Heavy Quark Energy Loss Status and Perspectives

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Outline

- Some data on light hadrons

  For illustration only

- Single inclusive particle quenching at high p_T:

  Review of e-loss
  Heavy flavor energy loss

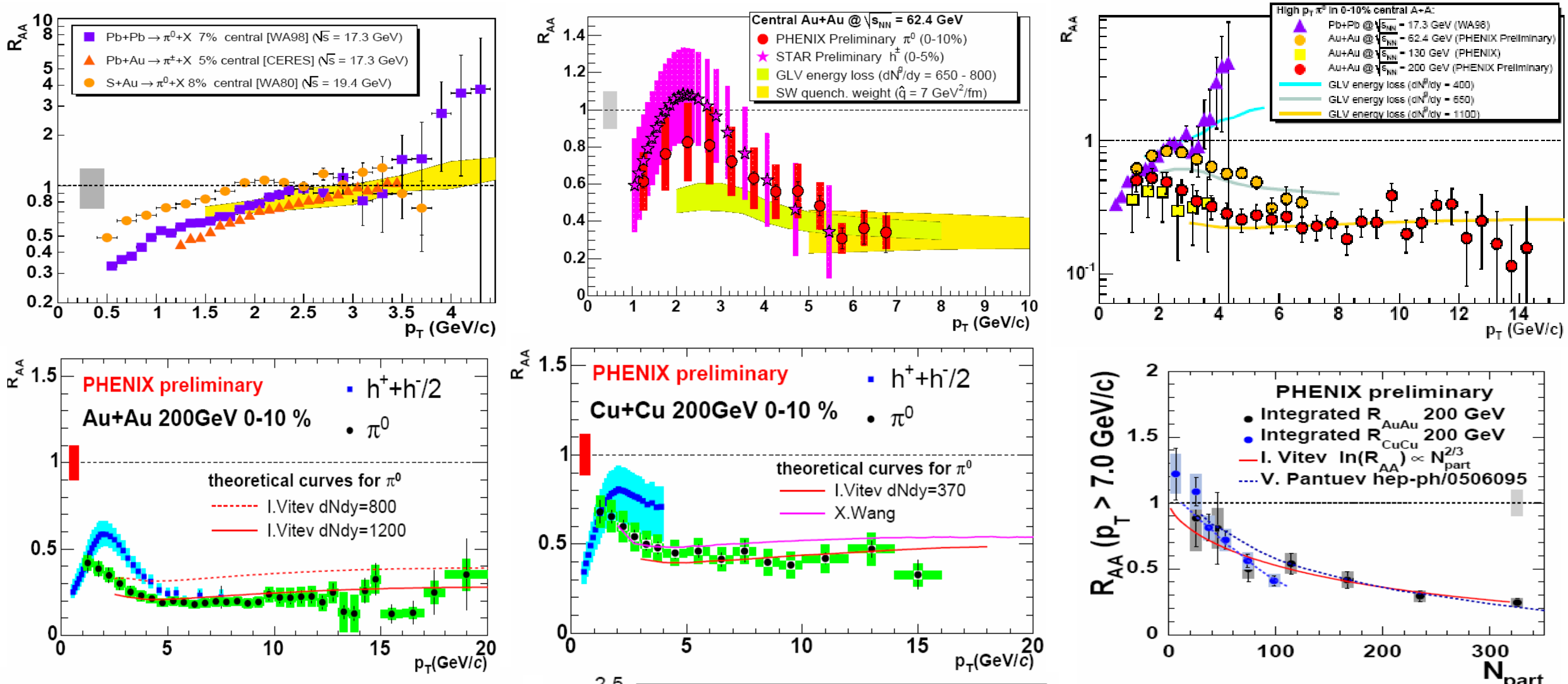
- Other heavy quark calculations:

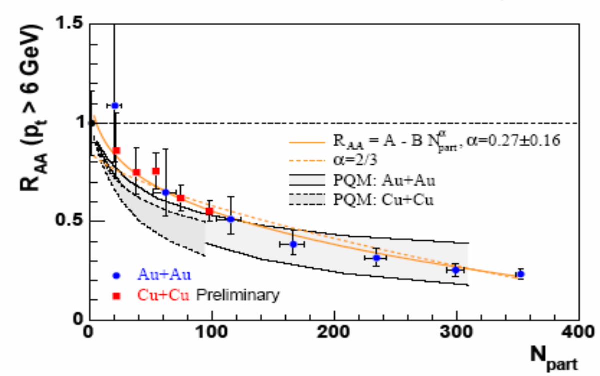
  Elastic energy loss, transport coefficients
  In medium resonances

