^{3}He Transport Simulations

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nEDM Collaboration

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The Governing Equations

Convection Diffusion, time dependent.

$$\frac{\partial c}{\partial t} + \boldsymbol{u} \cdot \nabla c = \nabla \cdot (D \nabla c).$$

Weakly Compressible Navier-Stokes equation.

$$\rho \boldsymbol{u} \nabla \boldsymbol{u} = \nabla \cdot [-p\boldsymbol{I} + \eta(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T) - (2\eta/3)(\nabla \cdot \boldsymbol{u})\boldsymbol{I}] + \boldsymbol{F},$$

$$\nabla \cdot (\rho \boldsymbol{u}) = 0.$$

The stationary Navier-Stokes is solved with pressure and velocity as dependent variables. These are converted to temperature and heat flux by:

$$dP = \rho_0 S\, dT,$$

and

$$\dot{Q} = 4 \int v_n\, p\, dA.$$

Boundary Conditions.

- Temperature (via pressure)
- Heat Flux (via normal velocity)
- Wall Slip (or no slip)

Why Navier-Stokes with a slip condition?

The Navier-Stokes gives accurate solutions for small yet non-negligible Knudsen numbers when a wall slip velocity is included as a boundary condition. [Kogan]

$$\textbf{Knudsen number} \equiv \frac{mean\ free\ path}{pipe\ length}$$

For the current nEDM apparatus, typical Knudsen numbers are around

$$\frac{\Lambda}{L} \approx 5 \times 10^{-3}.$$

From kinetic theory we find an equation for the slip, and simplify to find,

$$v_{slip} = \zeta \frac{dv}{dr} = s\,\Lambda \frac{dv}{dr}.$$

Dennis Greywall's 1982 paper experimentally found the magnitude of the phonon slip coefficient, s, in operating conditions similar to those in the current nEDM design. Below is a summary of reduced conductivity, to the right is his plot of the reduced conductivity. Which he uses to find s

$$\kappa_{slip} = \frac{d^2 S^2 T}{32\eta}\left(1 + 8s\frac{\Lambda}{d}\right), \qquad \kappa_{Casimir} = \frac{1}{3} C_{ph} V_{th} d.$$

$$y \equiv \frac{\kappa_{slip}}{\kappa_{Casimir}} = \frac{5}{32}\frac{d}{\Lambda} g + \frac{5}{4} s g.$$

From the plot we see:
s = 8/15.

