Analysis of ^{3}He T_1 Depolarization in Collection Volume, Transfer Tube, and Experimental Volume: Magnetic Field Homogeneity Requirements

This problem is different from the usual one encountered in low field magnetic resonance because we are working with very small fields and we are operating near the limit between ballistic and diffusive propagation of ^{3}He in the superfluid helium bath. Taking the diffusion coefficient for ^{3}He motion against phonons (^{3}He-^{3}He collisions do not contribute significantly at the concentrations we are considering), $D = 1.8T^{-7}$ cm^2/s from our previous measurement. For $T = 0.35$K, $D = 3 \times 10^3$ cm^2/s. Using

$$D = \lambda v/3$$

and $v = 3600$ cm/s (effective ^{3}He mass in superfluid ^{4}He is $2.2m_3$), implies a mean free path between collisions $\lambda = 2.3$ cm.

We imagine a 20 liter maximum volume region filled with superfluid helium that will collect polarized ^{3}He atoms from the ABP (atomic beam polarizer). Assume a cylinder, 30 cm diameter, 30 cm high, that contains the superfluid helium plus ^{3}He. The ballistic mean free path is determined by the ratio of volume to surface area,

$$\ell = 4V/S = 27\text{cm}.$$

From the known ABP flux, the time required to collect the polarized ^{3}He is about 200 sec.; we assume the collection time can be limited to this period (see [2] below). This volume then feeds the UCN measurement cell through a tube of 1 cm diameter, length 100 cm. The transfer should take less than 20 sec., so any particular ^{3}He atom will be in the transfer region for less than three seconds. After the main cells are filled, the spin flip occurs within 10 sec, so the total storage time is about 30 seconds in the main volume before spin flip. We therefore must analyze the field homogeneity requirements in three different regions in regard to homogeneity requirements.

Collection Volume

From the above considerations, the diffusive limit is applicable to the main collection volume. The time to travel a cell mean free path is

$$\ell = \sqrt{2D\tau_d}$$

which implies $\tau_d = 0.1$ sec. We are considering holding fields of at least B_0=10 mG, so $\omega_0\tau_d = 9 >> 1$. In this limit, Eq. (58) of [1] is applicable,

$$\frac{1}{T_1} = D\frac{|\nabla B_x|^2 + |\nabla B_y|^2}{B_0^2} \tag{1}$$

with B_0 assumed to lie along $\hat{z}$, and $B_{x,y}$ are the transverse (gradient) fields. This assumes that the gradients and B_0 are constant through the volume.

To have a 1% polarization loss in the collection volume, we need a lifetime of 20,000 sec. Defining $\sqrt{(dB_x/dx)^2 + (dB_y/dy)^2} = dB_r/dr = \alpha$ for a constant dB_z/dz, and $\Delta B_r = \alpha R$, Eq. (1) implies a maximum gradient of

$$5 \times 10^{-5} < (3 \times 10^3 \mathrm{cm}^2/\mathrm{s}) \frac{\alpha^2}{B_0^2} \rightarrow \alpha/B_0 < 1.3 \times 10^{-4}/\mathrm{cm}$$

This requirement is further relaxed if the time between velocity changing collisions τ_c is such that $\omega\tau_c >> 1$ the relaxation rate will be further suppressed as $1/(1+(\omega_0\tau_c)^2$. Taking $\lambda = v\tau_c$, we see, under the conditions assumed, that $\tau_c = 6 \times 10^{-4}$ s, which implies a field of 83 mG before this suppression factor kicks in.

Thus the requirement is that the field is homogeneous to 0.2% in the collection cell, assuming a holding field of less than 100 mG.

Transfer Tube

We assume ^{3}He will be kept out of the transfer tube until the transfer actually takes place. If the transfer tube is 1 cm diameter, the mean free path is about 1 cm, so $\tau_c = 3 \times 10^{-4}$ s. Again, due to the short time between wall collisions, we will likely always be in the low field regime. The T_1 lifetime should be more than 200 sec (1% polarization loss in less than 2 second that a ^{3}He atom spends in the transfer tube, as described in the introduction). In this region, because the phonon collision mean free path is longer than the ballistic mean free path, Eq. (66) of [3] is applicable; in the small τ_c limit

$$T_1^{-1} = \frac{1}{2}\gamma^2 R^2 \alpha^2 \tau_c$$

where γ is the gyromagnetic ratio, Hz/mG, and R is the tube radius. The limit for α is therefore

$$\alpha < 1.8\mathrm{mG/cm}$$

which should be easy to achieve.

On the other hand, if we want to tailor a strong holding field on the collection volume to a small field inside the experimental magnetic shields, there is a constraint on the maximum rate of change of B_0. If we imagine a long straight tube, and if we vary B_0 along the length, say by winding a solenoid with varying pitch, the radial gradient is given simply by $dB_z/dz = \alpha$ as described above. It therefore seems possible to have a relatively high field at the collection region that is tailored to the experimental cells.

Experimental Cell

The T_1 gradient requirements are similar to what is required for T_2 in the experimental cell. These limits are discussed elsewhere in this report.

Footnotes and References

1. G.D. Cates, S.R. Schaefer, and W. Happer, Phys. Rev. A **37**, 2877 (1988).

2. The limits for the collection volume size have not been addressed. We could imagine operating the experiment as follows: fill the experimental measurement nearly full (fraction β) of ultrapure superfluid ^{4}He, and have the ^{3}He collection volume equal to the unfilled fraction $(1-\beta)$. The limit for the collection cell volume is determined by: 1. maximum allowable concentration before phase separations sets in (concentration $x = 0.4$); 2. ^{3}He-^{3}He diffusion, which becomes equal to the phonon diffusion at 0.4 K when $x = 10^{-7}$. We need $x = 10^{-10}$ in the experiment cells so in fact the collection volume could be more than 100 times smaller than the experimental volume.

3. D. Kleppner, H.M. Goldenberg, and N.F. Ramsey, Phys. Rev. **126**, 603 (1962).