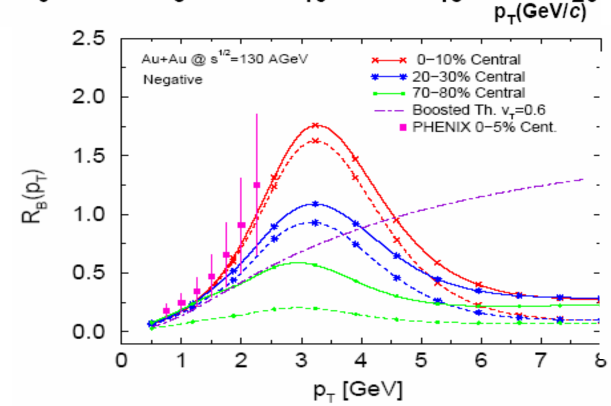
- Possible directions:

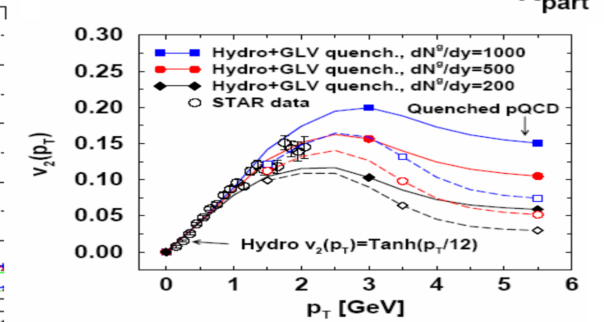
  Look at D and B meson dissociation
  Verify the calculations

How Does it Work?







