

Note on Analysis of UIUC and Duke ^3He T_1 Data

Mike Hayden

Physics Department, Simon Fraser University, Burnaby BC, Canada V5A 1S6

On May 15, 2008 Doug Beck sent a plot of T_1^{-1} as a function of ‘ S/V ’ for ^3He in liquid ^4He at concentrations of order 10^{-3} . The data clearly exhibit an unphysically large *negative* intercept, which when combined with my understanding of the geometry of the experiment suggests that the underlying model is incorrect. A quick summary of what I believe to be the correct picture is given below. When the data are reanalyzed, they fall on a straight line passing through the origin, as anticipated. Extrapolation to the conditions of the nEDM experiment yield a T_1 of order 10^4 s. On May 19, 2008 Bob Golub showed me analogous data from Duke University which likewise exhibited an unexplained negative intercept. The model described here seems to do an equally good job of resolving the problem. Extrapolation from the Duke data to the conditions of the nEDM experiment yield $T_1 \sim 5 \times 10^3$ s.

I believe that the UIUC cell is a cylinder of height $L = 8.6$ cm and radius $R = 0.79$ cm. It is filled to some depth h with liquid. I’ll refer to all quantities pertaining to bulk liquid with a subscript ℓ , those pertaining to the vapour with a subscript v , and those pertaining to the superfluid film with an f . Quantities without subscripts refer to totals for the cell. Thus, the total number of ^3He atoms is $N = N_\ell + N_v + N_f$, the total volume of the cell is $V = V_\ell + V_v + V_f$ (although V_f is tiny compared to both V_ℓ and V_v for all practical conditions), and the total area of the cell is $A = A_\ell + A_f$ (the vapour is never in contact with the substrate).

Based on the geometry of the cell outlined above and the range of values along the abscissa of the plot sent by Doug (copied below as Fig. 1), I interpret ‘Surface/Volume’ to mean the surface area of the cell wet by bulk liquid divided by the volume of bulk liquid. This implies that the underlying model for wall relaxation is one in which all ^3He atoms are confined to the bulk liquid.

My picture for this system is as follows. The experimental chamber consists of 3 communicating reservoirs: the bulk liquid, the vapour, and the film. At temperatures well below 1 K most of the ^3He atoms reside within the bulk liquid. In equilibrium, the ^3He density n satisfies [1]

$$\frac{n_v}{n_\ell} = \left(\frac{m}{m^*}\right)^{\frac{3}{2}} \exp\left(\frac{-E_B}{kT}\right) \quad (1)$$

or

$$\frac{N_v}{N_\ell} = \frac{V_v}{V_\ell} \left(\frac{m}{m^*}\right)^{\frac{3}{2}} \exp\left(\frac{-E_B}{kT}\right) \quad (2)$$

where $E_B \sim 2.8$ K is the solvation energy for ^3He in liquid ^4He , and $m^*/m \sim 2.2$ is the ^3He quasiparticle effective mass ratio [2]. This ratio is plotted as a function of typical experimental parameters in Fig. 2 to illustrate the point that $N_v/N_\ell \ll 1$. One can write down a similar condition for the film-vapour equilibrium, but ultimately

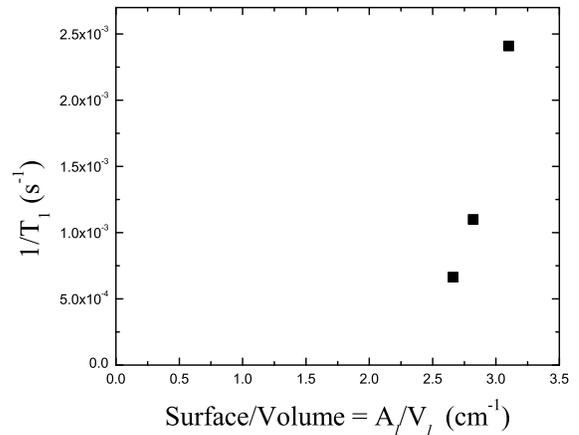


FIG. 1: Relaxation rate data from UIUC; points were extracted from the original plot by hand.