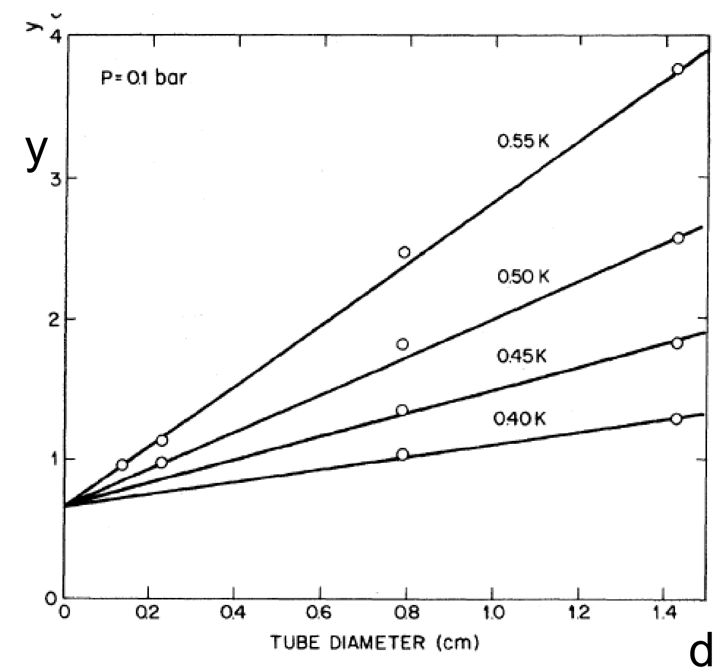


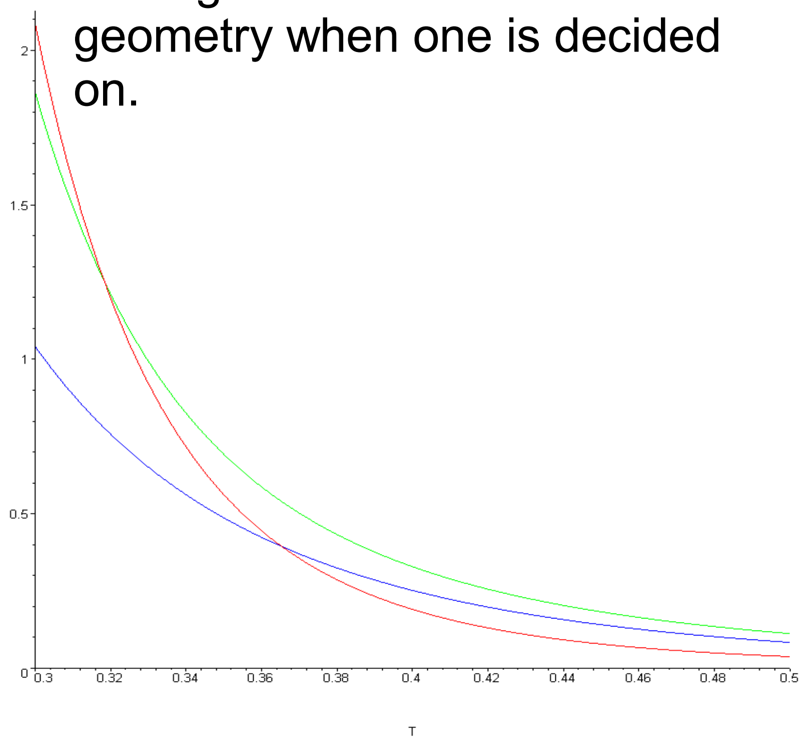
FIG. 13. The reduced conductivity y in the viscous flow regime plotted vs the tube diameter for several isotherms. The y-axis intercept is directly related to the slip coefficient.

Plot taken from Greywall Paper

Side note on Mean Free Path and Pipe Roughness.

Mean Free Path

The final solution strongly depends on the mean free path. Currently everyone working on this problem is using a similar yet different mean free path. Adding robustness to the final geometry when one is decided on.



Roughness

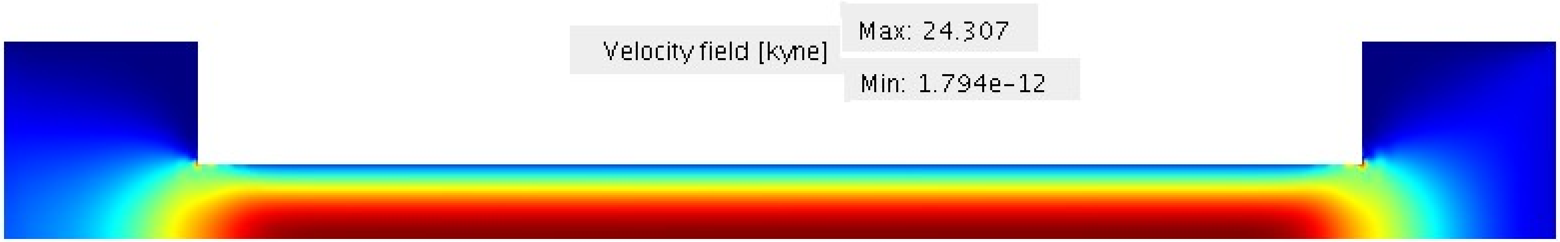
The slip coefficient drastically depends on the roughness of the pipe. Greywall's experiment used copper pipes. It is important to know how our acrylic pipes compare in roughness. Ultimately an experimental test of the same acrylic that will be used in the final apparatus is necessary.

$$f \equiv f_{\text{spec}}(\beta, T) = \int_0^\infty \frac{e^{-2\beta^2 k^2} \epsilon k^2 dk}{e^{\epsilon/k_B T} - 1} \Big/ \int_0^\infty \frac{\epsilon k^2 dk}{e^{\epsilon/k_B T} - 1} .$$

Above, a function for the fraction of spectrally reflected phonons as a function of the surface roughness β.

2-D axial symmetric FEM model

First the model solves for the stationary state of the Navier-Stokes equation from the chosen boundary conditions.



