Chapter 4. Analysis

4.1 Centrality

Centrality is characterized by the impact parameter, which is the minimum distance between centers of two colliding nuclei. This variable can not be measured experimentally, therefore total number of the secondary particle multiplicity or total transverse energy corresponding to the total number of participant nucleon are used as a tool to measure the magnitude of the collision. Total forward energy corresponding to the total number of the spectator nucleon are also used to measure the amount of energy which is not consumed at the collision. Both of those two approaches give an estimation of the impact parameter in order to know the centrality of the collision.

NA44 experiment uses one of the centrality measurement method 'charged particle multiplicity'. The T0 scintillation counter is one of the devices to measure the charged multiplicity in the backward pseudo-rapidity region. This device is also used in the trigger for on-line event selection. With requiring at least one hit of minimum ionizing particle on this device in on-line trigger, minimum biased trigger in centrality will be given. On the other hand, by triggering the T0 counter with higher threshold of the discriminator, the higher multiplicity events which means central collision will be selected. The fraction of the total hadronic cross section which corresponds to the centrality depends on the threshold value of the discriminator. In the off-line analysis, silicon multiplicity detector is also used to measure the charged multiplicity at backward pseudo-rapidity region ($1.5 < \eta < 3.3$). In the figure 4-1, correlation between charged multiplicity measured by the silicon pad detector and T0 ADC spectrum is shown for 200AGeV/c sulphur on sulphur collisions. Contour plot seen in (a) in the figure 4-1 shows the minimum bias events and solid box shows the central triggered events. A linear correlation between those two counters are clearly seen. A trigger threshold on the T0 ADC spectrum can be seen as a cut-off of the spectrum. Next two figures (b) and (c) in the figure 4-1 are projections on to those two axes, by tuning the normalization between two spectra (minimum bias spectrum and central spectrum), the centrality is calculated to be top 15% for both two projections, where 15% is a ratio hatched area from central triggered events over total area from minimum bias trigger.

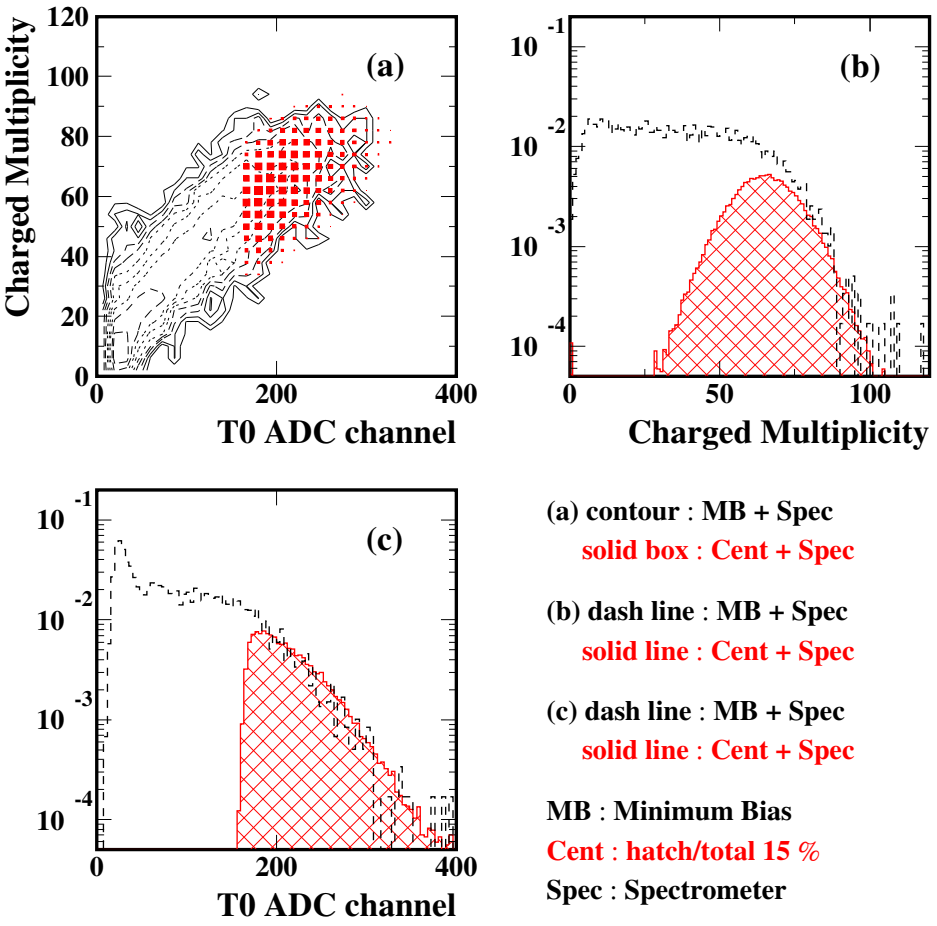

Figure 4-1: centrality measured with T0 and Silicon pad detectors

4.2 Particle identification

Particle identification is typically done with combining the time of flight measured by the hodoscopes and the two gas Cherenkov informations. The figure 4-2 shows typical example of particle identification done in the time of flight and momentum space at 4GeV/c setting. Here, Cherenkov C2 detector is already used as a veto mode to reject electrons in on-line trigger. Three bands for pions, kaons and protons can be seen clearly in (a) in the figure 4-2, however the separation between pions and kaons is not good enough to make clear particle separation. Therefore, by requiring Cherenkov C1 hit, pion is selected seen in (b) in the figure 4-2, on the other hand, kaon or more heavier particle can be selected by rejecting Cherenkov C1 hit seen in (c) in the figure 4-2.

By changing the Cherenkov gas into high pressure at higher momentum setting, for example at 8GeV/c, C1 will be fired by both pions and kaons with the different pulse height, because the clearly different velocities between pions and kaons at this momentum will make enough difference of the Cherenkov radiations between those two particles. C2 will be fired by pions, so one can also use C2 to select pion with requiring C2 hit. Proton or more heavier particle can be selected by rejecting both C1 and C2 hit. So the two Cherenkov C1 and C2 has a major role to identify pions, kaons and protons as shown in the top left figure in the figure 4-3. Proton and deuteron separation can be done with mass square given by the time of flight measurement with hodoscopes seen in the top right figure in the figure 4-3. By selecting the proper ranges of both two Cherenkov detectors C1 and C2, pions, kaons and protons are clearly separated seen in the mass square spectra shown in the bottom three figures in the figure 4-3.