that condition is equivalent to writing

$$\frac{N_f}{N_\ell} = \frac{V_f}{V_\ell} \quad (3)$$

since the film and bulk liquid must also be in equilibrium. Since V_f can be rewritten in terms of the microscopic film thickness t , one has

$$\frac{N_f}{N_\ell} = \frac{A_f t}{V_\ell} \ll 1. \quad (4)$$

The arguments presented above indicate that to good approximation $N = N_\ell$. That is, essentially all of the ^3He atoms are in the bulk liquid. Nevertheless, exchange between the three reservoirs is rapid and ^3He atoms regularly explore the entire cell. To see this, consider the limit where diffusion within the liquid and vapour is fast. The time scale for exchange of ^3He between the liquid and the

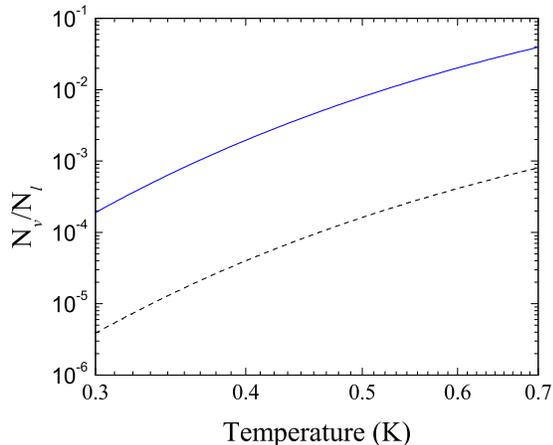


FIG. 2: Ratio of the number of ^3He atoms in the vapour relative to that in the liquid when the cell is 1/8 full (solid blue line) and 7/8 full (dashed blackline).

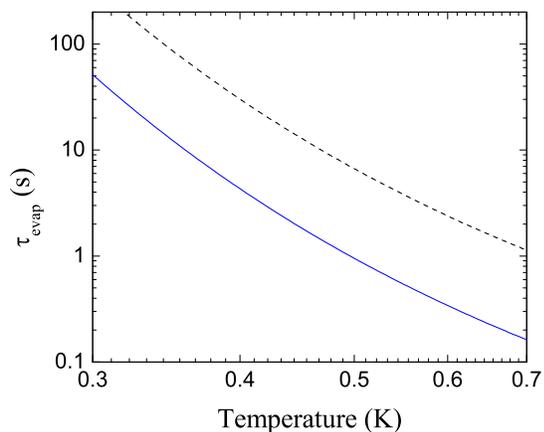


FIG. 3: Time constant for exchange between the liquid and vapour, assuming that diffusion is rapid. Solid blue line: cell 1/8 full. Dashed black line: cell 7/8 full. Note that $V_\ell/S = h$. My understanding is that the data I was sent were acquired at 0.4 K, and that the cell was never more than $\sim 45\%$ full. This implies that the exchange of ^3He between reservoirs is rapid compared to T_1 .

vapour is then given by [1]

$$\tau = \frac{4V_\ell}{S\bar{v}\alpha_{v\ell}} \left(\frac{m^*}{m}\right)^{\frac{3}{2}} \exp\left(\frac{E_B}{kT}\right) \quad (5)$$

where $S = \pi R^2$ (the free liquid surface area) and $\alpha_{v\ell} \sim 1$ is the thermally averaged probability that a ^3He atom in the vapour phase striking the interface will pass into the

liquid. A similar relationship holds for the vapour-film exchange. At temperatures relevant to the experiment this yields exchange times ranging from a few seconds to a few tens of seconds, as shown in Fig. 3. Even when the cell starts to become fairly full it seems that these times may well be fast compared to extrapolated relaxation rates. Thus, I will assume that exchange between reservoirs is rapid. In a more refined analysis one may want to explicitly include this exchange rate as a factor that influences T_1 . Likewise, diffusion within the liquid will also slow down the exchange of atoms between reservoirs. Here, I've been told that the effect of concentration on T_1 is only of order 10% for changes in ^3He concentrations by a factor of 3. This is small enough to ignore for now; it can be dealt with in a more complete analysis down the road.