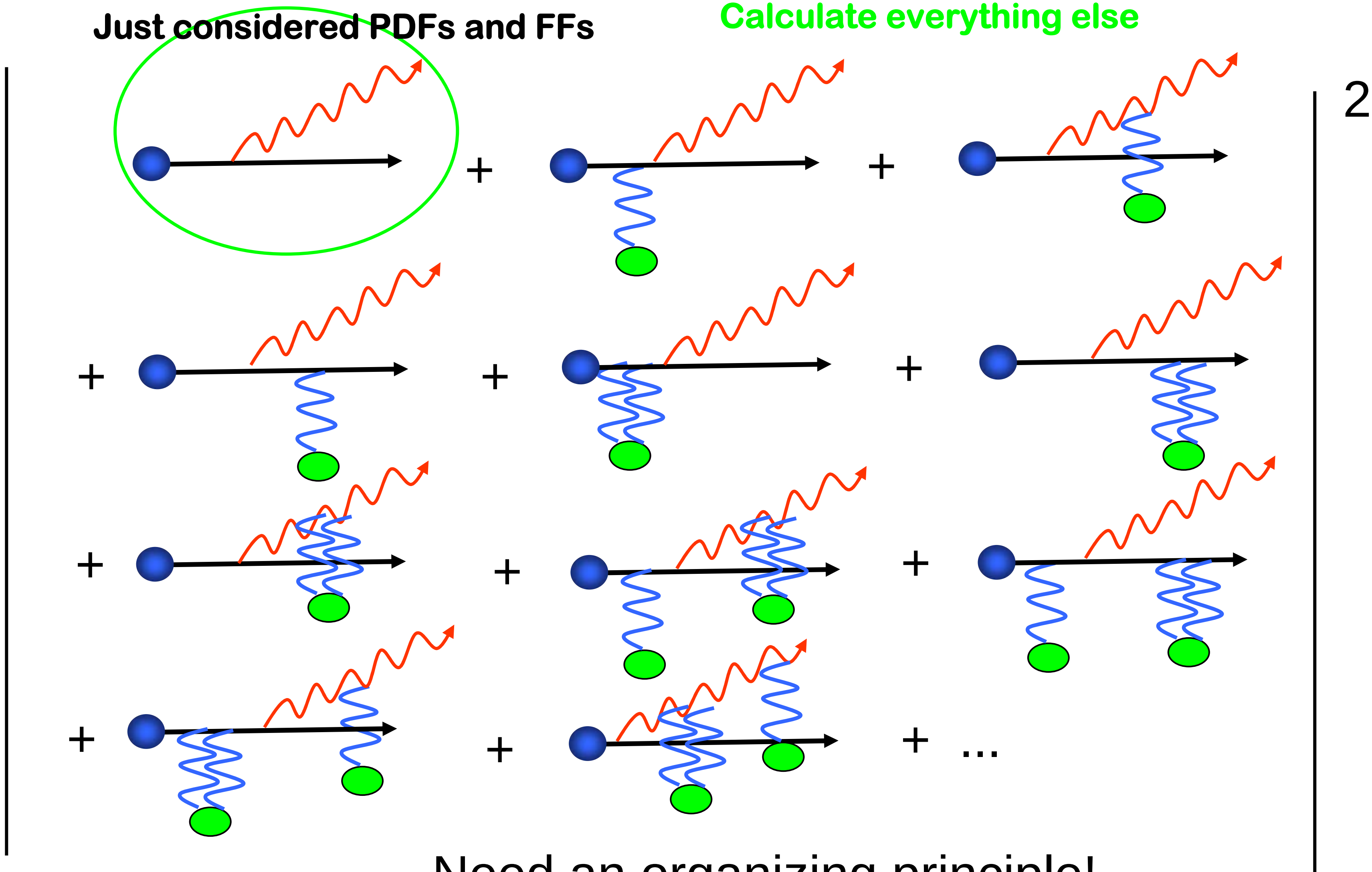


Not a proof of anything but ….

Induced Radiation

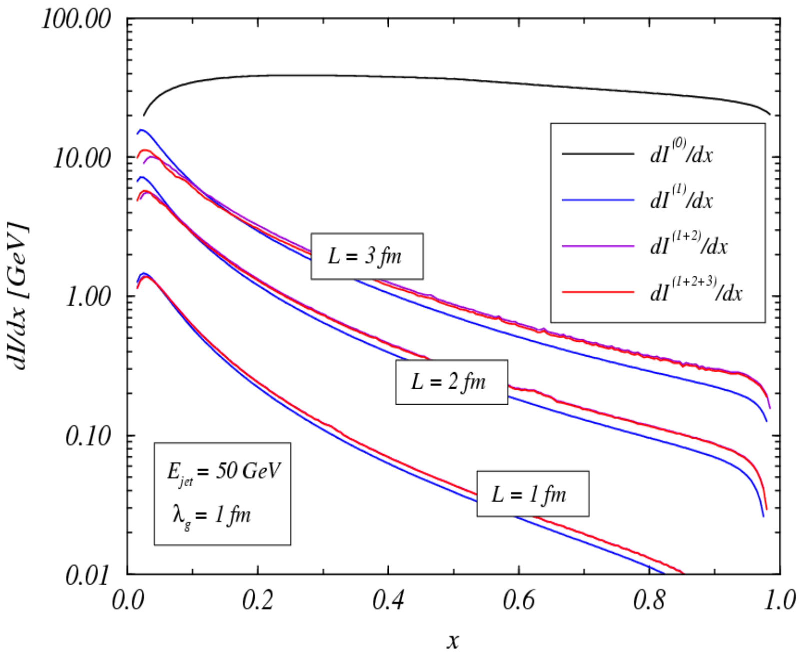
Just considered PDFs and FFs

Calculate everything else



Need an organizing principle!

No Vacuum E-Loss



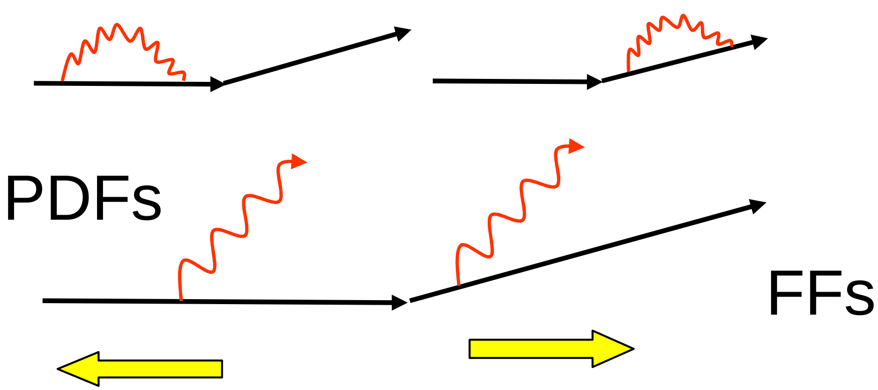
(As much as I like oxymorons)

What did I mean yesterday

$$\Delta E_{vac} \sim 80 - 90\%$$

- Clearly does not make sense

(Will never be a leading hadron of z > 0.1-0.2)



DGLAP evolution

Matrix equation

$$\frac{d}{d\ln Q^2} D(x,Q^2) = \int_x^1 \frac{dy}{y} P(z, a_s(Q^2)) D\left(\frac{x}{y}, Q^2\right)$$

