The FVTX Strip Location Project
(a work in progress)

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1 Introduction

The precise determination of the location of the strips in the FVTX detector is critical to the proper understanding of the track data recorded by it. This document will describe the steps involving in locating the strips in the PHENIX IR.

2 Outline

1. Specify the design locations of the strips on a sensor
2. Specify the design locations of the strips in the full detector
3. Locate the strips on a single sensor, with respect to the sensor reference marks
4. Locate the strips on all the sensors on a half-disk, using measured locations of reference marks
5. Locate the strips on the sensors on a half-disk using surveyed locations of the survey targets
6. Refinement of strip locations using track data

3 Naming and Numbering Scheme

A given strip is identified by the following of set of locators:

- Arm: North ($A = 1$) or South ($A = 0$); the North arm has $z > 0$, the South $z < 0$;
- Station: numbered $N = 0 - 3$, according to increasing value of $|z|$;
- Wedge: numbered $W = 0 - 47$, starting nearest $\phi = 0$ and then according to increasing value of ϕ;
- Column: numbered $C = 0 - 1$, according to increasing value of ϕ;
- Strip: numbered $I = 0 - 639$ (small sensor) or $0 - 1663$ (large sensor); strips nearest the beam are numbered 0.

So there are 2 arms, each arm has 4 stations, each station has 48 wedges, each wedge has 2 columns, and each column has 640 or 1664 strips. The orientation of the North and South arms in the PHENIX Coordinate System, and the numbering of the wedges, are illustrated in Figure 1.

4 Coordinate Systems, and Coordinates

Three coordinate systems will be discussed here.

- **Sensor Coordinate System** S: This is a two-dimensional system for a single sensor, defined by the manufacturer's production drawings.
- **Disk Coordinate System** D: This is a two-dimensional system describing the sensors and survey targets on a half-disk.
- **PHENIX Coordinate System** P: This is the system defined in the PHENIX IR. All of the sensors have expected locations in this system according to the design drawings, but the actual locations will be determined first by a combination of survey and CMM data. Eventually we expect to refine our knowledge of the sensor locations using track data.

In this document, the unit of measure in all these coordinate systems will be microns.

There are also a variety of coordinates defined within these coordinate systems.

- **Strip locations on a Sensor**: These are described by the sensor design drawings, are defined in the sensor coordinate system, S, and are labeled (x_I^S, y_I^S).

- **Design strip locations on a Half-Disk**: These are described by the mechanical design of the half-disk, are defined in the disk coordinate system, D, and are labeled as (x_I^D, y_I^D).

- **Measured strip locations on a Half-Disk**: These are determined by measurements using a Coordinate Measuring Machine (CMM) on a half-disk. They are defined in the disk coordinate system, D, and are labeled as (x_I^{MD}, y_I^{MD}). Each half-disk will be measured by the CMM in the same way. This description of the half-disk can be treated as a solid body which we then follow through the rest of the installation.

- **Design strip locations in PHENIX**: These are described by the mechanical design of the FVTX, are defined in the PHENIX coordinate system, P, and are labeled as $(x_I^{(0)}, y_I^{(0)}, z_I^{(0)})$. This is the zeroth-order estimate of the location of the strips.

- **Measured strip locations in PHENIX**: These are determined by CMM measurements on a half-disk, (x_I^{MD}, y_I^{MD}), combined with design information about the location and orientation of half-disks in the FVTX. They are defined in the PHENIX coordinate system, P, and are labeled as $(x_I^{(1)}, y_I^{(1)}, z_I^{(1)})$. This is our next-best estimate of the location of the strips, after the design locations.

- **Survey strip locations in PHENIX**: These are determined by a combintaion of CMM disk measurements and surveys of the survey targets in the PHENIX IR. They are defined in the PHENIX coordinate system, P, and are labeled as $(x_I^{(2)}, y_I^{(2)}, z_I^{(2)})$. This is our best estimate of the location of the strips, before looking at track data.

What are the relationships between all these sets of coordinates?

- **The Designed Detector**: The strip locations on a single sensor (x_I^S, y_I^S) are propagated to the *design* locations of the sensors on a half-disk to produce the coordinates (x_I^D, y_I^D) and then onto the *design* construction of the half-cages to produce the coordinates $(x_I^{(0)}, y_I^{(0)}, z_I^{(0)})$. These describe a detector which has been constructed exactly according to the design.

- **The Measured Detector**: The strip locations on a single sensor (x_I^S, y_I^S) are propagated to the *measured* locations of the sensors on a half-disk to produce the coordinates (x_I^{MD}, y_I^{MD}); these should not differ very much from the design values (x_I^D, y_I^D). These CMM measurements of the half-disks can then be propagated to the design construction of the half-cages to produce the coordinates $(x_I^{(1)}, y_I^{(1)}, z_I^{(1)})$. The survey in the IR will tell us how to transform the $(x_I^{(1)}, y_I^{(1)}, z_I^{(1)})$ (which describe a rigid body) to the $(x_I^{(2)}, y_I^{(2)}, z_I^{(2)})$.

5 Design Location of the Strips

An good understanding of the information in this section is probably not possible without having a look at Figures 2, 3, 4, 5, 6 and 7 while you are reading.

5.1 Tabular Information on the Design Locations of the Wedges and Strips

Each station has four planes of sensors; in the drawing package, the mid-plane $|z|$ of the station is given, and then the distance Δz of the sensor from the mid-plane is specified. The Δz values are different for North and South, because the ordering of the wedges is reversed between the two arms.

Station Number	$\|z\|$ of mid-plane	
0	194000	microns
1	251000	microns
2	305980	microns
3	360480	microns

Table 1: Location of the mid-plane of each station.

Wedge Number (Modulo 4)	Δz of sensor South ($A = 0$)		North ($A = 1$)	
0	-8860	microns	-8860	microns
1	+8860	microns	+4920	microns
2	-4920	microns	-4920	microns
3	+4920	microns	+8860	microns

Table 2: Δz of sensors, with respect to the mid-plane of each station. The sign of these values are with respect to the IP; a positive value means the sensor is farther away from the IP than the mid-plane of the station.