With these approximations in hand, the time evolution of the ^3He nuclear magnetization density M resulting from interactions with the wall can be written

$$\frac{\dot{M}}{M} \approx \frac{\dot{M}_\ell + \dot{M}_f}{M_\ell}. \quad (6)$$

Note that \dot{M}_v has been set to zero since the vapour does not ever interact with the walls. Rapid diffusion in the vapour in the presence of magnetic field gradients *could* result in $\dot{M} \neq 0$. This can be evaluated most effectively through further experimentation. If the relaxation process is described in terms of a simple kinetic theory attempt frequency and a relaxation probability P , one obtains

$$\frac{\dot{M}}{M} = \frac{-n_\ell \bar{v}_\ell A_\ell P/4 - n_f \bar{v}_\ell A_f P/4}{N_\ell} \quad (7)$$

where $\bar{v}_\ell = \sqrt{8kT/\pi m^*}$ denotes the mean thermal speed of ^3He quasiparticles, $n_\ell = N_\ell/V_\ell$ and $n_f = N_f/V_f = n_\ell$. Simplifying this and recognizing $1/T_1 = \dot{M}/M$ yields

$$\frac{1}{T_1} = \frac{\bar{v}_\ell A P}{4V_\ell}. \quad (8)$$

Thus, the relevant surface area that should be used in the analysis is the full cell area A rather than simply the area wet by bulk liquid.

To reanalyze the data, I will use the fact that

$$\frac{A_\ell}{V_\ell} = \frac{2}{R} + \frac{1}{h} \quad (9)$$

to infer the height h of liquid in the cell for each datum, and then replot $1/T_1$ as function of

$$\frac{A}{V_\ell} = \frac{2 + 2L/R}{h} \quad (10)$$

as suggested by Eq. 8. The result is shown in Fig. 4. It clearly shows a more plausible dependence of $1/T_1$ on the surface-area-to-volume ratio. Extracting the slope of a

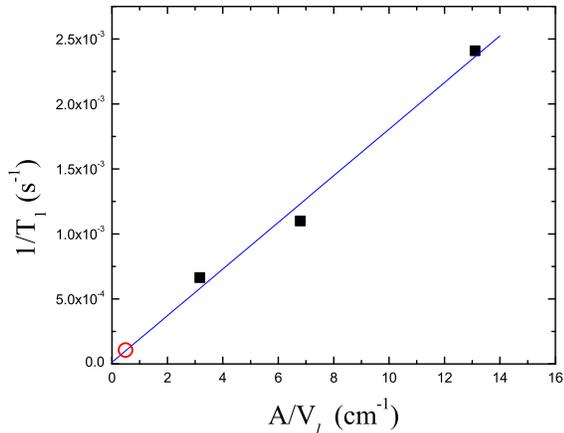


FIG. 4: UIUC ^3He relaxation rate data (*cf.* Fig. 1) replotted as a function of A/V_ℓ as suggested by Eq. 8. The best fit line (not constrained to pass through the origin) shown in the figure has a slope of 1.8×10^{-4} cm/s, implying that T_1 would be 1.1×10^4 s for an EDM experiment performed with $A/V = 0.5 \text{ cm}^{-1}$ (red circle).

best-fit line to these data allows one to extrapolate to the anticipated conditions for an nEDM experiment. For a *completely* full $8 \text{ cm} \times 10 \text{ cm} \times 50 \text{ cm}$ rectangular cell one has $A/V \sim 0.5 \text{ cm}^{-1}$ suggesting $T_1 = 1.1 \times 10^4$ s. This extrapolation is indicated by the red circle shown in Fig. 4. I can't evaluate the uncertainty in this number without knowing more about the experimental conditions, but I view the result as very promising.

Note that the decrease in $1/T_1$ as the cell is filled up (*i.e.* smaller and smaller A/V) illustrates a point I tried to make during the 2002 EDM Collaboration meeting in Los Alamos [3]. I was asked to speculate on what sort of T_1 s one might obtain for an EDM experiment. In doing so I identified experiments on sealed superfluid film-coated cells by Chris Lusher at Sussex [4] and Marc Himbert at the ENS [5] as being critical, and claimed that one could extrapolate from their results in two different ways. In what I called the 'pessimistic viewpoint' T_1 would simply scale as A/V as cell dimensions were changed, and the quantity of liquid would play no role. In what I called the '(overly) optimistic viewpoint' the concentration of ^3He near the walls would be also diluted by a factor equal to the inverse of Eq. 1. This uncertainty effectively launched the effort to measure T_1 , and the underlying physics is the reason that I have so emphatically insisted over and over again that the test had to be done at the correct temperature *and* in a cell full of bulk liquid. Life is often not kind to experimentalists, but in this case it seems that the optimistic scenario has won out.

