tr>
<td>Total</td>
<td>$-32.6$</td>
<td>$\pm 8.4$</td>
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</table>

(statistical uncertainty of $\lambda_{\mu^-}$: $12$ s$^{-1}$)
In situ detection of $Z > 1$ captures

TPC (side view)

Capture Time

$Z > 1$ Capture (recoil nucleus)

$\mu$ Stop

$\mu$ Beam

---

Graph showing CHUPS Flow (slpm) vs. CHUPS Connection Hours and Capture Yield (ppm) vs. Capture Time (ns).

---

40
The final $Z > 1$ correction $\Delta \lambda_Z$ is based on impurity-doped calibration data.

Lifetime deviation is linear with the $Z>1$ capture yield.

Some adjustments were made because calibration data with the main contaminant, oxygen ($H_2O$), were taken in a later running period (2006).
Internal corrections to $\lambda_{\mu^-}$

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(statistical uncertainty of $\lambda_{\mu^-}$: $12$ s$^{-1}$)
Residual deuterium content is accounted for by a zero-extrapolation procedure.

\[ \lambda \]

Production Data (d-depleted Hydrogen)

Calibration Data (Natural Hydrogen)

\[ \lambda \text{ from fits to data } (f = N\lambda e^{-\lambda t} + B) \]

Extrapolated Result

0

d Concentration (\(c_d\))

This must be determined.
$c_d$ Determination: Data Analysis Approach

$\mu_d$ can diffuse out of acceptance region:
- signal proportional to number of $\mu_d$, and therefore to $c_d$.

Fits to Lifetime Spectra

- Natural hydrogen ($c_d \approx 120$ ppm)
- d-doped target ($c_d \approx 17$ ppm)
- Production target ($c_d \approx 2$ ppm)

$\frac{c_d(\text{Production})}{c_d(\text{Natural H2})} = 0.0125 \pm 0.0010$

*after accounting for $\mu_p$ diffusion
Measurements with New ETH Zürich Tandem Accelerator:

- 2004 Production Gas,
  \[ c_D = 1.44 \pm 0.13 \text{ ppm D} \]
- 2005 Production Gas,
  \[ c_D = 1.45 \pm 0.14 \text{ ppm D} \]
- 2006 Production Gas (isotope separation column),
  \[ c_D < 0.06 \text{ ppm D} \]

The “Data Analysis Approach” gives a consistent result:

- 2004 Production Gas,
  \[ c_D = (0.0125 \pm 0.0010) \times (122 \text{ ppm D}) \]
  \[ = 1.53 \pm 0.12 \text{ ppm} \]
Consistency Checks

• lifetime vs. variations in parameters not expected to change the results
Lifetime vs eSC segment

\[ p_0 = 455431.4 \]
\[ \chi^2/NDF = 15.07/15 \]

Sum over all segments

Beam view of MuCap detector
Lifetime vs. Non-Overlapping Fiducial Volume Shell

Example TPC fiducial volume shells (red areas)

outside the standard fiducial cut

Included in standard fiducial cut

$\lambda_{fit} (s^{-1})$

$p_0 = 455429.8$

$\chi^2$/NDF = 40.05/35
Fit Start Time Scan

\[ \lambda_{\text{fit}} (s^{-1}) \times 10^3 \]

\[ \chi^2 / \text{Dof} \]

Fit Start Time (μs)
Lifetime vs. Chronological Subdivisions

\[ p_0 = 455429.1 \]

\[ \chi^2/\text{NDF} = 93.14/95 \]

Oct. 9, 2004

Nov. 4, 2004
MuCap $\Lambda_S$ from the $\mu^-$ lifetime $\lambda_{\mu^-}$

$$\lambda_{\mu^-} = \lambda_0 + \Lambda_S + \Delta \lambda_{p\mu p}$$

- $\lambda_0$: baseline lifetime
- $\Lambda_S$: bound-state effect
- $\Delta \lambda_{p\mu p}$: molecular formation