The azimuthal angle locations ϕ of the wedges in the north and south arms are different, because of the $3.75°$ overall rotation of the assembly. In order to maintain a rule that "wedges facing towards the IP should be even-numbered" then the ϕ value for wedge-0 is different in north and south.

ϕ_0^A: ϕ of wedge-0 in arm A	
South ($A = 0$)	North ($A = 1$)
+3.75 degrees	-3.75 degrees

Table 3: ϕ of wedge-0, in the north and south arms.

A number of other parameters are needed in order to specify the locations of the wedges and strips.

$\Delta\phi$ between wedges	7.5	degrees
Radial location of the wedge "base point"	44000	microns
Perpendicular distance between strips (the "pitch")	75	microns
Angle of the centerline of the strips, with respect to the sensor centerline	1.875	degrees
Δx between strips (in the sensor system) $= 75\sin(1.875°)$	2.45	microns
Δy between strips (in the sensor system) $= 75\cos(1.875°)$	74.96	microns

Table 4: Parameters for specifying the locations of the strips.

5.2 Strip locations in the Sensor Coordinate System

This is the easiest and most precise part of the process, because the strips are located on the sensor by the manufacturer with a precision on the order of a micron. The information in this subsection is based entirely on drawings provided by the manufacturer Hamamatsu; see Figures 6 and 7.

The coordinate system defined in the sensor drawing will be called the *sensor coordinate system* and denoted by S. The origin of S is at the very bottom center of the sensor and is called the "base point (0,0)". The units in this coordinate system are microns.

The drawing identifies a line, called the "1.875° line", which goes through the center of every strip. There is also a "center line" going through the center of the sensor and a "3.75° line" — all of these lines converge at a center point located at $(x, y) = (0, -44000)$. In principle, this center point is the location of the beam axis.

The strip at the bottom of each ladder is located in S at $y = 1000$. The pitch (distance between strips) along the 1.875° line is 75; so the distance between strip centers in (x, y) is

$$(\Delta x, \Delta y) = (\pm 75 \sin 1.875^\circ, 75 \cos 1.875^\circ) = (\pm 2.45, 74.96)$$

where the "+" refers to the strips on the $+x$ side (column $C = 0$) and "$-$" refers to strips on the -x side (column $C = 1$). In terms of C this can be rewritten

$$\Delta x = (1 - 2C) 75 \sin 1.875^\circ.$$

An equation for the 1.875° line is

$$y = -44000 + (1 - 2C)(\cot 1.875^\circ)x,$$

where the $(1-2C)$ factor gives the correct sign for the slope. If the bottom strip is numbered $I = 0$, then the locations of the centers of the strips in S are

$$y_I^S = 1000 + I \cdot \Delta y \qquad x_I^S = (1 - 2C)(y_I^S + 44000) \tan 1.875^\circ$$

where $I = 0...1663$ for a large sensor and $I = 0...639$ for a small sensor.

5.3 Design Strip Locations throughout the Detector

Combining the information about the expected wedge locations in Section 5.1 with the information about the strip locations on a single sensor in Section 5.2, we can translate and rotate the strip locations in S to their design locations in the full detector.

For a strip located in arm A, station N, Wedge W, Column C, and Strip I, we start with the location (x_I^S, y_I^S) in the S system. First we translate this to the correct radius of the sensors; starting from the S system this means we add 44000 to the y coordinate. Then we rotate to the ϕ of the wedge. The resulting transformation looks like this:

$$x_I = x_I^S \cos(90 - \phi_W) + (y_I^S + 44000) \sin(90 - \phi_W)$$
$$y_I = -x_I^S \sin(90 - \phi_W) + (y_I^S + 44000) \cos(90 - \phi_W)$$
$$z_I = z_W$$

where $\phi_W = \phi_0^A + W\Delta\phi$ with ϕ_0^A determined from Table 3, and z_W is determined from Tables 1 and 2.

Note that the two-dimensional coordinates (x_I, y_I) computed above are the design locations for the strips in a half-disk, (x_I^D, y_I^D). Once you have decided exactly where a given half-disk will be placed in the full assembly, then the full three-dimensional coordinates (x_I, y_I, z_I) are the design locations $(x_I^{(0)}, y_I^{(0)}, z_I^{(0)})$ in the PHENIX system P.

6 Strip Locations with respect to Sensor Reference Marks

Each sensor has six reference marks, which I will call $R0$ through $R5$. (See Figure 8.) Three are near the bottom of the sensor and are located in S at

$$(x^S_{R0}, y^S_{R0}) = (-a_x, a_y) \qquad (x^S_{R1}, y^S_{R1}) = (0, a_y) \qquad (x^S_{R2}, y^S_{R2}) = (+a_x, a_y).$$

Three are located near the top, at

$$(x^S_{R3}, y^S_{R3}) = (-b_x, b_y) \qquad (x^S_{R4}, y^S_{R4}) = (0, b_y) \qquad (x^S_{R5}, y^S_{R5}) = (+b_x, b_y).$$

The values of the reference mark location parameters, for the large sensor, are

$$a_x = 3000 \qquad b_x = 10000 \qquad\qquad a_y = 200 \qquad b_y = 126585.$$

After sensors have been placed in wedges, and wedges have been placed in a half-disk, then the location of the reference marks on each sensor in a half-disk will be measured using a Coordinate Measuring Machine (CMM).

For half of the sensors the reference marks at $(\pm a_x, a_y)$ ($R0$ and $R2$) will be obscured by overlapping wedges and thus will not be observable by the CMM; we will ignore those two marks and use only the remaining four reference marks so that all sensors are treated the same way.

The use of the CMM data is broken into the following four steps:

1. Determine the design locations of the reference marks on a half-disk;
2. Measure the location of the reference marks on an actual half-disk using the CMM;
3. Compare the design and CMM locations of the reference marks to find a coordinate system in which the half-disk has the correct origin and orientation; this is the D coordinate system; transform all the CMM data to this new coordinate system; these are now the MD coordinates discussed in Section 4;
4. Compute the strip locations in the D system; these are the MD coordinates for the strips.