When(if) the solver converges, the velocity field is used as the convection current in the Convection Diffusion equation.

t = 0 sec

t = 30 sec

Find the BEST Operating Conditions

A parameter study was done in the 2-D Geometry. The geometries were ranked to maximize the total number of moles in the final volume while minimizing the total number of moles in the starting volume.

A set of requirements were forced (roughly) on the solutions, these are:

$T_{min} \geq 345$ mK; $V_{max} \leq 80$ cm/sec; $Q_{max} \leq 8$mW.

- The Knudsen numbers become to large at T < 345 mK for the Navier-Stokes to give accurate solutions.
- Quantum Turbulence dominates laminar flow at higher velocities. 80 cm/sec is very generous, calculations imply velocity could go to 200 cm/sec without much effect.(at temperatures ≤ 450 mK)
- Cooling power is a hot commodity.

2-D Results

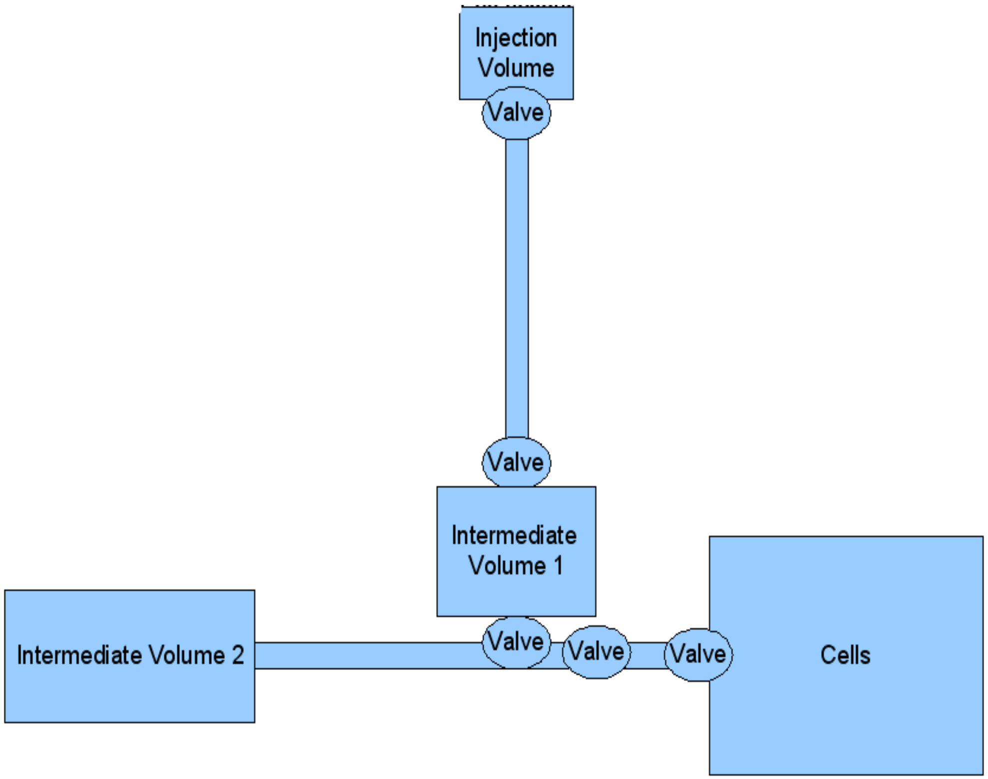
Run	Inj to IV1	IV1 to Cells	Cells to IV2
r of tube [cm]	1.1	1.4	2.2
L of tube [cm]	140.0	150.0	300.0
Vol_in [cc]	393.0	1570.0	7800.0
Vol_out [cc]	1570.0	7800.0	7800.0
Vmax [cm/sec]	62.8	75.0	54.7
Tin	0.380	0.424	0.400
Tout	0.345	0.400	0.363
Q [mW]	4.240	4.920	5.910
N vol_in	0.006	0.0003	0.0035
N_pipe	0.086	0.013	0.080