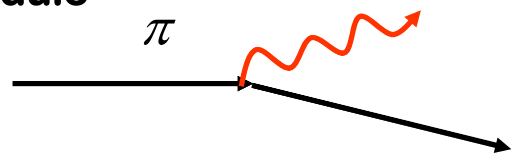
$$\frac{d}{d\ln Q^2} a_s(Q^2) = \beta(a_s(Q^2))$$

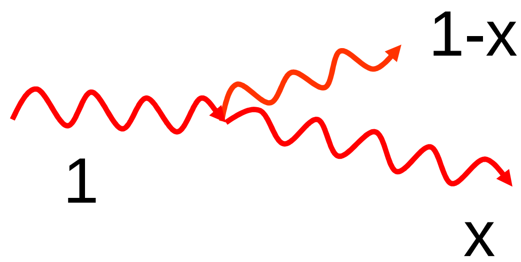
Very slow

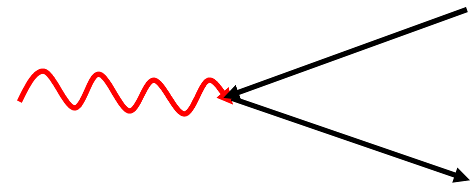
Splitting that counts

Splitting Functions

Modulo $\dfrac{\alpha_s}{\pi}$







And the corresponding anti-quark

$$C_F = \frac{4}{3},\ \ C_A = 3,\ \ T_F = \frac{1}{2}$$

$$P_{qq}^{(1)}(x) = C_F\left[(1+x^2)\left[\frac{1}{1-x}\right]_+ + \frac{3}{2}\delta(1-x)\right],$$

$$P_{qg}^{(1)}(x) = T_F[(1-x)^2 + x^2],$$

$$P_{gq}^{(1)}(x) = C_F\frac{(1-x)^2+1}{x},$$

$$P_{gg}^{(1)}(x) = 2C_F\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right]$$

$$+\left[\frac{11}{6}C_A - \frac{2}{3}T(F)n_f\right]\delta(1-x),$$

The "+" prescription regulates a divergent integral with a divergent subtraction

$$\int_z^1 dx\, f(x)\left[\frac{g(x)}{1-x}\right]_+ = \int_z^1 dx\frac{(f(x)-f(1))g(x)}{1-x}$$

Give the Altarelli-Parisi evolution kernels:

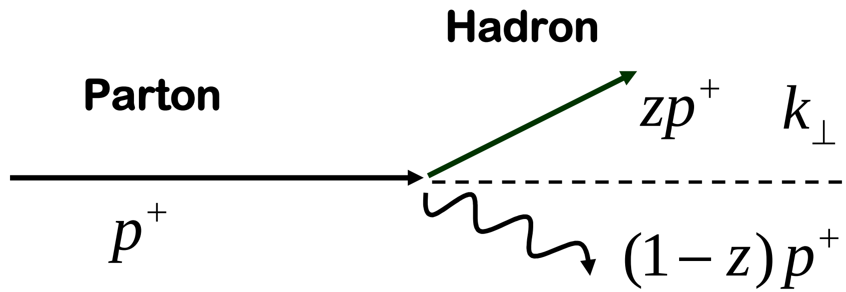
$$\mu^2\frac{d}{d\mu^2}\phi_{i/j}(x,\alpha_s(\mu^2)) = P_{ij}(x) + O(\alpha_s^2)$$

For real radiation leading to energy loss the loop corrections do not contribute

Formation Time

- **From the uncertainty principle:** $\tau_f \simeq 1/\Delta Q$

$$p_q = \left[p^+, \frac{M_q^2}{2p^+}, 0 \right]$$



$$k_\perp \sim \Lambda_{QCD}$$

$$p_h = \left[zp^+, \frac{k_\perp^2 + m_h^2}{2zp^+}, k_\perp \right]$$

$$p_g = \left[(1-z)p^+, \frac{k_\perp^2}{2(1-z)p^+}, -k_\perp \right]$$

$$\Delta y^+ = \frac{1}{\Delta p^-} = \frac{(0.2\ GeV.fm)\ 2z(1-z)p^+}{k_\perp^2 + (1-z)m_h^2 - z(1-z)M_q^2}$$

Formation time (p_T = 5 GeV):

π	D	B
12 fm	**1.5 fm**	**0.25 fm**

The simpler argument: "the gamma boosted size of the hadron" gives qualitatively the same results

$$\tau_f = \gamma_h R_h \approx \gamma_h \times 1\ fm$$

The incorrect argument: $k_T \to p_T \quad \Delta y \sim \dfrac{2z(1-z)}{p_T}$ NOT correct