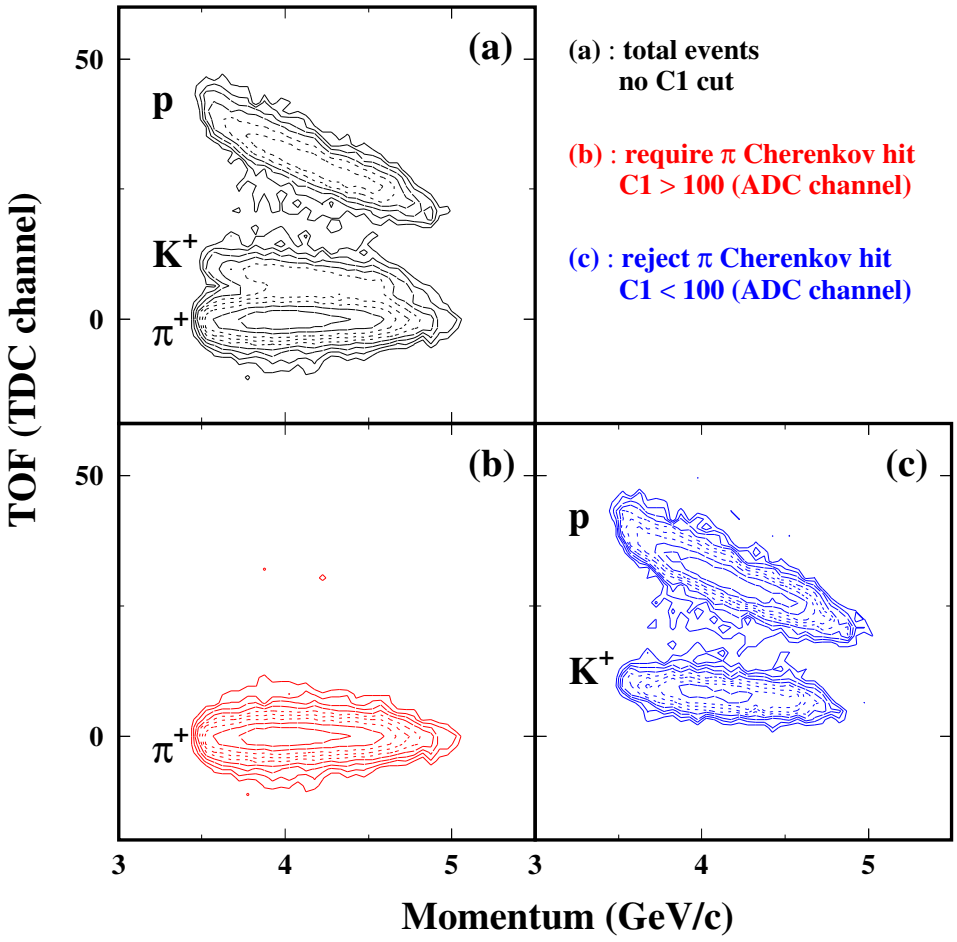

Figure 4-2: PID at 4GeV/c setting

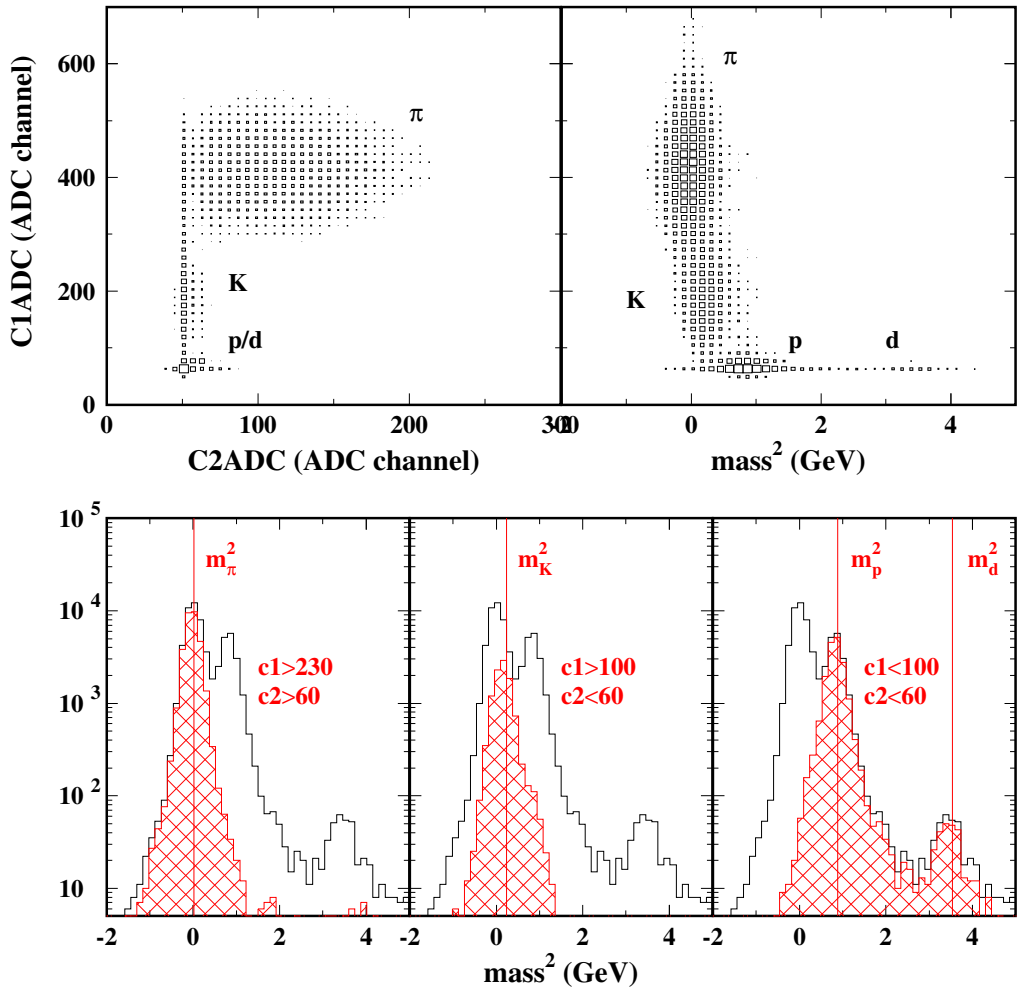

Figure 4-3: PID at 8GeV/c setting

4.3 Single particle distributions

After selecting the required centrality and the particle identification, several momentum distributions will be created. The following is the list of the necessary kinematical variables,

P : absolute momentum,

P_X : X (horizontal) component of the momentum,

P_Y : Y (vertical) component of the momentum,

P_Z : Z (beam direction) component of the momentum,

P_T : momentum (perpendicular to the beam) $P_T = \sqrt{{P_X}^2 + {P_Y}^2}$,

M_T : transverse mass $M_T = \sqrt{{P_T}^2 + M^2}$,

K_T : transverse kinetic energy $K_T = M_T - M$,

E : energy $E = \sqrt{{M_T}^2 + {P_Z}^2}$,

y : rapidity $y = \log(\frac{E + P_Z}{M_T})$,

where M is mass of the particle in question. These distributions are made from the reconstructed events accepted within the NA44 acceptance. The number of filled events are integrated over all triggered and accepted collisions during the run. For example, several single particle momentum distributions measured at large angle 8GeV/c setting for pion are shown in the figure 4-4. The measured spectra are compared with the Monte Carlo simulation, which are shown as superimposed histogram in the figure 4-4. One can see that the shape between experimental spectra and Monte Carlo are similar. The spectrum have to be corrected for the spectrometer acceptance and to be normalized to one unit collision in order to know the production of the secondary particle per collision. In the following two sections, the experimental acceptance correction will be applied and the spectrum will be normalized to one collision.