6.1 Design Locations of the Reference Marks

We gave the locations of the reference marks in S earlier. Then the reference marks on a wedge **facing the $+z$-direction** with azimuthal angle ϕ_W are located at

$$x^D_{W,Ri} = x^S_{Ri} \cos(90 - \phi_W) + (y^S_{Ri} + 44000) \sin(90 - \phi_W)$$

$$y^D_{W,Ri} = -x^S_{Ri} \sin(90 - \phi_W) + (y^S_{Ri} + 44000) \cos(90 - \phi_W).$$

For sensors **facing the $-z$-direction**, we must switch the sign of the x-coordinates in S before making the transformation:

$$x^D_{W,Ri} = (-x^S_{Ri}) \cos(90 - \phi_W) + (y^S_{Ri} + 44000) \sin(90 - \phi_W)$$

$$y^D_{W,Ri} = (+x^S_{Ri}) \sin(90 - \phi_W) + (y^S_{Ri} + 44000) \cos(90 - \phi_W).$$

An expression which gives this "z-facing sign factor" is

$$F_z = (2A - 1)[1 - 2(W \bmod 2)].$$

Then we can have a single expression for the transformation:

$$x^D_{W,Ri} = F_z x^S_{Ri} \cos(90 - \phi_W) + (y^S_{Ri} + 44000) \sin(90 - \phi_W)$$

$$y^D_{W,Ri} = -F_z x^S_{Ri} \sin(90 - \phi_W) + (y^S_{Ri} + 44000) \cos(90 - \phi_W).$$

6.2 Transformation to the MD Coordinates in the D Coordinate System

The origin and orientation of the Disk Coordinate System, D, is defined in terms of the PHENIX coordinate system. Its origin is on the beam axis; it has x and y axes parallel to that of PHENIX; and each half-disk has a given location in z. However, there is no artefact on the half-disk itself that locates the origin or orientation, so we will determine the origin and orientation in terms of the locations of whole set of sensor reference marks on the sensors mounted on the half-disk in question; that is, we will depend on the machine accuracy of the assembly fixtures and the sensors themselves to provide a best estimate for the D system origin and orientation.

It will be difficult to place each half-disk on the CMM platform such that the proper disk origin and orientation are in alignment with that of the CMM. So, the approach taken here will be to measure the sensor reference marks in a temporary CMM coordinate system C, and then establish the disk system D from those data.

For a given half-disk, the reference marks of each sensor have an expected location in the disk coordinate system. We can use the CMM data combined with these expected locations to find the best translation and rotation that converts the CMM data to the disk system D. The coordinates in that system are now the measured disk coordinates which we call MD.

Let the expected locations of the reference marks in D be $(x^D_{i,Rj}, y^D_{i,Rj})$, where the subscript i labels the sensor and j labels the reference mark Rj on sensor i. The measured locations in the temporary CMM system C will be labeled $(x^C_{i,Rj}, y^C_{i,Rj})$. The transformation from the CMM system C to the disk system D will be defined in terms of a translation $\vec{V}^{CD} = (V^{CD}_x, V^{CD}_y)$ to the new origin followed by a rotation through an angle ϕ^{CD} about the new origin:

$$
\begin{aligned}
x^{MD} &= +(x^C + V^{CD}_x)\cos\phi^{CD} + (y^C + V^{CD}_y)\sin\phi^{CD} \\
y^{MD} &= -(x^C + V^{CD}_x)\sin\phi^{CD} + (y^C + V^{CD}_y)\cos\phi^{CD}.
\end{aligned}
$$

We can define a deviation function (a χ^2) to help us find the best values for $\vec{V}^{CD}$ and ϕ^{CD}:

$$
\chi^2 = \sum_{i,j}(x^D_{i,Rj} - x^{MD}_{i,Rj}(\vec{V}^{CD}, \phi^{CD}))^2 + \sum_{i,j}(y^D_{i,Rj} - y^{MD}_{i,Rj}(\vec{V}^{CD}, \phi^{CD}))^2.
$$

Minimization of this function will determine the best values for the transformation from the CMM system C to the disk system D. This establishes the disk system D. We now convert all the CMM data to be MD coordinates in the D system.

6.3 Measured Strip Locations in the MD Coordinates

Now we have the locations of the reference marks in the MD coordinates. The next step is to locate the strips on each sensor. We need to be able to specify the location of the sensor base point, and the orientation of the sensor, in order to determine the strip locations from the CMM measurements.

6.3.1 Determination of the Sensor Orientation

The orientation is specified as an azimuthal angle ϕ. There are two methods to determine ϕ, depending on whether the sensor faces along the $+z$-direction or along the $-z$-direction. We can make use of the "z-facing sign factor" F_z defined in Section 6.1 to take care of the difference. The two methods are related by a phase change of π. For a sensor facing along the $-z$-direction, the reference mark $R3$ will be on the "low ϕ" side of the sensor, and $R5$ will be on the "high ϕ" side. Then these two expressions for the orientation angle should give the same result:

$$\phi_{1,4} = \arctan \frac{y_{R4} - y_{R1}}{x_{R4} - x_{R1}} \qquad \phi_{5,3} = \arctan \frac{y_{R3} - y_{R5}}{x_{R3} - x_{R5}} - \frac{\pi}{2}.$$

In case the sensor is facing along the $+z$-direction, then $R3$ will be on the "high ϕ" side and $R5$ will be on the "low ϕ" side, and the two expressions are

$$\phi_{1,4} = \arctan \frac{y_{R4} - y_{R1}}{x_{R4} - x_{R1}} \qquad \phi'_{5,3} = \arctan \frac{y_{R5} - y_{R3}}{x_{R5} - x_{R3}} - \frac{\pi}{2}.$$

The calculation of ϕ from $R1$ and $R4$ is unchanged; it is the calculation using $R3$ and $R5$ that is adjusted. Using the factor F_z we can write a single set of expressions:

$$\phi_{1,4} = \arctan \frac{y_{R4} - y_{R1}}{x_{R4} - x_{R1}} \qquad \phi_{5,3} = \arctan \frac{F_z(y_{R3} - y_{R5})}{F_z(x_{R3} - x_{R5})} - \frac{\pi}{2}.$$

N.B. — in all cases, we use the function atan2(y,x) to get the right phase for the arctangent $\arctan(y/x)$. That's why the factor F_z doesn't actually "cancel out" in the expression for $\phi_{5,3}$ above.