Cross Sections

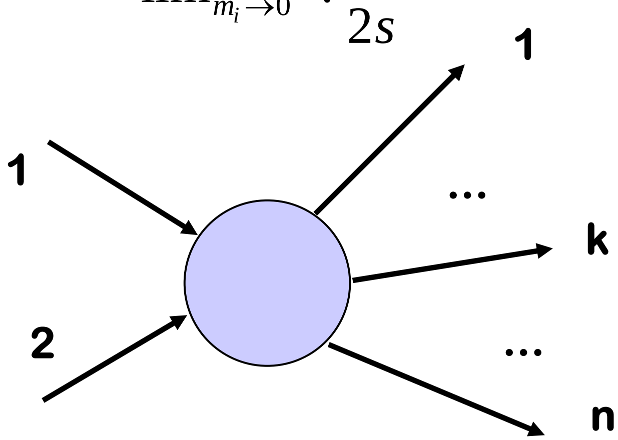
S - matrix

$$S = I + iT, \ \langle f \,|T|\, i \rangle = (2\pi)^4 \delta^4 (P_i - P_f) M_{fi}$$

- **The Feynman diagrams automate the calculations of the contributions to the invariant scattering amplitude between definite initial and final states**

$$d\sigma = \frac{1}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \frac{|M_{fi}|^2}{\prod_i k_i!} \prod_n \frac{d^3 p_n}{2E_n (2\pi)^3} (2\pi)^4 \delta^4 (P_f - P_i)$$

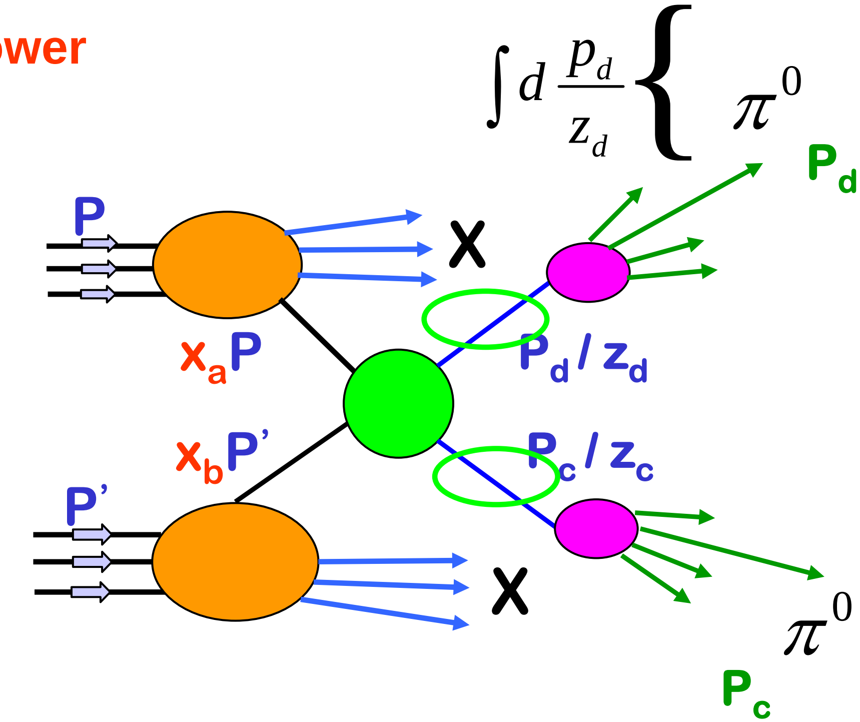
$$\lim_{m_i \to 0} : \frac{1}{2s}$$



- **Initial particle flux factor (2 particles)**
- **Identical final state symmetry factor**
- **Phase space factor (final state)**
- **Energy-momentum conservation factor**