The Injection Pipe has too much ^{3}He

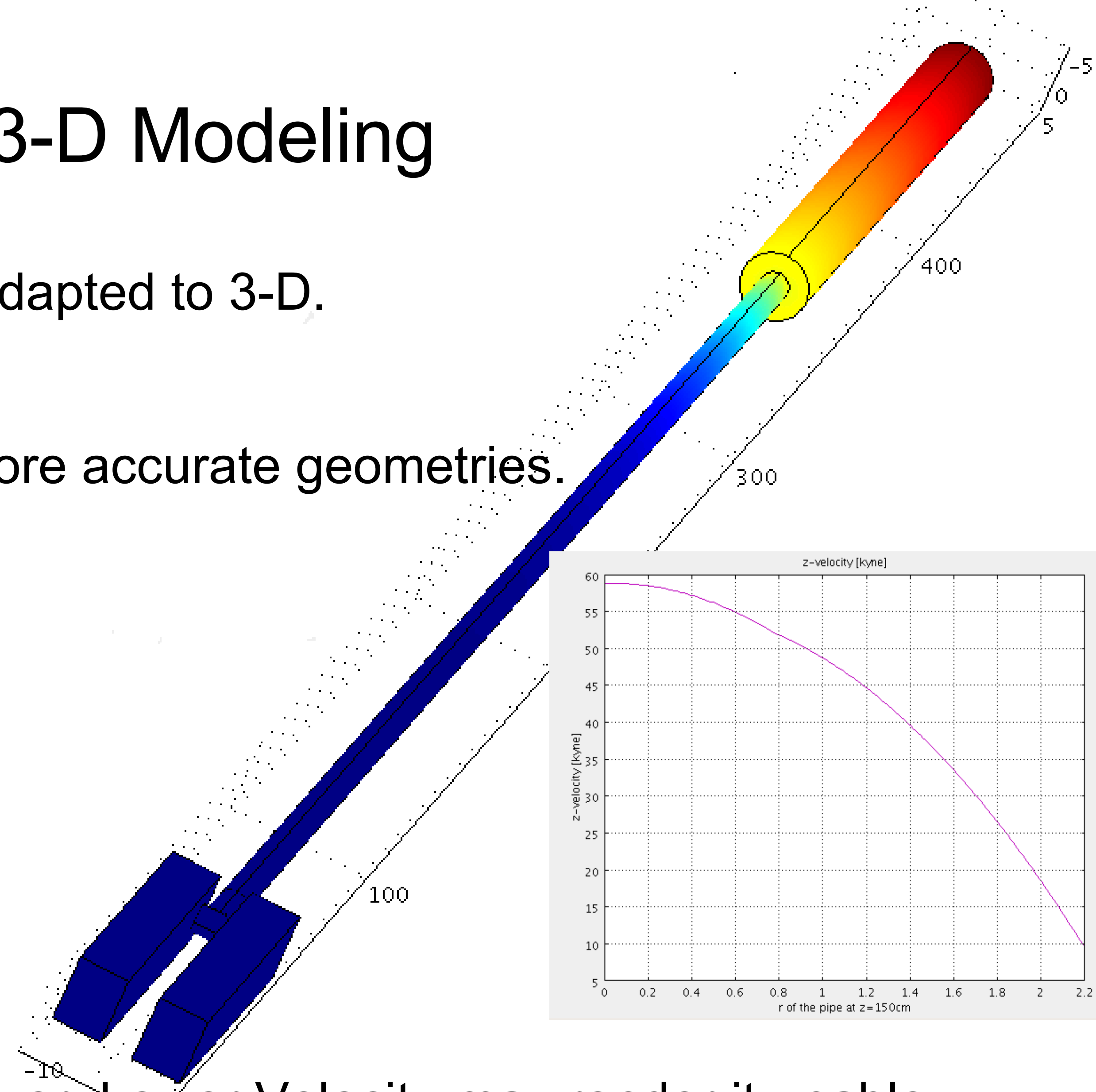
Possible Solution:

IV1 empties very quickly during the flush from IV1 to the Cells. A valve between IV1 and the Cell could be closed off shortly after the flush. Then the pipes, IV1 and IV2 could be allowed to equilibrate reducing the concentration to below the required level.

However this adds unwanted heat into the injection volume, and increases the number of Valves.



3-D Modeling



The 2-D model was adapted to 3-D.

The Up side:
•Rectangular cells, more accurate geometries.
•Looks better.

The Down Side:
• Less converging power. Lower Velocity-max render it unable to converge. (Maybe a result of from turbulent effects due to the more realistic geometries.)
•Requires longer time to solve and adjust geometries.