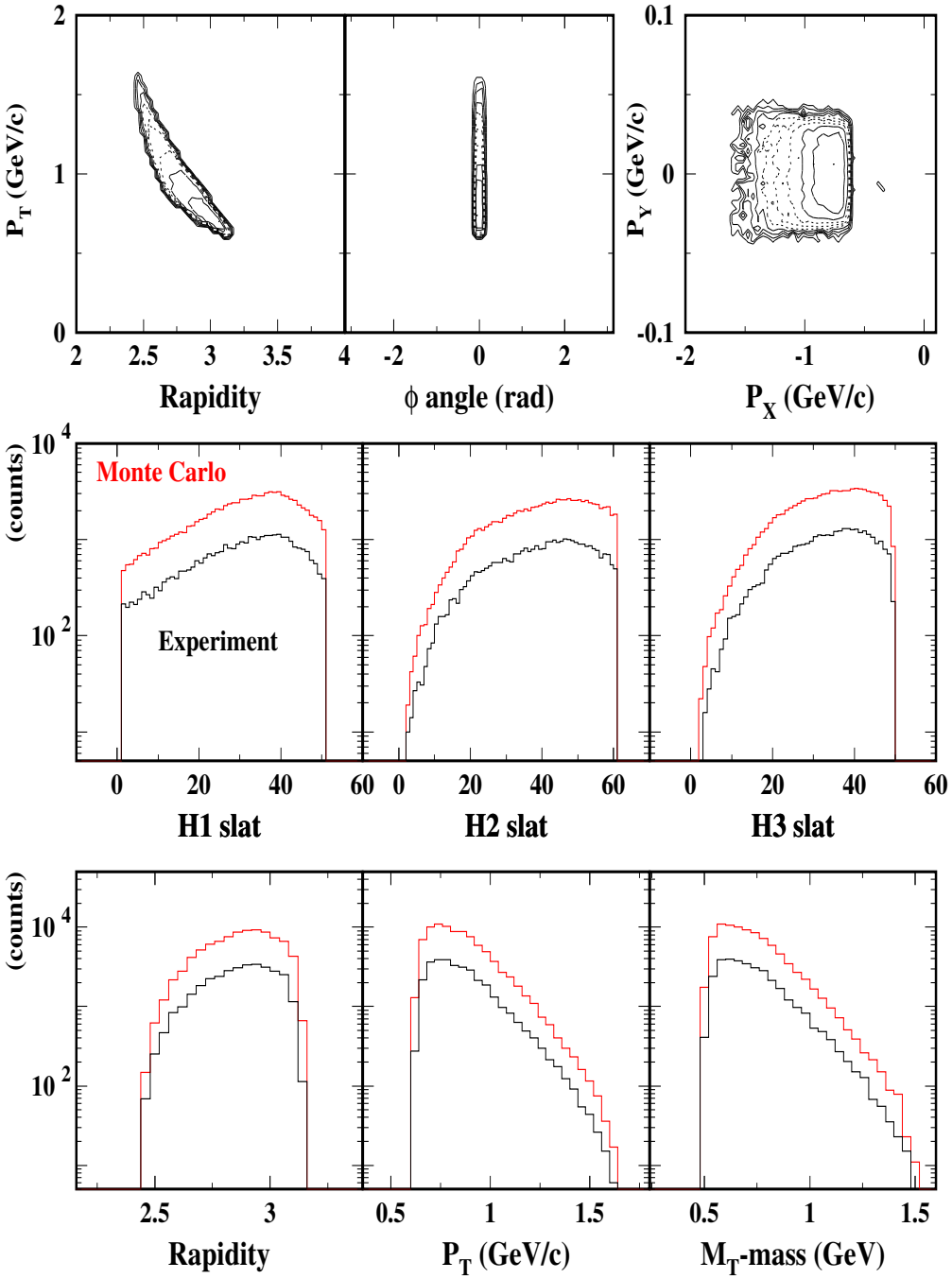

Figure 4-4: single particle distributions for both experiment and Monte Carlo

4.4 Acceptance correction

To correct the spectrometer acceptance effect to the measured spectrum, a Monte Carlo simulation is necessary. For an input momentum distribution to the Monte Carlo, a single exponential spectrum is assumed as a formula of invariant cross section $1/M_T\ dN/dM_T$ in the function of M_T, and a Gaussian shape spectrum is assumed as an rapidity distribution dN/dy for given particle species. Generated momentum spectrum is characterized as the following formula.

$$\frac{1}{M_T}\ \frac{d^2N}{dM_T dy} = \exp(-\frac{M_T}{A})\ \exp\{-(\frac{y-B}{2C})^2)\},$$

where A is an inverse slope parameter of the exponential function, B, C are a peak and a width of the Gaussian distribution, respectively. Those simulated secondary particles are transported though inside the aperture of the magnetic spectrometer with the DECAY TURTLE program, which has multiple scattering effects and decaies. By using the detector Monte Carlo program, the digitized signals on the hodoscopes are generated and recorded. Then using the digitized detector informations, the tracking and momentum reconstruction program, which has been used for the experimental data reduction, will be executed to calculate the reconstructed momentum distribution as a output from the Monte Carlo.

A definition of the acceptance correction factor CF is the ratio of a generated momentum spectrum which is an input to the Monte Carlo over a reconstructed momentum spectrum which is an output from the the Monte Carlo. This correction factor will be multiplied to an experimental momentum spectrum to get an original physical momentum distribution as following formulas. Here, the rapidity axis is already integrated to one unit of rapidity for each factor.

$$CF = \frac{\exp(-\frac{M_T}{A})\ \{= \frac{1}{M_T}\frac{dN}{dM_T}(\text{generated})\}}{\frac{dN}{dM_T}(\text{Monte Carlo output})},$$

$$\frac{1}{M_T}\frac{dN}{dM_T}(\text{corrected}) = CF \times \frac{dN}{dM_T}(\text{experiment}).$$

By fitting the corrected spectrum $1/M_T\ dN/dM_T$ with a single exponential function, an inverse slope parameter can be determined from the experimental measurement. The steps of the acceptance correction are shown in the figure 4-5. Since the exponential function $\exp(-M_T/A)$ is used as an input spectrum, the normalization of the corrected spectrum is arbitrary, so the slope parameter can only be discussed with this correction. Acceptance correction factor for the normalization will be taken into account in the following section.

Because of the shape of NA44 spectrometer acceptance which is narrower in the rapidity, especially for kaons and protons, the input parameters for the rapidity distribution such as the peak and the sigma of the Gaussian distribution will not affect

to the corrected spectrum $1/M_T\ dN/dM_T$ too much. However, the spectrometer acceptance for pion, which has strong correlation between P_T and rapidity, the corrected spectrum will depend on those input parameters. Therefore, one needs to use realistic rapidity distribution as an input to the monte Carlo particle generation at least for pion. In this analysis, the results from the other experiments (WA80, NA35 and NA49) at SPS with the same colliding system are used as the input parameters for the pion rapidity distribution in order to extract transverse mass spectra.