The pQCD Formalism

- Reliable formalism with predictive power



QCD factorization

- To LO (2 to 2 scattering) - single and double inclusive hadron production

Can also incorporate Cronin effect: $\int d^2k_T\, f_{med}(k_T)$

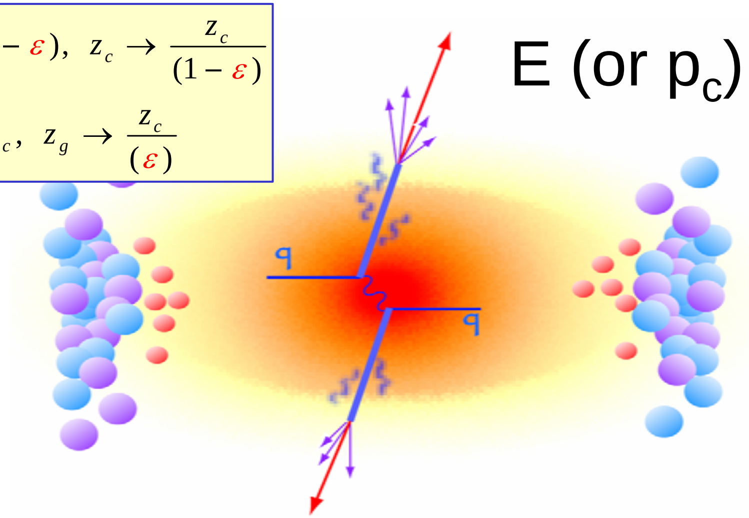
$$\frac{d\sigma^{h_1}_{NN}}{dy_1 d^2p_{T1}} = \sum_{abcd} \int_{x_a\,\min}^{1} dx_a \int_{x_b\,\min}^{1} dx_b\, \phi(x_a)\phi(x_b)\, \frac{\alpha_s^2}{(x_a x_b S)^2} \left|\overline{M}\right|^2_{ab\to cd} \frac{D_{h_1/c}(z_1)}{z_1}$$

$$\frac{d\sigma^{h_1h_2}_{NN}}{dy_1 dy_2 d^2p_{T1} d^2p_{T2}} = \frac{\delta(\Delta\varphi-\pi)}{p_{T1}p_{T2}} \sum_{abcd} \int_{z_1\,\min}^{1} dz_1\, \frac{D_{h_1/c}(z_1)}{z_1} D_{h_2/d}(z_2)\, \frac{\phi(\overline{x}_a)\phi(\overline{x}_b)}{\overline{x}_a\overline{x}_b}\, \frac{\alpha_s^2}{S^2} \left|\overline{M}\right|^2_{ab\to cd}$$

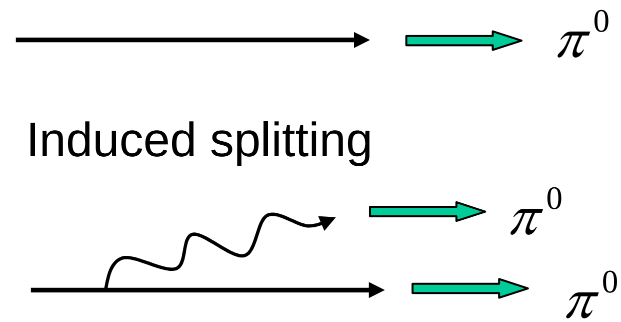
Derivation and Verification

$$p_c \to p_c(1-\varepsilon),\ \ z_c \to \frac{z_c}{(1-\varepsilon)}$$

$$p_c \to (\varepsilon)\, p_c,\ \ z_g \to \frac{z_c}{(\varepsilon)}$$



E (or p_c)



"Modified fragmentation" is also a bad choice since it is not universal

$$D_{h_1/d}(z_1) \;\to\; \frac{1}{1-\varepsilon} D_{h_1/d}\left(\frac{z_1}{1-\varepsilon}\right) + \frac{p_{T_1}}{z_1}\int_0^1 \frac{dz_g}{z_g} D_{h_1/d}(z_g)\,\frac{dN^g}{d\omega}$$

Quenched parent parton — Feedback gluons

- **Use energy conservation to verify the fragmentation sum rule**

$$\int dz_1\ z_1 \left(\frac{1}{1-\varepsilon} D_{h_1/d}\left(\frac{z_1}{1-\varepsilon}\right) + \frac{p_{T_1}}{z_1}\int_0^1 \frac{dz_g}{z_g} D_{h_1/d}(z_g)\,\frac{dN^g}{d\omega} \right)$$

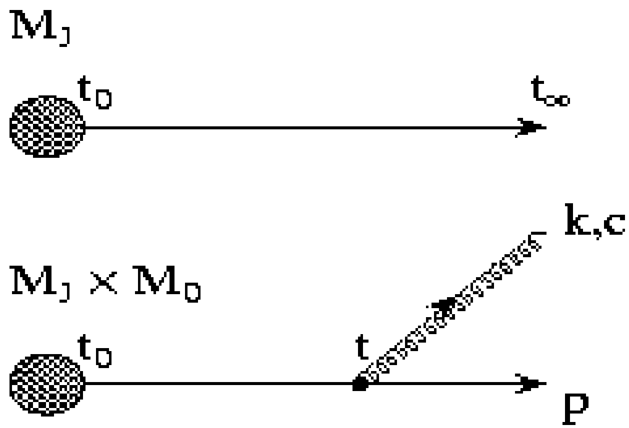
$$= (1-\varepsilon) + \varepsilon = 1$$

- **May worry about Q^2 – from yesterday large changes, small effect (does not introduce large uncertainties)**