Now we have established the orientation ϕ of the sensor.

6.3.2 Determination of the Sensor Base Point

The base point lies along the sensor centerline, which is determined by reference marks $R1$ and $R4$. The equation for that line is

$$(y - y_{R1}) = (x - x_{R1}) \tan \phi.$$

The base point is located a distance of $a_y = 200$ microns from the $R1$ point, in a direction opposite to that of the $R4$ point. Starting at $R1$, the shifts in x and y needed to reach the base point are

$$\Delta x = -200 \cos \phi \qquad \Delta y = -200 \sin \phi$$

and so the location of the base point is

$$(x_B, y_B) = (x_{R1} - 200 \cos \phi, y_{R1} - 200 \sin \phi).$$

6.3.3 Determination of Strip Locations

Once the orientation ϕ and the base point location (x_B, y_B) of the sensor are known, then we can rotate the S-system strip locations to the proper orientation and translate the origin to the new base point. For a strip with location (x_I^S, y_I^S) in S, the location determined from the CMM measurements is

$$x_I^{MD} = [x_I^S \cos(90 - \phi) + y_I^S \sin(90 - \phi)] + x_B$$
$$y_I^{MD} = [-x_I^S \sin(90 - \phi) + y_I^S \cos(90 - \phi)] + y_B.$$

6.4 Locations in the Detector based on the CMM Measurements

At this point we can propogate the MD strip coordinates to the $(x_I^{(1)}, y_I^{(1)}, z_I^{(1)})$ values by selecting the design value for the z locations and rotating (if necessary) the half-disk to its design orientation.

7 Strip Locations with respect to Survey Targets

The half-disks will include, in addition, a set of survey targets; precision pins that converge to a sharp point. Right now the number and location of these targets is not known, but let us say for now that there are N_T survey targets on a half-disk with design locations (x^D_{Ti}, y^D_{Ti}). Their locations will be measured with the CMM along with the reference marks. Therefore, the relative locations of the reference marks and survey targets on a single half-disk will be known very well, about 5 microns in x and y, the precision of the CMM. Combined with the design locations of the half-disks, we can then produce a first guess at where the survey targets will be located in the IR, $(x^{(1)}_{Ti}, y^{(1)}_{Ti}, z^{(1)}_{Ti})$.

These survey targets will be used to locate the half-disks in the PHENIX IR by the BNL survey crew. The survey crew will provide us with coordinates $(x^P_{Ti}, y^P_{Ti}, z^P_{Ti})$ for the targets in the PHENIX IR coordinate system. We will use these values to determine the locations of the strips in the PHENIX IR.

The relative locations of the survey targets on a half-disk, as mentioned above, will be very well known from the CMM measurements. In this respect, we can regard each half-disk as a single rigid body. The BNL survery will provide relatively poor measurements of the location of several points (the survey targets) on this rigid body. We can use this data to provide a best guess as to the location of the half-disk in the IR. We will express this as a transformation from the CMM-measured survey target positions $(x^{(1)}_{Ti}, y^{(1)}_{Ti}, z^{(1)}_{Ti})$ to "fitted" values of the surveyed positions $(x^{(2)}_{Ti}, y^{(2)}_{Ti}, z^{(2)}_{Ti})$. In general this is a translation of the origin $(\Delta X^P, \Delta Y^P, \Delta Z^P)$ followed by a rotation defined in terms of Euler angles (ϕ, θ, ψ):

$$\begin{bmatrix} x^{(2)}_{Ti} \\ y^{(2)}_{Ti} \\ z^{(2)}_{Ti} \end{bmatrix} = \begin{bmatrix} c_\psi c_\phi - c_\theta s_\phi s_\psi & c_\psi s_\phi + c_\theta c_\phi s_\psi & s_\psi s_\theta \\ -s_\psi c_\phi - c_\theta s_\phi c_\psi & -s_\psi s_\phi + c_\theta c_\phi c_\psi & c_\psi s_\theta \\ s_\theta s_\phi & -s_\theta c_\phi & c_\theta \end{bmatrix} \begin{bmatrix} x^{(1)}_{Ti} + \Delta X^P \\ y^{(1)}_{Ti} + \Delta Y^P \\ z^{(1)}_{Ti} + \Delta Z^P \end{bmatrix}$$

where $c_\theta = \cos\theta$, etc. The Euler angles (ϕ, θ, ψ) are defined in the "x-convention"; see this weblink for a useful reminder.

`http://mathworld.wolfram.com/EulerAngles.html`

The best values for the origin translation and Euler angles are determined by a χ^2 comparison between the actual survey of the target locations and the fit values:

$$\chi^2 = \sum_{i=1}^{N_T} \left[(x^P_{Ti} - x^{(2)}_{Ti})^2 + (y^P_{Ti} - y^{(2)}_{Ti})^2 + (z^P_{Ti} - z^{(2)}_{Ti})^2 \right].$$

Then the CMM-determined strip locations $(x_I^{(1)}, y_I^{(1)}, z_I^{(1)})$ can be transformed to the surveyed strip locations $(x_I^{(2)}, y_I^{(2)}, z_I^{(2)})$ using that transformation:

$$\begin{bmatrix} x_I^{(2)} \\ y_I^{(2)} \\ z_I^{(2)} \end{bmatrix} = \begin{bmatrix} c_\psi c_\phi - c_\theta s_\phi s_\psi & c_\psi s_\phi + c_\theta c_\phi s_\psi & s_\psi s_\theta \\ -s_\psi c_\phi - c_\theta s_\phi c_\psi & -s_\psi s_\phi + c_\theta c_\phi c_\psi & c_\psi s_\theta \\ s_\theta s_\phi & -s_\theta c_\phi & c_\theta \end{bmatrix} \begin{bmatrix} x_I^{(1)} + \Delta X^P \\ y_I^{(1)} + \Delta Y^P \\ z_I^{(1)} + \Delta Z^P \end{bmatrix}.$$

This process needs to be carried out for each half-disk.