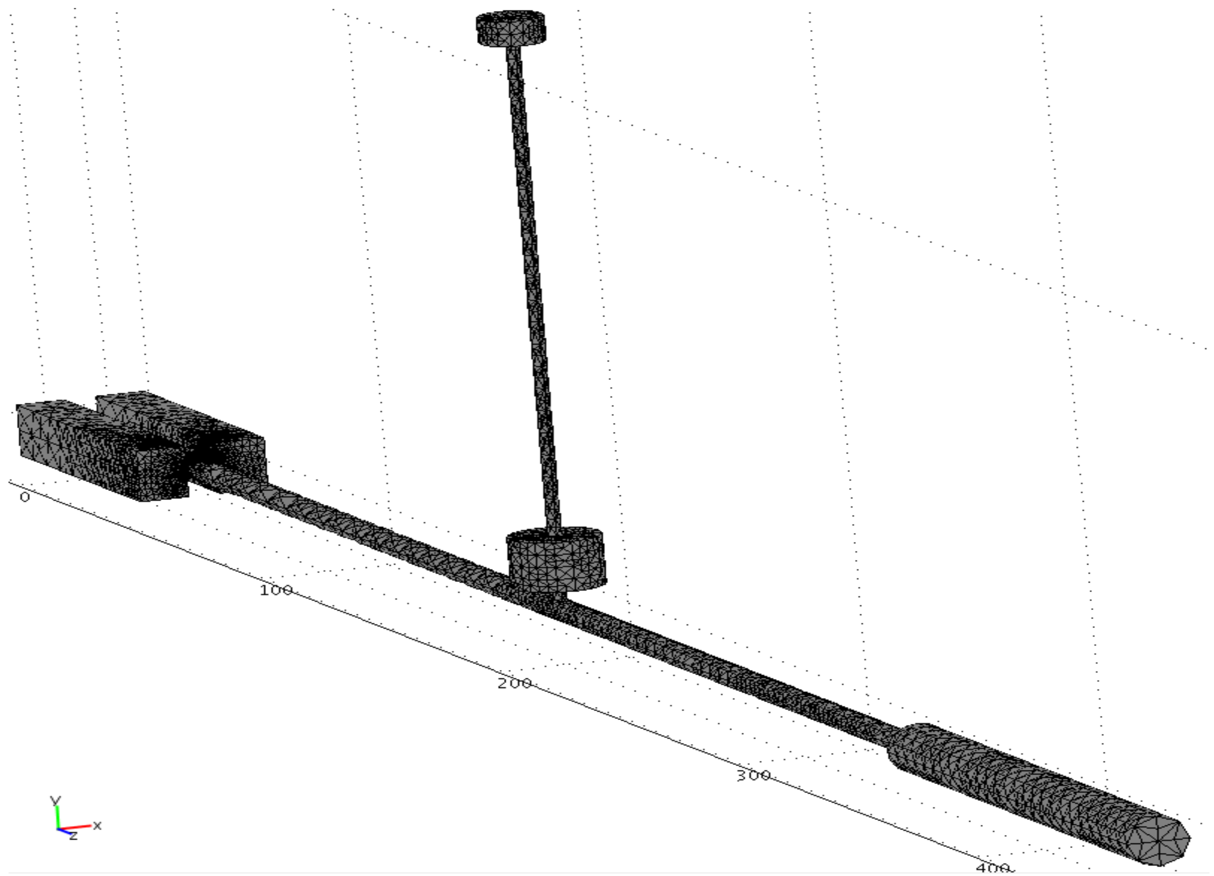
3-D Results.

The results for the 3-D Model strongly resembled the 2-D model.

Concentrations to and from the rectangular cells made the final concentrations in the cell smaller than the 2-D model. This can be attributed by the split in the tube, which increases the conductivity.

Full 3-D model.

- may aide in solving the problem of excess concentration in the pipe.
- looks neat.
- currently unable to solve, maybe due to improper meshing from complicated geometry.



Conclusion

- Clearing the Tubes to the required 1% concentration seems unlikely without bypass tubes, or including more valves and complex Flush patterns. Or Heat Flux > 10mW and Vmax > 200 cm/sec.
- Experimentation of Heat Flux should proceed to test the model's validity. Or the similarities of our pipe to Greywall's pipe.

Thank you.