Gluon Radiation in Hard Processes

Hard Jet Production

Soft Gluon Radiation



$x = 1 - z$

$$k^\mu = \left[xE^+, \frac{k_\perp^2}{x E^+}, k_\perp \right]$$

$$p^\mu = \left[(1-x)\, E^+, \frac{p_\perp^2}{(1-x)E^+}, p_\perp \right]$$

$$M_0 = iJ(p+k)e^{i(p+k)x_0}(ig_s)(2p+k)_\mu \epsilon^\mu(k) i\Delta(p+k)c$$

$$\approx J(p+k)e^{i(p+k)x_0}(-2ig_s)\frac{\epsilon\cdot k}{k^2}c \approx J(p)e^{ipx_0}(-2ig_s)\frac{\epsilon\cdot k}{k^2}\, e^{i\omega_0 z_0}\, c$$

We have assumed the gluon to be sufficiently soft and collinear

$$\mathcal{M}_J = J(P)e^{iPx_0},$$

$$\mathcal{M}_0 = -2ig_s\frac{\vec{\epsilon}_\perp\cdot\vec{k}_\perp}{k_\perp^2}e^{it_0\frac{k_\perp^2}{2xE_0}}c$$

$$\Delta(p) = \frac{1}{p^2 + i\varepsilon}, \quad c = T_c$$

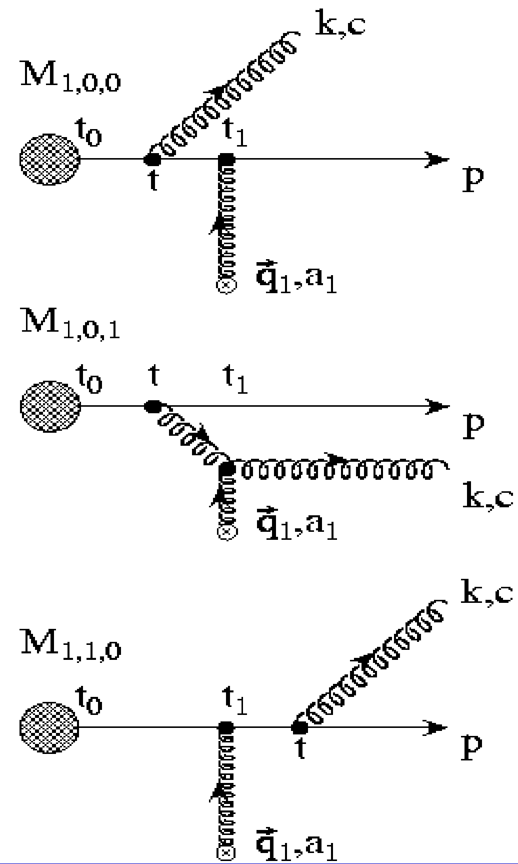
$$x\frac{d^3N_g^{(0)}}{dx\, d^2\vec{k}} \approx \frac{C_R\alpha_s}{\pi^2}\frac{1}{k_\perp^2}$$