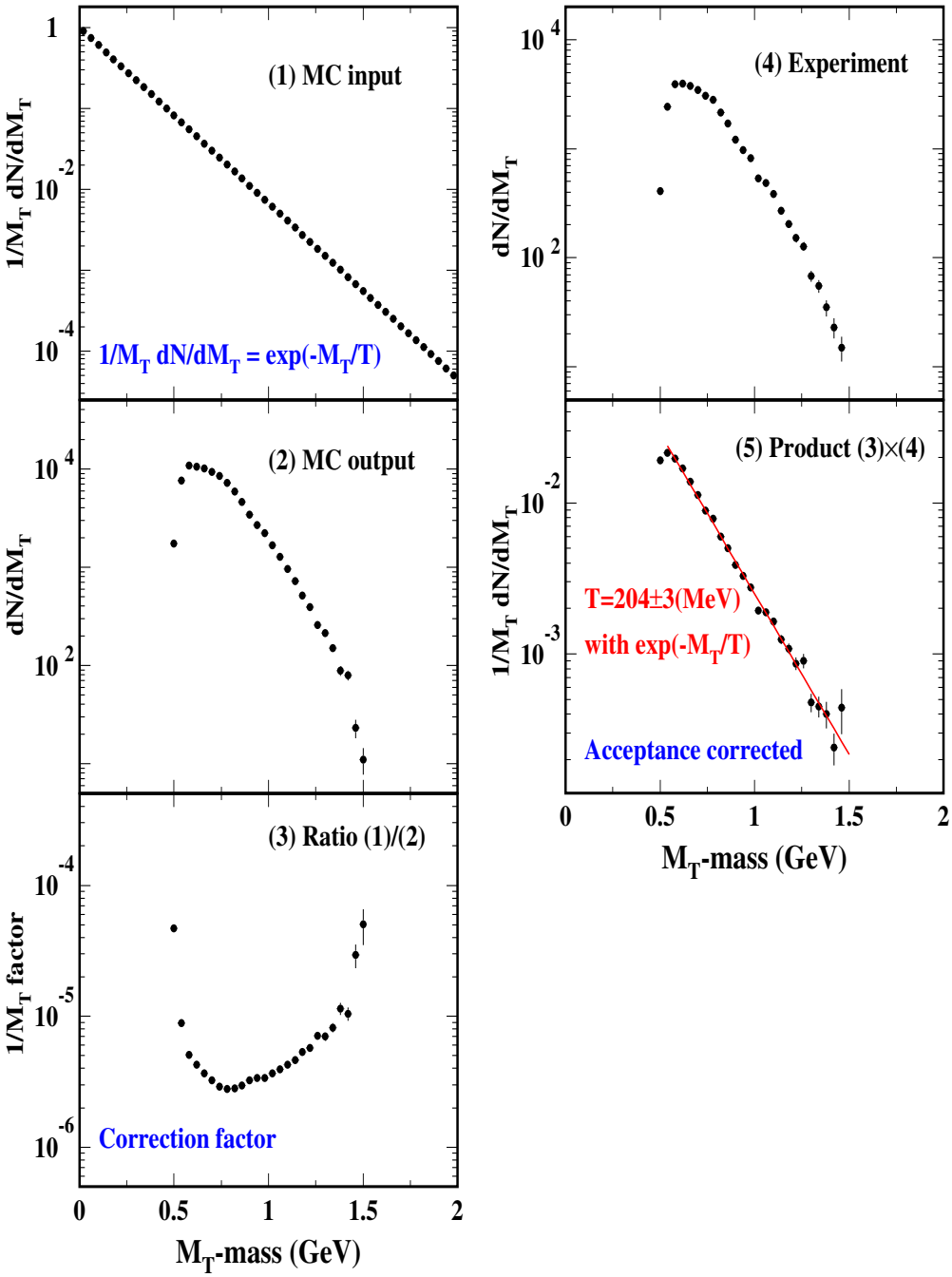

Figure 4-5: acceptance correction

4.5 Normalization

Now, an absolute production cross section, $1/(2\pi M_T)\ d^2N/(dM_T dy)$, which is normalized to one unit collision, will be calculated. The normalization factor is described as follows,

$$F = \frac{1}{N_{beam}\sigma_{int}\sigma_{trig}} \times \frac{N_{pre}}{N_{acc}} \times N_{mc} \times F_{cv} \times F_{mul}.$$

where N_{beam} is number of beam particles during the data taking. N_{pre} is the number of presented triggers, N_{acc} is the number of accepted triggers, so N_{pre}/N_{acc} gives the dead time correction factor for DAQ. σ_{int} is total geometrical interaction probability, therefore $N_{beam}\sigma_{int}$ gives the number of minimum bias collision. σ_{trig} is trigger centrality fraction, so that $N_{beam}\sigma_{int}\sigma_{trig}$ is the number of central collision for given centrality. N_{mc} is an intersection value at $M_T = 0$ of the generated Monte Carlo exponential spectrum, which is integrated over ϕ angle and normalized to unit rapidity. F_{cv} is Cherenkov veto correction factor. Kaons and protons coming through the experimental acceptance with pions or electrons in the same event will be rejected in on-line trigger, because of the on-line particle selection with the two gas Cherenkov detectors. This factor is to correct those loss in the on-line trigger selection or in the off-line Cherenkov cut and depends on the particle multiplicity and the ratios between those particles. F_{mul} is a correction factor for multi-particle event. Because the particle identification with single read-out gas Cherenkov detector can only be done for single particle event on the experimental acceptance, therefore more than two tracks events are thrown away when making the M_T distribution. This factor is to correct a fraction of multi-particle events to the single particle events. F_{cv} and F_{mul} is tabled for 4, 6 and 8GeV/c horizontal small and large angle setting with S+S collisions in the following.

angle setting	small	small	small	large	large	large
momentum setting	4	6	8	4	6	8
F_{cv}	1.17	1.24	1.35	1.04	1.03	1.06
F_{mul}	1.08	1.11	1.12	1.01	1.01	1.01

Then, multipling $F/(2\pi)$ to $1/M_T\ dN/dM_T$ given by the previous section, the absolute production cross section will be calculated in the following.

$$\frac{1}{2\pi M_T}\ \frac{d^2N}{dM_T dy} = \frac{F}{2\pi}\ \times \frac{1}{M_T}\frac{dN}{dM_T}.$$

Absolutely normalized production cross sections are seen in the figure 4-6,4-7 and 4-8 showing the several different settings at a time. By assuming the single exponential for the cross section, the exponential fitting with $A\exp(-M_T/B)$ can

be done. Using the fitting parameters A and B, dN/dy will be calculated with integrating over M_T as the following,

$$\frac{dN}{dy} = 2\pi A \int_M^\infty M_T \exp(-\frac{M_T}{B}) dM_T$$

$$= 2\pi AB \ (M + B) \ \exp(-M/B),$$

where M is the mass of the particle in question.