**$\mu^+$ decay rate**

$\lambda_{\mu^+} + \Delta \lambda_{\mu p}$

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<td>MuCap $\lambda_{\mu^-}$</td>
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<td>12.4 8.4</td>
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<tr>
<td>Molecular Formation ($\lambda_{OF}$) Correction</td>
<td>17.3</td>
<td>4.7</td>
</tr>
<tr>
<td>Molecular Transitions ($\lambda_{OP}$) Correction</td>
<td>5.7</td>
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<tr>
<td>Bound State Correction ($\Delta \lambda_{\mu p}$)</td>
<td>12.3</td>
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<tr>
<td>World Average $\lambda_{\mu^+}$</td>
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<tr>
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<th>Value ($s^{-1}$)</th>
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<tbody>
<tr>
<td>MuCap $\Lambda_S^a$</td>
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<td>13.6 10.6</td>
</tr>
</tbody>
</table>

Averaged with UCB result gives

$$\Lambda_S^{MuCap} = 725.0 \pm 13.7_{\text{stat}} \pm 10.7_{\text{syst}} \, s^{-1}$$
$\Lambda_S$ Calculations and MuCap (2007) Result

MuCap agrees within $\sim 1\sigma$ with $\Lambda_S$ theory

rad. corrections

- Czarnecki Marciano Sirlin (2006) $\Delta_R = 2.8\%$
Updated $g_P$ vs. $\lambda_{op}$

- MuCap 2007 result (with $g_P$ to 15%) is consistent with theory.
- This is the first precise, unambiguous experimental determination of $g_P$
Summary

• MuLan:
  – First $G_F$ update in > 23 years - no surprise
  – Factor 10 additional improvement on the way (more events; WFDs)

• MuCap:
  – First $g_P$ with non-controversial interpretation
  – Agrees with $\chi$PT expectation
  – Factor 2.5 additional improvement on the way (more events; systematics studies)
“Calibrating the Sun” via Muon Capture on the Deuteron

Motivation for the MuSun Experiment:

• First precise measurement of basic Electroweak reaction in 2N system,
• Impact on fundamental astrophysics reactions (ν’s in SNO, pp fusion)
• Comparison to modern high-precision calculations
Extra Slides
Muon-On-Demand

- Single muon requirement (to prevent systematics from pile-up)
- limits accepted $\mu$ rate to $\sim 7$ kHz,
- while PSI beam can provide $\sim 70$ kHz

Muon-On-Demand concept

- Muon-On-Demand concept
  - Kicker Plates
  - $\mu$ detector
  - TPC
  - $+12.5$ kV $-12.5$ kV
  - $50$ ns switching time

- Beamline
  - $\mu$Lan kicker
  - TRIUMF rf design
  - $2$-Dec-$2005$ kicked
  - $\sim 3$ times higher rate
μCap Experimental Strategy

- Unambiguous interpretation
  - capture mostly from F=0 μp state at 1% LH₂ density

- Lifetime method
  - $10^{10} \mu^- \rightarrow e\nu\nu$ decays
  - measure $\tau_{\mu^-}$ to 10ppm
  - $\Lambda_S = 1/\tau_{\mu^-} - 1/\tau_{\mu^+}$ to 1%

- Clean μ stop definition in active target (TPC)
  to avoid μZ capture, 10 ppm level

- Ultra-pure gas system and purity monitoring
  to avoid: $\mu p + Z \rightarrow \mu Z + p$, ~10 ppb impurities

- Isotopically pure “protium” to avoid
  $\mu p + d \rightarrow \mu d + p$, ~1 ppm deuterium

---

fulfill all requirements simultaneously
unique μCap capabilities
MuCap Collaboration


Petersburg Nuclear Physics Institute (PNPI), Gatchina, Russia
Paul Scherrer Institute (PSI), Villigen, Switzerland
University of California, Berkeley (UCB and LBNL), USA
University of Illinois at Urbana-Champaign (UIUC), USA
Université Catholique de Louvain, Belgium
TU München, Garching, Germany
University of Kentucky, Lexington, USA
Boston University, USA
Sensitivity of $\Lambda_S$ to Form Factors

$$\frac{\delta \Lambda_S}{\Lambda_S} = 2 \frac{\delta V_{ud}}{V_{ud}} + 0.466 \frac{\delta g_v}{g_v} + 0.151 \frac{\delta g_m}{g_m} + 1.567 \frac{\delta g_a}{g_a} - 0.179 \frac{\delta g_p}{g_p}$$