8 Figures

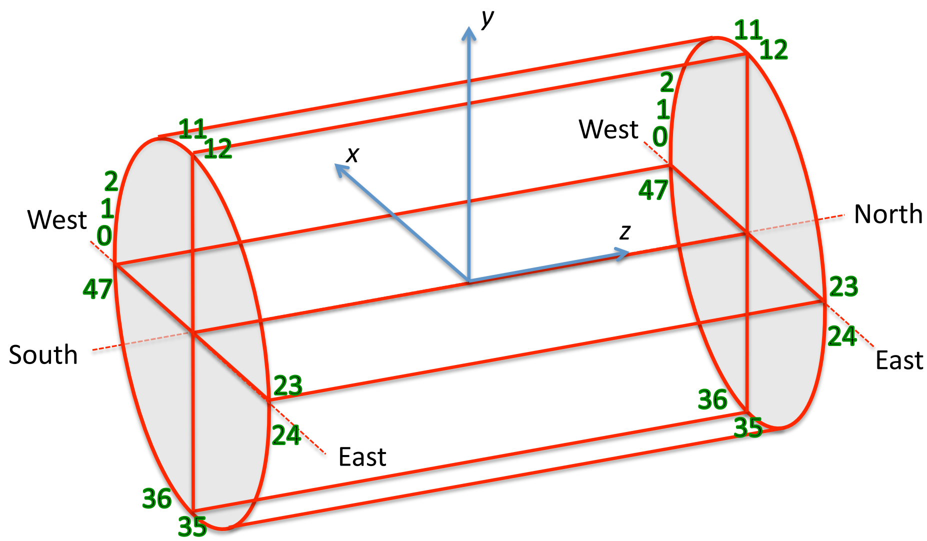

Figure 1: FVTX endcaps (North and South) and wedge numbering (0-47) in the PHENIX Coordinate system.

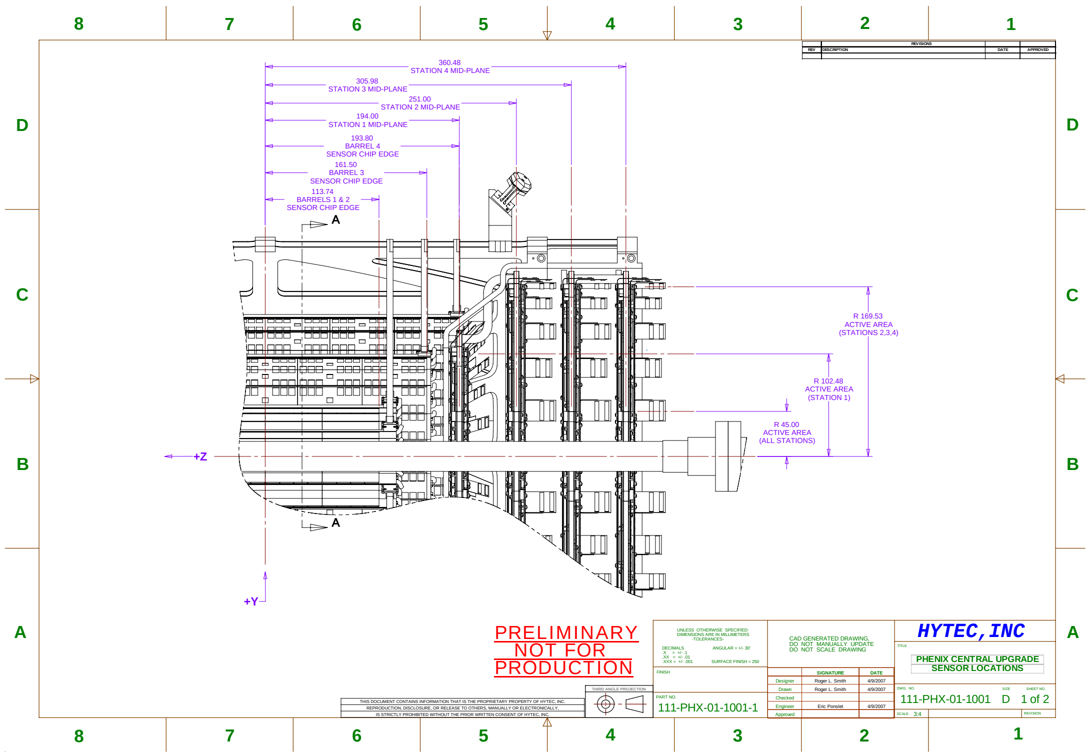

Figure 2: FVTX Stations, specifying z locations.