Finally, it is also worth commenting on the relaxation

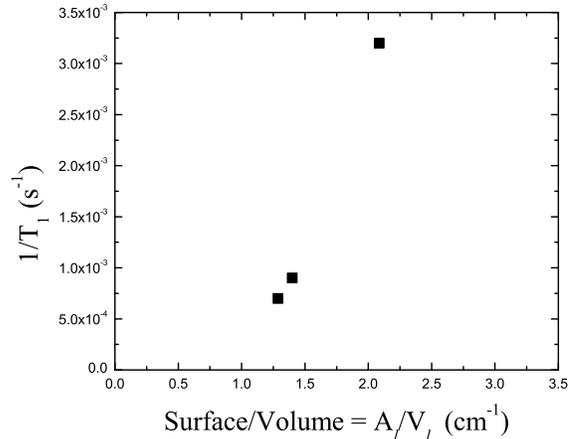


FIG. 5: Relaxation rate data from Duke; T_1 values were extracted from the original plot by eye, and Bob gave me values for the liquid depth. Note that the original plot I saw may have used $A_\ell + S$ (instead of A_ℓ as shown here) for the relevant surface area. The data were acquired at a temperature of order 0.5 K. The cell radius and height are $R = 1.84$ cm and $L = 5.08$ cm, respectively.

probability P , which from these data is of order 2×10^{-7} per attempt. This is comparable to the value observed at 1.9 K a few years ago at Duke [6].

ADDENDUM

On May 19, 2008 Bob Golub showed me the T_1 data from Duke University. It was being analyzed in the same manner as the UIUC data, and showed the same unphysical negative intercept (Fig. 5). This problem is also resolved when the data are reanalyzed within the framework of the model presented above, as shown in Fig. 6. Extrapolation to the conditions of the nEDM experiment yields $T_1 \sim 4.7 \times 10^3$ seconds. The relaxation rate is roughly double the rate observed at UIUC, but still encouraging. The corresponding relaxation probability is of order $P = 5 \times 10^{-7}$ per attempt.

-
- [1] M.E. Hayden, S.K. Lamareaux, and R. Golub, 'Evaporative Isotopic Purification of Superfluid Helium-4', in Y. Takano, S.P. Hershfield, S.O. Hill, P.J. Hirschfeld, and A.M. Goldman Eds, *Low Temp. Phys: 24th Intl. Conf. Low Temp. Phys., AIP Conf. Proc.* **850**, 147 (2006).
 - [2] E. Krotscheck, M. Saarela, K. Schrkhuber, and R. Zillich, *Phys. Rev. Lett.* **80**, 4709 (1998).
 - [3] M.E. Hayden, 'Experiments relevant to transporting ^3He into the superfluid He and the ^3He polarization lifetime,'

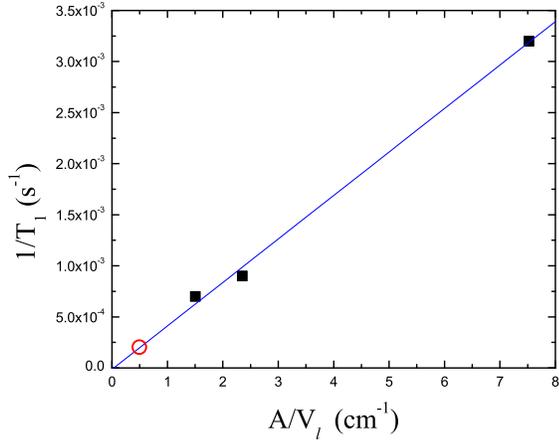


FIG. 6: Duke ${}^3\text{He}$ relaxation rate data (*cf.* Fig. 5) replotted as a function of A/V_ℓ as suggested by Eq. 8. The best fit line (not constrained to pass through the origin) shown in the figure has a slope of 4.3×10^{-4} cm/s, implying that T_1 would be 4.7×10^3 s for an EDM experiment performed with $A/V = 0.5 \text{ cm}^{-1}$ (red circle).

EDM Collaboration Meeting July 19-20, 2002 Los Alamos NM; LA-UR 02-4760.

- [4] C.P. Lusher *et al.*, *J. Low Temp. Phys.* **72**, 71 (1988).
- [5] M. Himbert and J. Dupont-Roc, *J. Low Temp. Phys.* **76**, 435 (1989).
- [6] Q. Ye *et al.* *Phys. Rev. A* **77**, 053408 (2008).