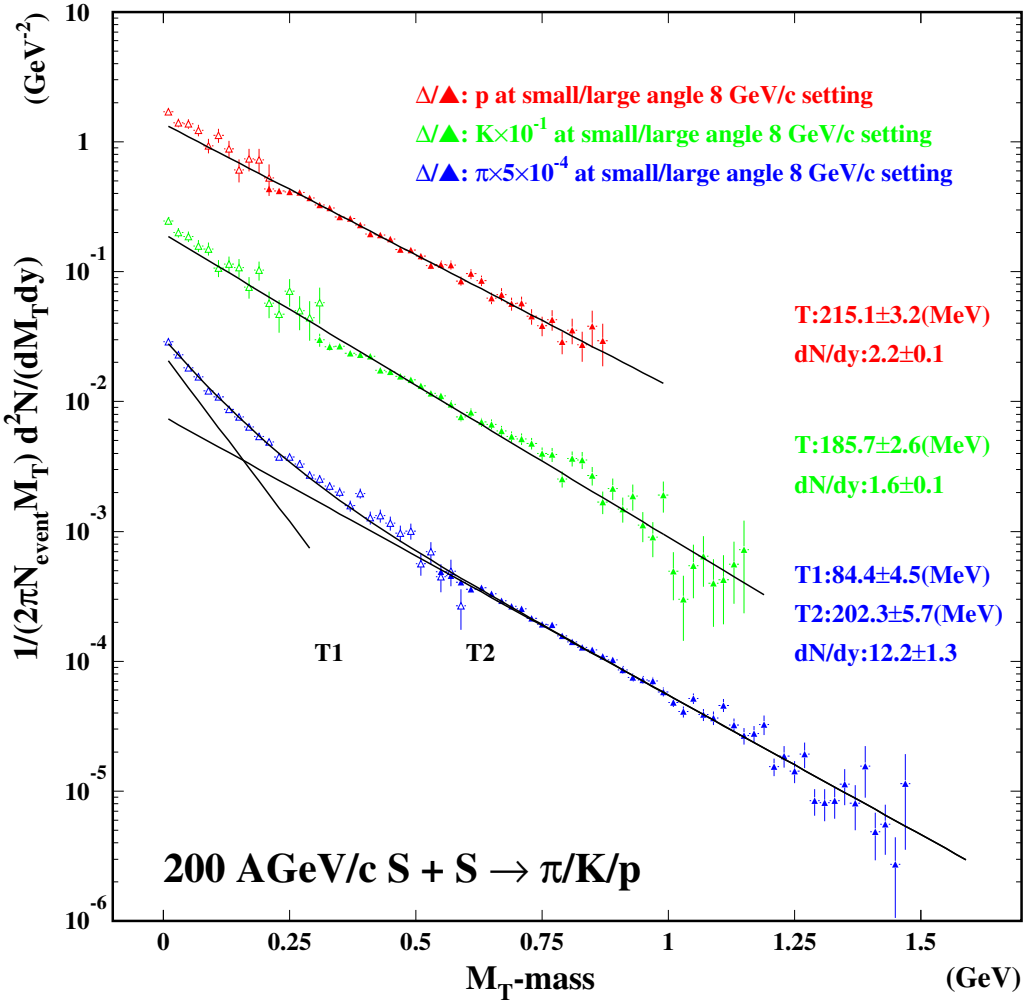

Figure 4-6: normalized production cross sections at 8GeV/c

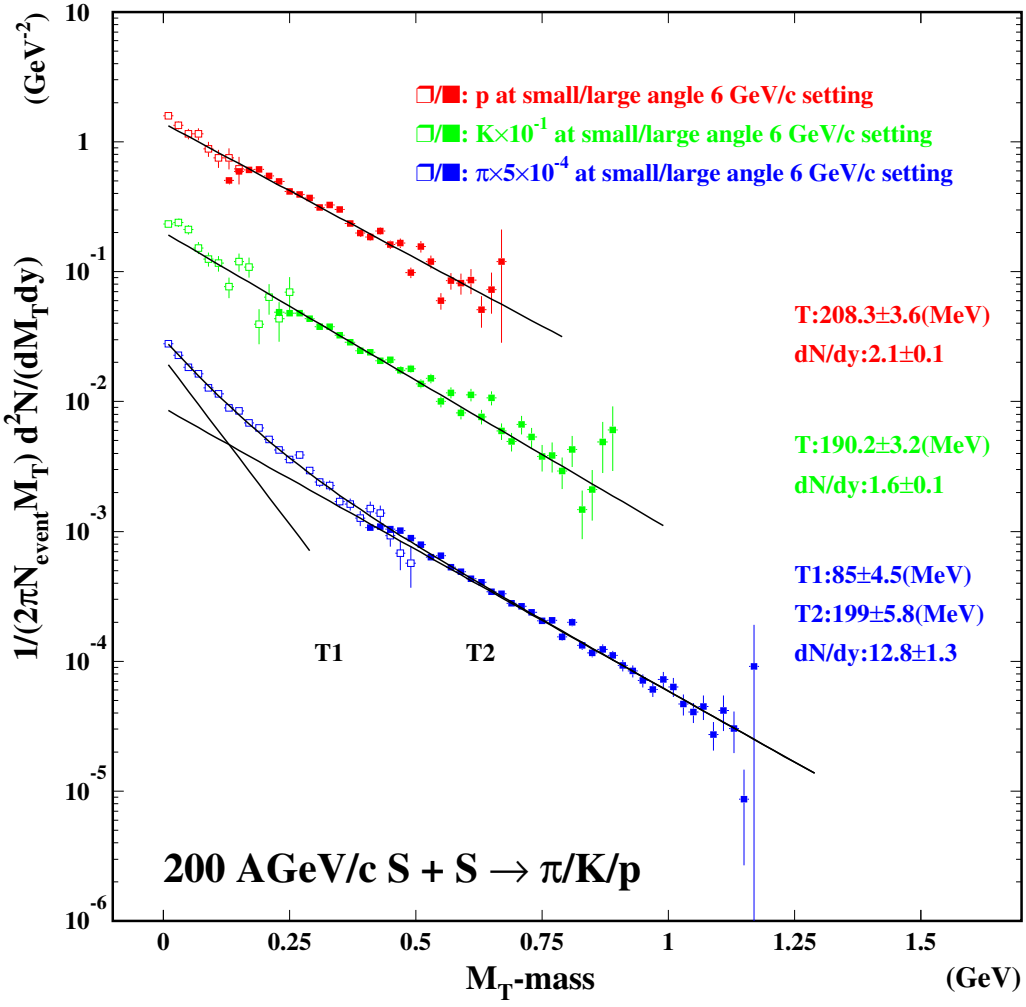

Figure 4-7: normalized production cross sections at 6GeV/c

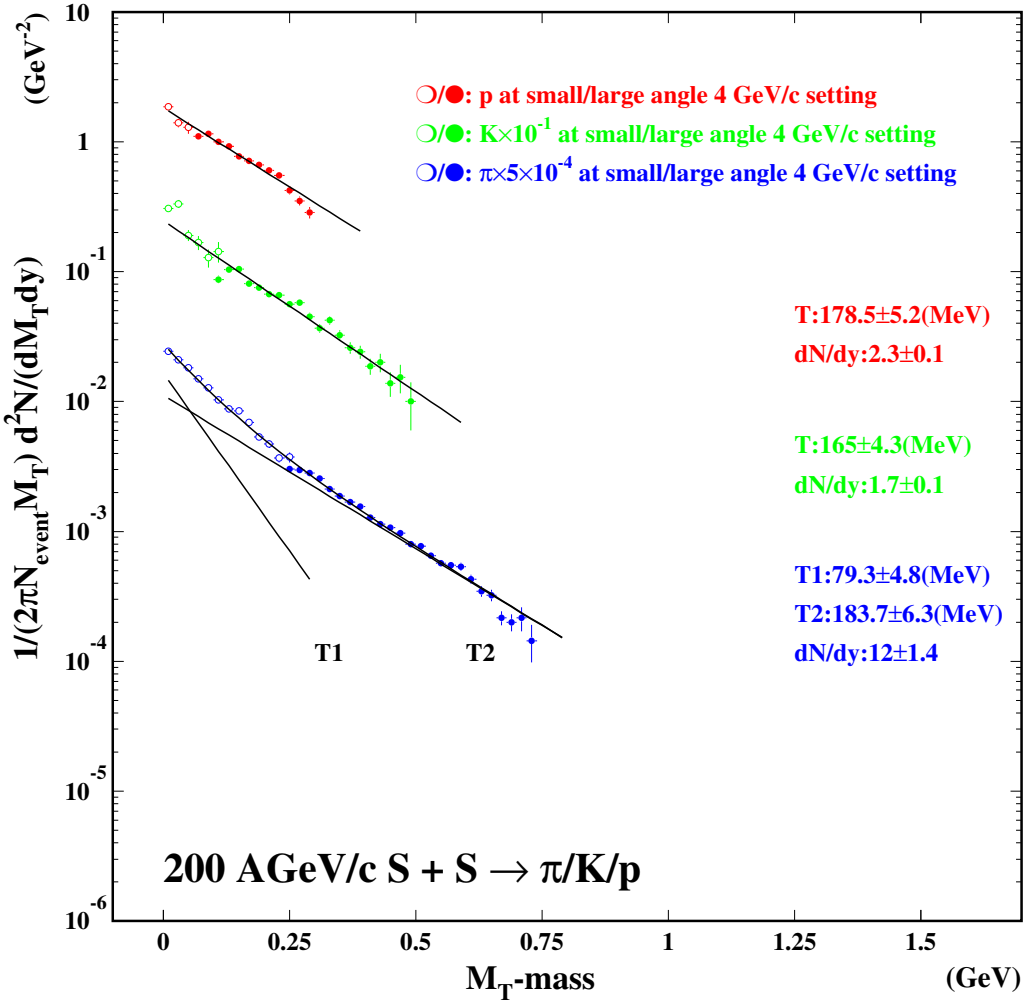

Figure 4-8: normalized production cross sections at 4GeV/c

4.6 Two particle interferometry

To study two particle interferometry, which is an enhancement at small relative momentum difference between measured two identical particles simultaneously, parameterization of those relative momentum differences is first discussed. Invariant momentum difference is defined as the following.

invariant momentum difference $Q_{inv} = \sqrt{(Q^2 - {Q_0}^2)}$

Transverse momentum difference $Q_T = \mid \vec{P_{T1}} - \vec{P_{T2}} \mid$

Longitudinal momentum difference $Q_L = \mid P_{Z1} - P_{Z2} \mid$

where $Q = \mid \vec{P_1} - \vec{P_2} \mid$, $Q_0 = \mid E_1 - E_2 \mid$, and P_Z is measured at LCMS (longitudinal center-of-mass system) flame, which the total longitudinal momentum of the two particle is equal to zero ($P_{Z1} + P_{Z2} = 0$). Q_T is separated into two orthogonal directions Q_{TO} and Q_{TS} as mentioned in the previous chapter.

Experimentally, two particle correlation function will be constructed by a ratio of a real momentum difference spectrum over a fake momentum difference spectrum.

$$C(\vec{P_1}, \vec{P_2}) = \frac{R(\vec{P_1}, \vec{P_2})}{B(\vec{P_1}, \vec{P_2})},$$