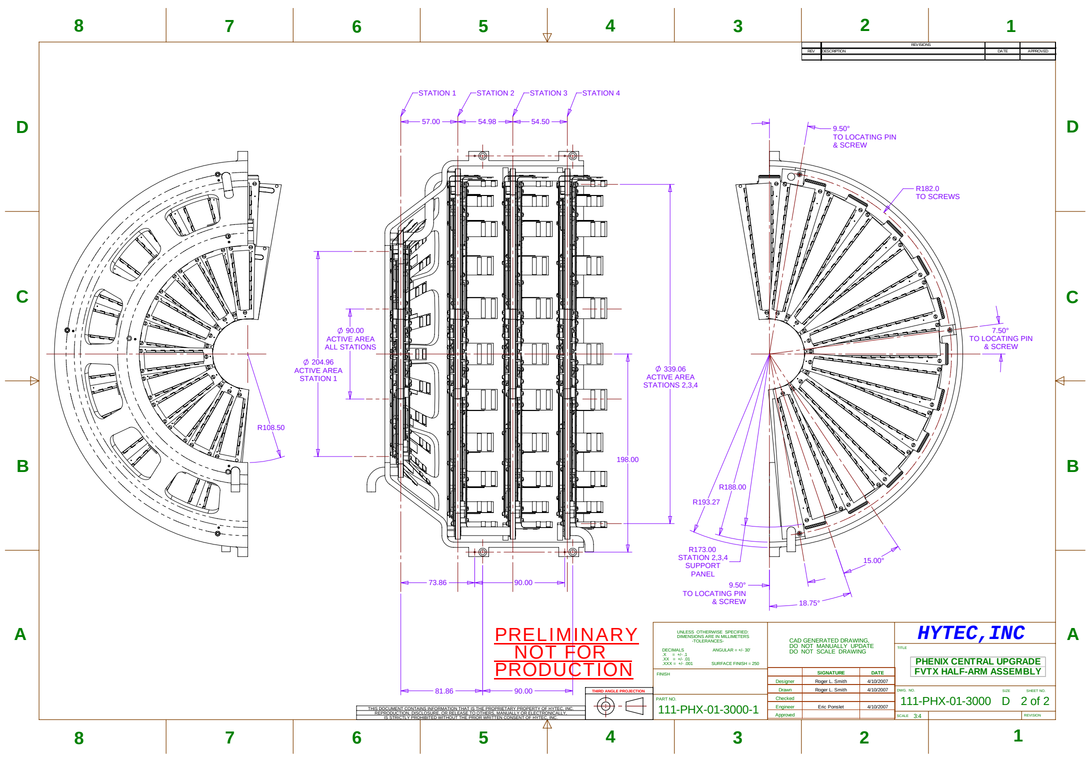

Figure 3: FVTX Half Cage, in three views.

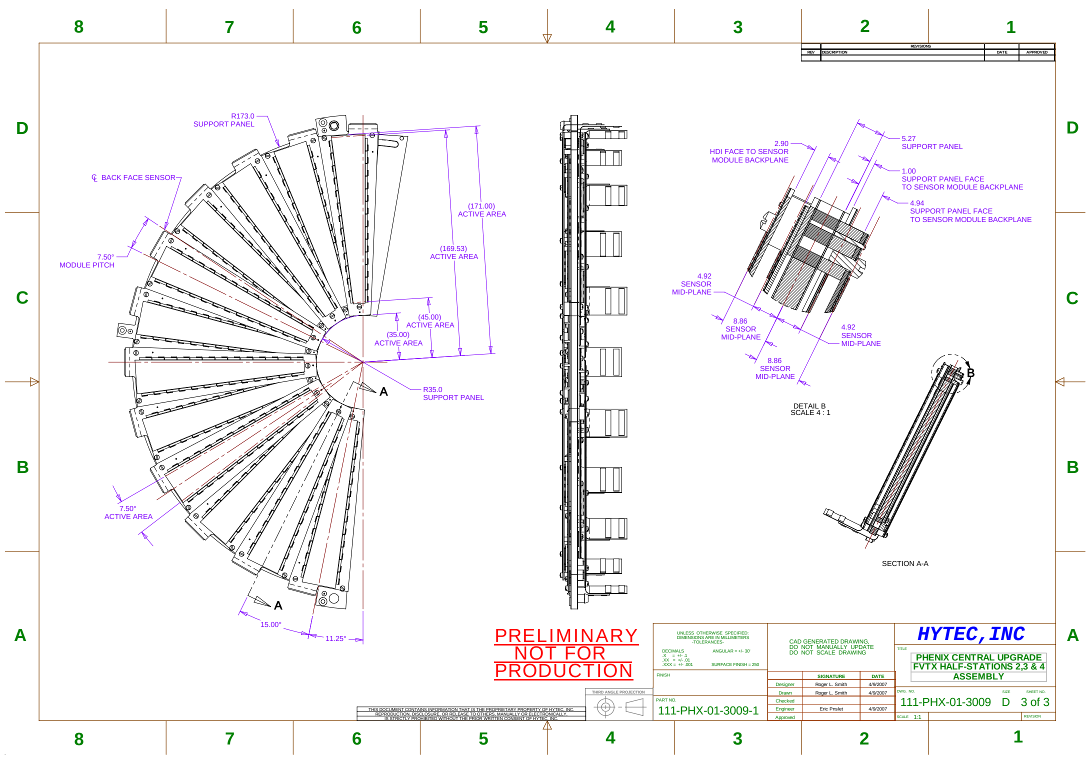

Figure 4: FVTX Half-Disk, with specifications.

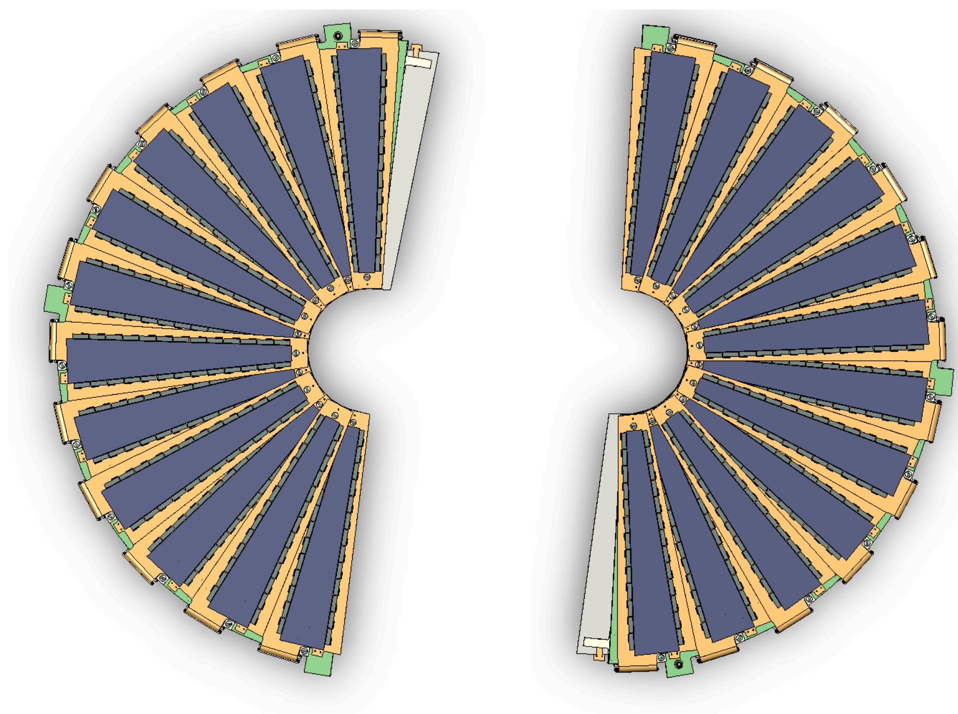

Figure 5: Two half-disks shown next to each other, indicating how they fit together.