Contributes 0.45% uncertainty to $\Lambda_S$(theory)

Examples:

<table>
<thead>
<tr>
<th>$\frac{\delta \Lambda_S}{\Lambda_S}$</th>
<th>$\frac{\delta g_p}{g_p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4%</td>
<td>13.6%</td>
</tr>
<tr>
<td>1.0%</td>
<td>6.1%</td>
</tr>
<tr>
<td>0.5%</td>
<td>3.8%</td>
</tr>
</tbody>
</table>

$g_p$ vs. $\Lambda_S$

slope = -0.065 s (w/rad. corr.)

$\Lambda_S$ (s$^{-1}$)

\( g_p \) from \( \Lambda_S^{\text{MuCap}} = 725.0 \pm 17.4 \text{ s}^{-1} \)

\[
g^\text{MuCap}_p = g^\text{theory}_p + \frac{\partial g_p}{\partial \Lambda_S} (\Lambda_S^{\text{MuCap}} - \Lambda_S^{\text{theory}})
\]

Average HBChPT calculations of \( \Lambda_S \):
\[
\frac{687.4 \text{ s}^{-1} + 695 \text{ s}^{-1}}{2} = 691.2 \text{ s}^{-1}
\]

Apply new rad. correction (2.8%):
\[
(1 + 0.028)691.2 \text{ s}^{-1} = 710.6 \text{ s}^{-1}
\]

\[
g^\text{MuCap}_p = 8.26 + (-0.065 \text{ s}) \left( (725.0 \pm 17.4 \text{ s}^{-1}) - (710.6 \text{ s}^{-1}) \right)
\]

\[= 7.3 \pm 1.1 \quad \text{(MuCap 2007, Final)}
\]

Note: uncertainty in theory (~0.5%) not propagated.
Later decays are less likely than early decays to pass the impact parameter cut.

The effect is calculated based on:
1) the observed $F(b)$,
2) a thermal diffusion model,
3) the requirement of consistency of the $c_d$ ratio vs. $b_{cut}$ (prev. slide).
Lifetime vs. e-definition

- **CathOR**
- **CathAND**
- eSC Only

- **b<12cm**
- **no b-cut**

- All e accepted
- One e gated

(treatment of detector planes)
(impact par. cut)
(e-multiplicity)

\( \lambda_{\text{e-Def.}} - \lambda_{\text{MuCap}} \) (s\(^{-1}\))
Lifetime deviations $\Delta \lambda_Z$ due to $Z > 1$ impurities can be calculated.

Based on full kinetics solution: $y_e(t)$

Fit $y_e(t)$ to a single exponential or calculate first moment.

Fit Function:
$f(t) = A \exp(-\lambda t)$

$\lambda^{-1} = \lambda_{1st}^{-1} \equiv \frac{\int_0^\infty ty_e(t) \, dt}{\int_0^\infty y_e(t) \, dt}$

$\Delta \lambda_Z = \lambda_{1st}(c_Z = 0) - \lambda_{1st}(c_Z)$
Impurity correction scales with $Z > 1$ capture yield.

$$\beta_Z = \frac{\Delta \lambda_Z}{Y_Z}$$ is similar for C, N, and O.

We can correct for impurities based on the observed $Z > 1$ capture yield, if we know the detection efficiency $\varepsilon_Z$. 
$\mu$Cap Method: Clean $\mu$ Stop Definition

Each muon is tracked in a time projection chamber.

Only muons stopped well-away from non-hydrogen are accepted.

$0 < \Delta T_i < 22 \mu s$
Tracking in the Time Projection Chamber

1) \( \mu \) entrance, Bragg peak at stop.