The real momentum difference spectrum $R(\vec{P_1}, \vec{P_2})$ will be filled as a function of one of the above mentioned variables, for example Q_{inv}, using two momenta $\vec{P_1}$, $\vec{P_2}$ from two track reconstructed event samples. The fake momentum difference spectrum $B(\vec{P_1}, \vec{P_2})$ is created by mixing two reconstructed momenta from two different events, here one can suppose no Bose-Einstein effect between two particles at small momentum difference in the first order approximation, therefore this fake spectrum can be used as a normalization spectrum for the real spectrum to get the two particle correlation function with the ratio of those two spectra. The correlation functions in one-dimensional as a function of Q_{inv}, and in two-dimensional as a function of Q_T and Q_L are shown in the figure 4-9.

Correlation functions will be described in the following parameterizations.

$C(Q_{inv}) = 1 + \lambda \exp(-{Q_{inv}}^2 {R_{inv}}^2)$,
$C(Q, Q_0) = 1 + \lambda \exp(-Q^2 R^2 - {Q_0}^2 \tau^2)$,
$C(Q_T, Q_L) = 1 + \lambda \exp(-{Q_T}^2 {R_T}^2 - {Q_L}^2 {R_L}^2)$,
$C(Q_{TS}, Q_{TO}, Q_L) = 1 + \lambda \exp(-{Q_{TS}}^2 {R_{TS}}^2 - {Q_{TO}}^2 {R_{TO}}^2 - {Q_L}^2 {R_L}^2)$,

where λ is a chaoticity parameter, R_{inv} is a measured source size in the Q_{inv} scale, R and τ are a geometrical source size and particles emission time duration, R_T and R_L are a transverse and a longitudinal geometrical source size, R_{TS} and R_{TO} are

a sideward source size which is purely geometrical transverse size and a outward source size which is a mixture of the geometrical transverse size and a emission time duration. Those functions are used to fit the experimental correlation function in order to extract the chaoticity and source size parameters.