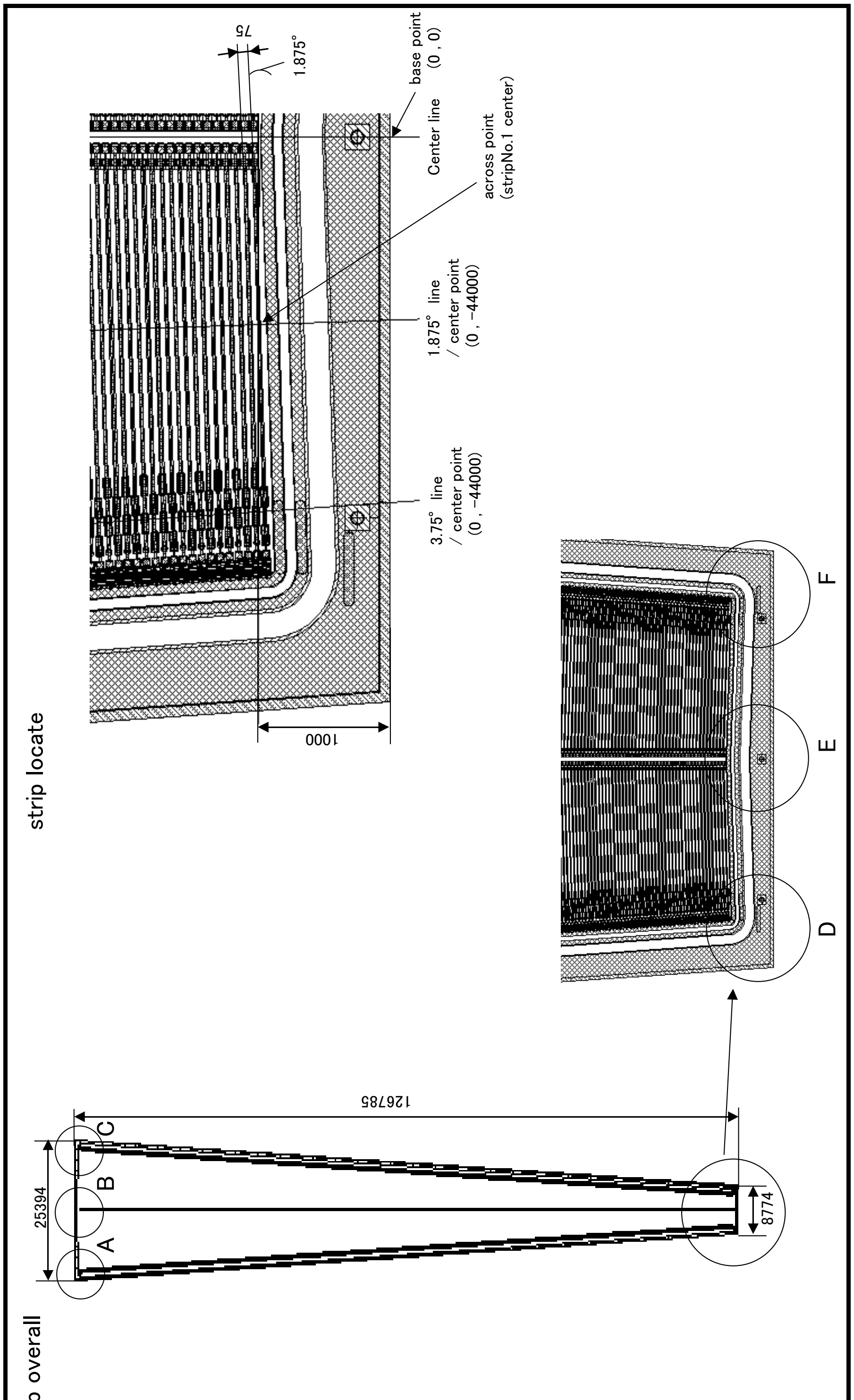

Figure 6: Large Sensor drawing from Hamamatsu.