2) ionization electrons drift to MWPC.

3) projection onto zx plane from anodes and strips.

4) projection onto zy plane from anodes and drift time.

5) projection onto zy plane from strips and drift time.
Muon Definition

- 2D clustering
- Stop identification
- Fiducial vol. cut

Cathode Number

Anode Number

time of signal arrival at MWPC (μs)

time of signal arrival at MWPC (μs)
Electron Definition

- timing from scintillator (eSC)
- temporal and spatial coincidences with wire chamber planes:
  full 3D tracking
Gas impurities ($Z > 1$) are removed by a continuous $H_2$ ultra-purification system (CHUPS).

Commissioned 2004

$c_{N_2}, c_{O_2} < 0.01$ ppm
Axialvector Form Factor $g_A$

Exp. History

Lattice QCD

Axial radius

$\nu+N$ scattering

$M_A = (1.026 \pm 0.021) \text{ GeV}$

$\sqrt{\langle r_A^2 \rangle} = (0.666 \pm 0.014) \text{ fm}$

consistent with $\pi$ electroproduction
(with ChPT correction)

PDG 2006


Bernard et al. (2002)

$g_A(q^2) = g_A(0)(1 + \frac{1}{6} \langle r_A^2 \rangle q^2)$

$g_A(0) = -1.2695 \pm 0.0029$

$g_A(-0.88m_\mu^2) = -1.245 \pm 0.003$

introduces 0.4% uncertainty to $\Lambda_S$ (theory)
Muon decay gives us unique access to the electroweak scale

The muon decays only via the weak interaction

The V-A theory factorizes into a pure weak contribution, and non-weak corrections, essentially uncontaminated by hadronic uncertainties.

\[ \frac{1}{\tau_{\mu^+}} = \frac{G_F^2 m_{\mu}^5}{192 \pi^3} \]

D. Hertzog
The Fermi constant is an implicit input to all precision electroweak studies.

\[
\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M^2_W} (1 + \Delta r(m_t, m_H, \ldots))
\]

Contains all weak interaction loop corrections.
CKM Summary: New $V_{us}$ & $\tau_n$ ?

- $V_{us}$ & $V_{ud}$
- New 0+ info?
- $0^+ - 0^+$ Nuclear Decay
- CKM Unitarity

New $\tau_n$!!

- UCNA Proposed
- PNPI 97
- PERKEO 02
- ILL-TPC
neutron (J. Nico, CIPANP 06)

\[ dW \propto (g_V^2 + 3g_A^2)F(E_e)\left[1 + a\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + \sigma_n \cdot \left(A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu} + D\frac{\vec{p}_e \times \vec{p}_\nu}{E_e E_\nu}\right)\right] \]

Jackson, Treiman, Wyld, Nucl. Phys. 4, 206 (1957)

**Lifetime**

\[ \tau = \frac{1}{f(1 + \delta_R) (1 + \Delta_R^V g_V^2 + 3g_A^2)} = (885.7 \pm 0.8) \text{s} \]

**Coupling ratio**

\[ \lambda = \frac{|g_A|}{|g_V|} e^{i\phi} = (-1.2695 \pm 0.0029) \]

**Electron-antineutrino asymmetry**

\[ a = \frac{1 - |\lambda|^2}{1 + 3|\lambda|^2} = (-0.103 \pm 0.004) \]

**Spin-antineutrino asymmetry**

\[ B = 2\frac{|\lambda|^2 - |\lambda|cos\phi}{1 + 3|\lambda|^2} = (0.983 \pm 0.004) \]

**Spin-electron asymmetry**

\[ A = -2\frac{|\lambda|^2 + |\lambda|cos\phi}{1 + 3|\lambda|^2} = (-0.1173 \pm 0.0013) \]

**Triple correlation**

\[ D = 2\frac{|\lambda|sin\phi}{1 + 3|\lambda|^2} = (-4 \pm 6) \times 10^{-4} \]

PDG, 2005 update
neutron
Unpublished analysis of MuCap $\mu^+$ data taken in 2004