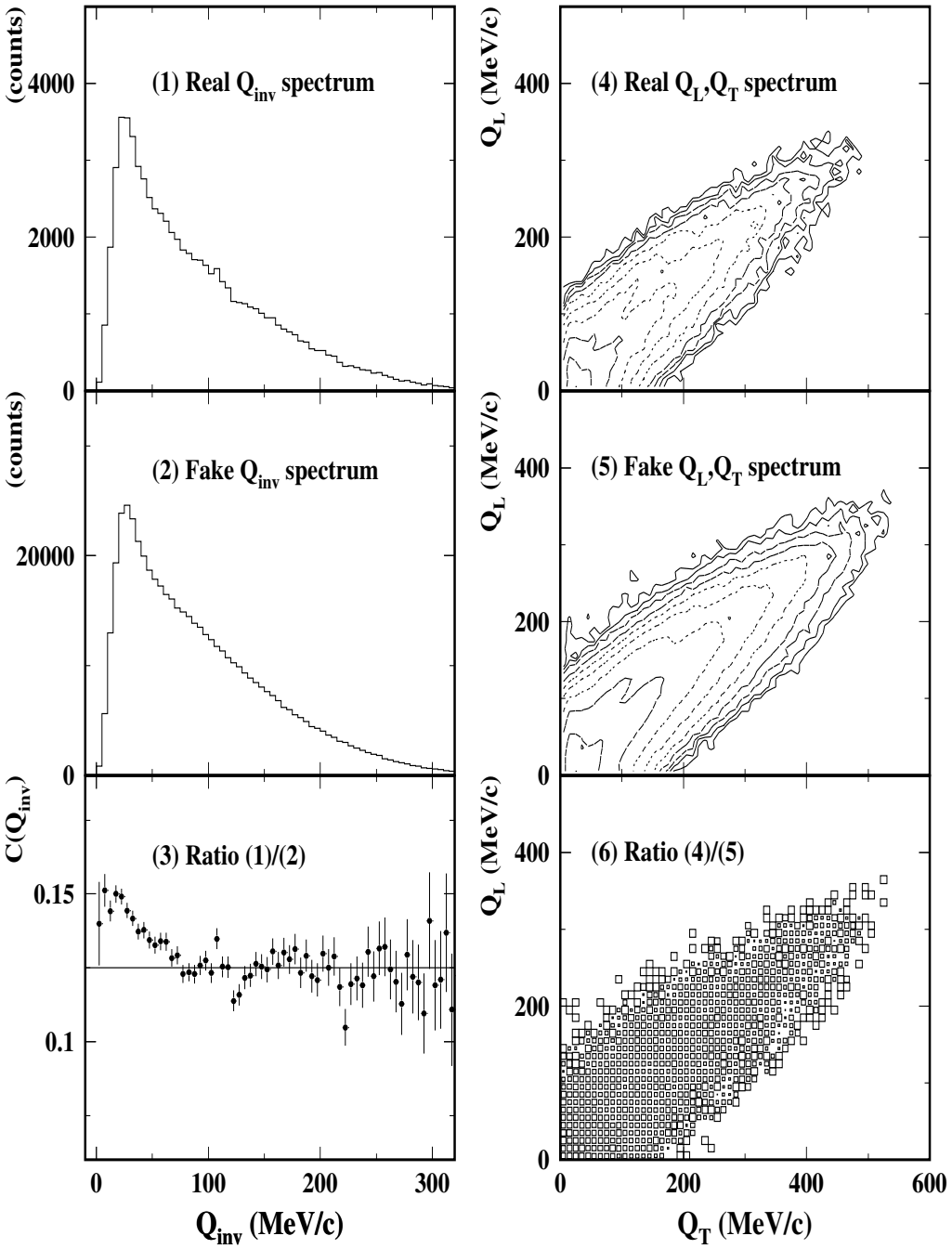

Figure 4-9: construction of correlation function

4.7 Chi-square mapping method

To extract an original correlation function, several experimental effects have to be corrected from the measured correlation function $C(\vec{P}_1, \vec{P}_2)$. Three major corrections for the measured correlation function are a coulomb correction (K_{coul}), which corrects coulomb repulsion effect especially at a small momentum difference, an acceptance correction (K_{acc}), which is an experimental detector resolution effect, a single particle correction (K_{spc}), which is a residual effect into the fake momentum difference spectrum coming from the 2 particle correlation, because of the event mixing technique using the real events. Therefore corrected correlation function can be given as follows,

$$C(\text{corrected}) = C(\text{raw}) \times K_{coul} \times K_{acc} \times K_{spc}.$$

However, those correction factors depend on the original correlation function, which will be the fitted results such as the source size and a chaoticity parameter of the corrected correlation function. To solve this parameter dependence, an iterative way of calculation will be used, until a difference between the input parameters and the output fitted results becomes negligibly small [2-4].

In this section, an approach different from the iterative way will be shown to analyze the two particle interferometry. This approach is named as a 'chi-square mapping method', where a basic idea of this approach is a comparison between an experimental data without any correction and a Monte Carlo simulated data which include every effects in order to find a best input parameter to the Monte Carlo giving the smallest chi-square between the experimental data and the Monte Carlo output data. Important points are, Monte Carlo simulation should reproduce the experiment as close as possible to the reality, the exactly same analysis procedure have to be used for the Monte Carlo events as the experimental analysis, chi-square map will be created by changing the input parameters, and the minimum chi-square will be searched to find the best input parameters as a results. The step of the approach is described in the following and shown in the figure 4-10.

Two particle events sample (typically 5 to 10 times larger than the amount of experimental data sample), where the two particles are going through the experimental acceptance, are generated, here physical single particle distribution for each particle will be assumed (exponential shape of invariant cross section in M_T and Gaussian shape in rapidity).

Those two particles will be transported into the magnetic spectrometer by the DECAY TURTLE program which has decays and multiple scattering inside and the digitized hits information of each detector will be recorded with taking into account the energy loss in the detector.

Using the digitized informations of the detectors, tracking and momentum reconstruction will be done by the same program which has been used by the experimental data analysis. Two generated momenta of generated two particles and reconstructed

momenta and the used slat number on each hodoscope of the reconstructed particles will be saved as the Monte Carlo momentum file.

Using a given set of input parameters (chaoticity parameter and radii of Gaussian source distribution) and two generated momenta of the simulated particle from the Monte Carlo momentum file, a weight which is used to decide to drop (throw away) this event or include it into the correlation function will be calculated. This weight is a product of the correlation function described in the previous section and a coulomb repulsion effect factor (which is an inverse of the Gamow correction factor in this analysis), this weight is normalized to one, where the product is maximum. The Gamow factor is shown in the following formula.

$$G(x) = \frac{x}{\exp(x) - 1} \quad : \quad x = \frac{2\pi\alpha M}{Q_{inv}},$$

which is defined as a function of Q_{inv} for given particle mass M. Then one real random number for each event will be generated between zero and one. If this random number is smaller than the weight, the event will be used to construct a real and a fake momentum difference spectra by using the two reconstructed momenta of this event, otherwise (if the random number is greater than the weight), this event will be dropped (thrown away, not used for the spectrum). The real and fake momentum difference spectra will be created with the same method as used in the experimental data analysis, especially for an event mixing method to make the fake spectrum, and for the slat cut selection to treat the neighbouring slats hit in the same way of the experimental data analysis. The correlation function is also made by the ratio of the real over the fake momentum difference spectrum in the same way. Two important points are that the generated momenta have to be used to calculate the weight and the reconstructed momenta have to be used for correlation function as an output of the Monte Carlo data to be compared with the experimental data. In this step, one Monte Carlo correlation function will be created by scanning the hole data set of the Monte Carlo momentum file with one set of the input parameters.