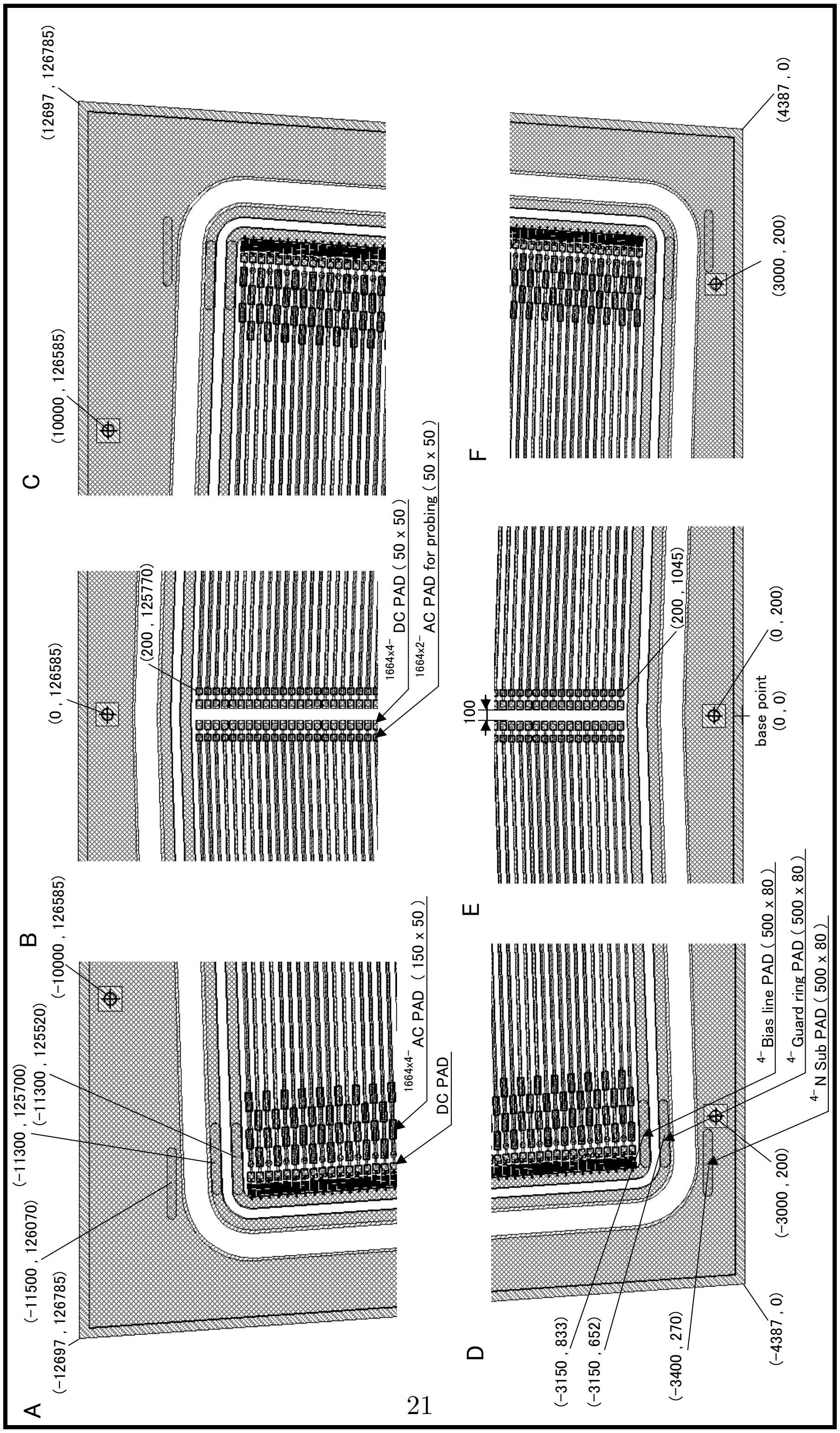

Figure 7: Large Sensor drawing from Hamamatsu.

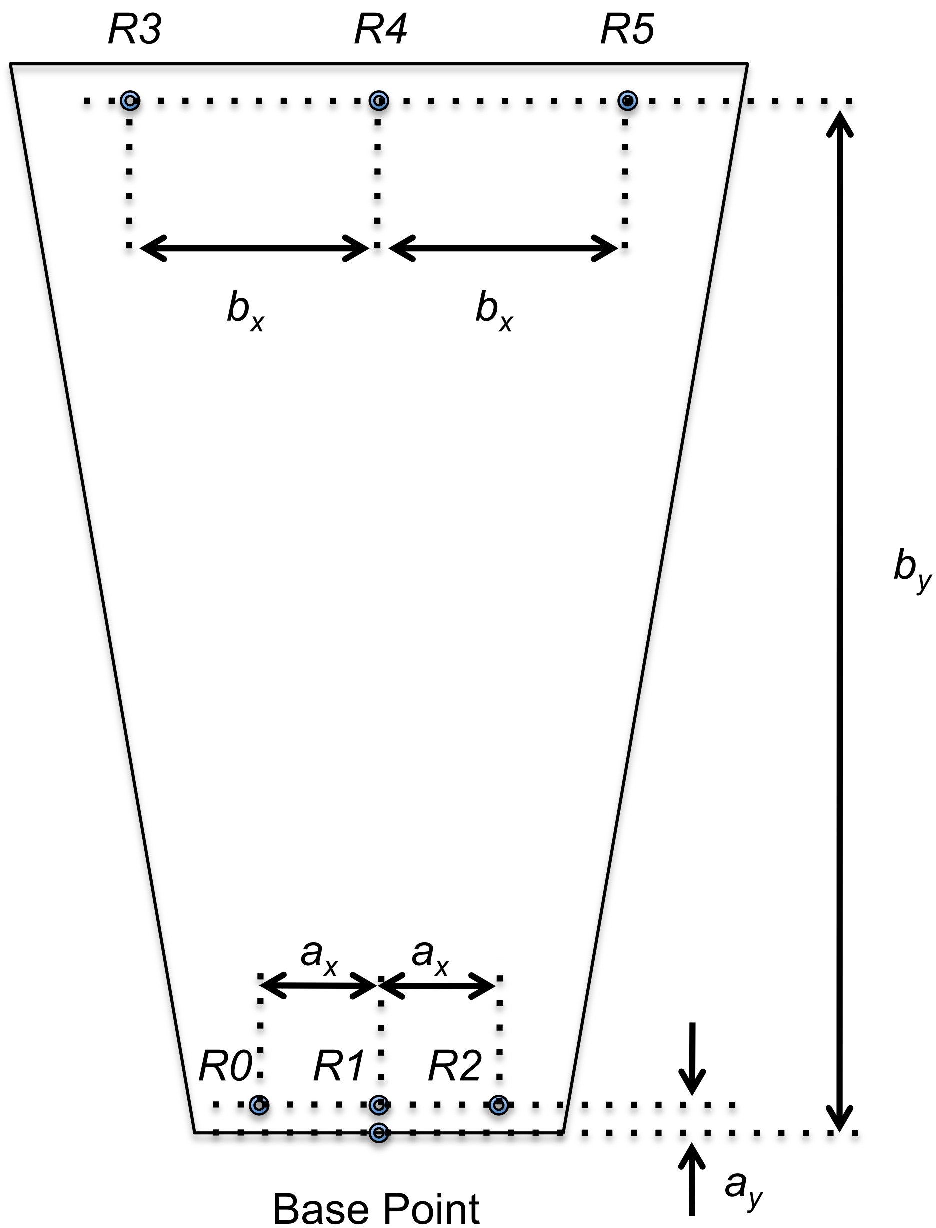

Figure 8: Schematic of reference mark locations on a sensor.