By changing the input parameters and doing the previous step over and over again, several Monte Carlo correlation functions will be prepared to scan those over the input parameter space. Therefore, in order to analyze one-dimensional correlation function like Q_{inv}, the Monte Carlo correlation function has to be prepared at each point of the two-dimensional parameter space in R_{inv} and λ. To find the nearest Monte Carlo correlation function to the experimental correlation function, a chi-square between the experimental correlation function and each Monte Carlo correlation function will be calculated to make chi-square map over the two-dimensional parameter space for one-dimensional correlation function with the following formula.

$$\chi^2(R_{inv}, \lambda) = \sum_{i=1}^{n} \frac{\{C(i)^{EX} - C(i : R_{inv}, \lambda)^{MC}\}^2}{\{E(i)^{EX}\}^2 + \{E(i : R_{inv}, \lambda)^{MC}\}^2},$$

where $C(i)^{EX}$ and $E(i)^{EX}$ are the experimental correlation function and corresponding error, $C(i : R_{inv}, \lambda)^{MC}$ and $E(i : R_{inv}, \lambda)^{MC}$ are Monte Carlo correlation function and corresponding error for given input parameters R_{inv} and λ, i is the bin

number, n is the total number of the bin. Therefore a chi-square map over the three-dimensional parameter space is necessary to analyze the two-dimensional correlation function like $C(Q, Q_0)$ or $C(Q_T, Q_L)$, because those function have three free parameters to be searched. Finding the minimum chi-square point in the input parameter space is done to get the best resulting fit parameters of the chaoticity and radii of Gaussian source distribution. The statistical error of those parameters are given from the region of 'the minimum chi-square + 1' as a definition.

As seen from above steps of the procedures, the effects corresponding to the correction factors, which are normally used to correct the experimental data and are mentioned as K_{coul}, K_{acc} and K_{spc} before, are included as inverses of those factors into the Monte Carlo correlation function automatically in this method. $1/K_{coul}$ can be seen in the weight for the event dropping, $1/K_{acc}$ can be seen in the distinguished usage of the generated and reconstructed momenta to create the Monte Carlo correlation, and $1/K_{spc}$ can be seen in the usage of the same event mixing method for the Monte Carlo fake momentum difference spectrum as for the experimental spectrum. The merit of this approach is that by scanning the whole parameter space to make chi-square contour map, one can know the global feature of the chi-square contour and local minimum can be found easily. On the other hand, the iteration method can be trapped by the local minimum. The demerit of this approach is that the calculation takes long time to scan over whole parameter space, so that an increase of the dimension or the segmentation of the chi-square map is not easy, compared to the iteration method. The both chi-square mapping method and the iteration method give comparable results, since there is no local minimum compare to the statistical fluctuation.

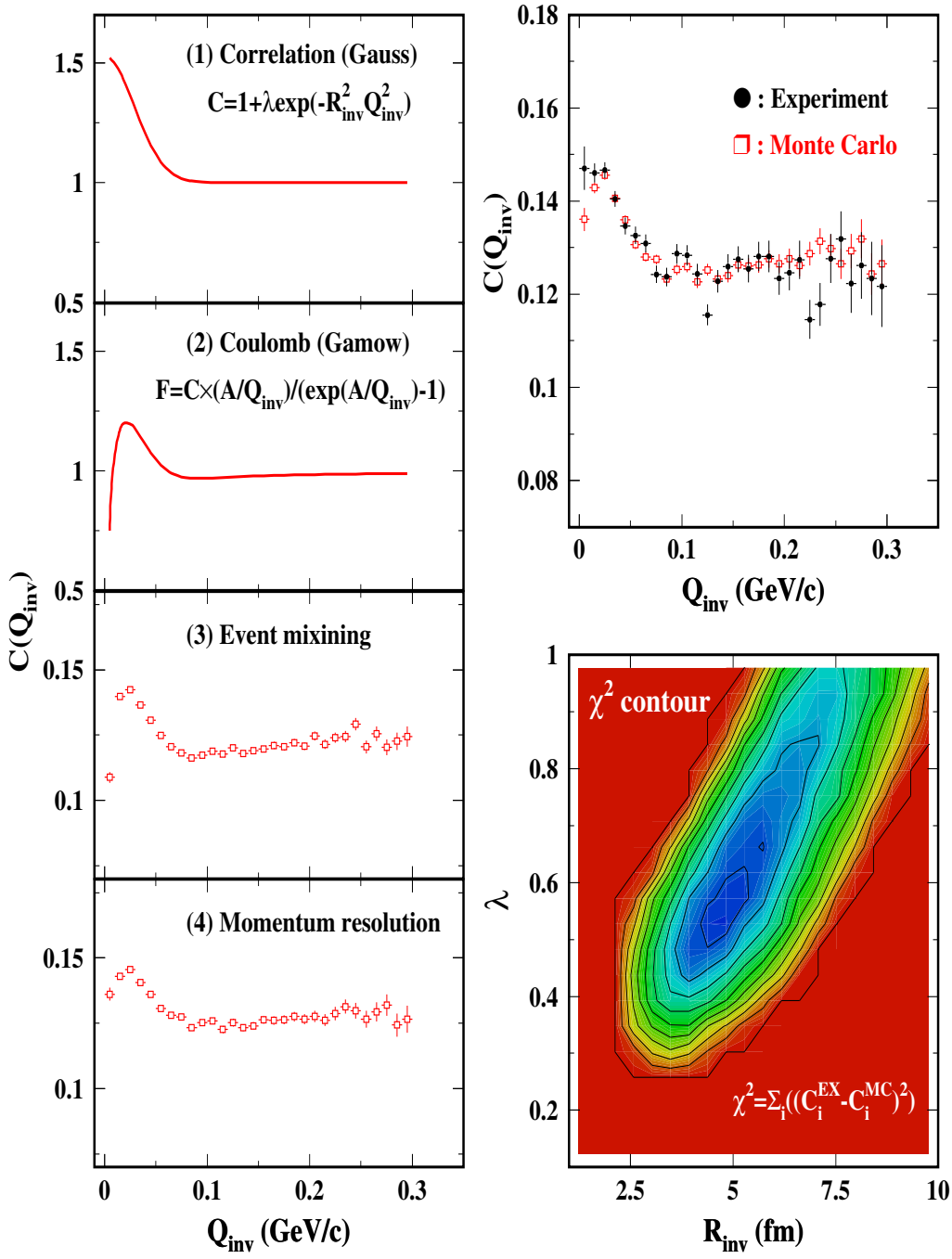

Figure 4-10: chi-square contour mapping method