$x \ll 1$ and $k_\perp \ll E_0$

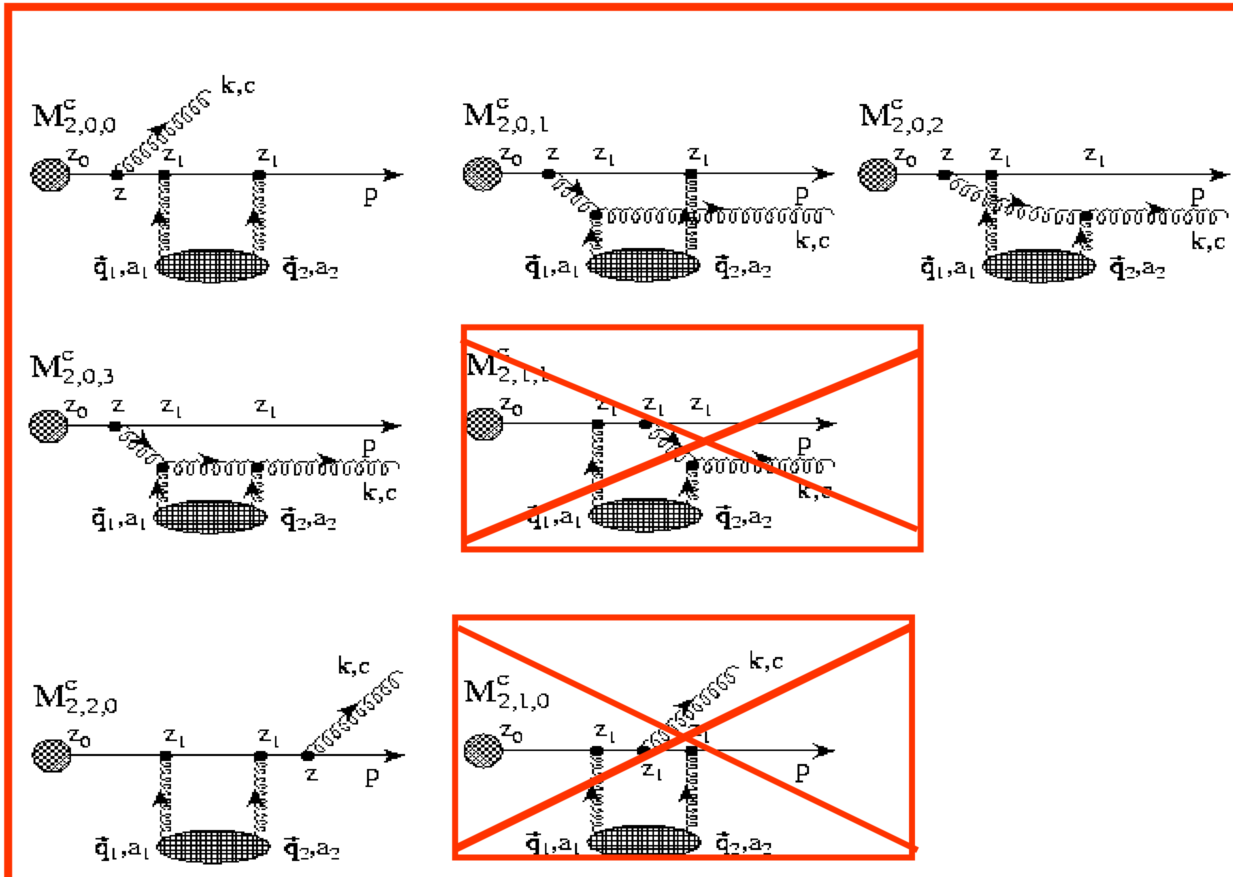
$$dN_g^{(0)} = \frac{C_R\alpha_s}{\pi}\frac{dx}{x}\frac{dk_\perp^2}{k_\perp^2}$$

Medium Induced Radiation

Direct $\hat{D}_1 A_0$



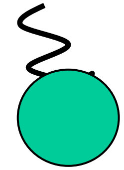
Virtual $\hat{V}_1 A_0$



Clearly similar **Recursion Method** is needed to go toward a large number of scatterings!

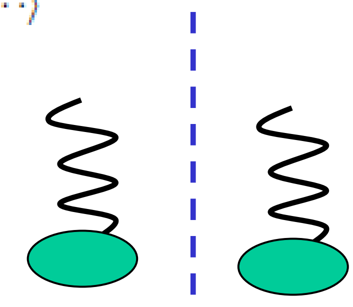
Elastic Scattering Part

$$|\bar{v}(\mathbf{q})|^2 \equiv \frac{1}{\sigma_{el}} \frac{d^2\sigma_{el}}{d^2\mathbf{q}} = \frac{1}{\pi} \frac{\mu_{eff}^2}{(\mathbf{q}^2+\mu^2)^2}$$



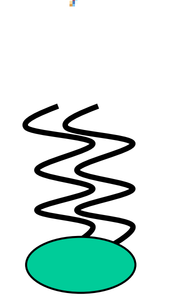
Normalized elastic potential

$$\begin{aligned}\langle \cdots \rangle_{A_\perp} &= \langle \cdots \int \frac{d^2\mathbf{b}_i}{A_\perp} (-i) \int \frac{d^2\mathbf{q}_i}{(2\pi)^2} v(0,\mathbf{q}_i)\, e^{-i\mathbf{q}_i\cdot\mathbf{b}_i} (+i) \int \frac{d^2\mathbf{q}_i'}{(2\pi)^2} v^*(0,\mathbf{q}_i')\, e^{+i\mathbf{q}_i'\cdot\mathbf{b}_i} \cdots \rangle \\ &= \cdots \int \frac{d^2\mathbf{q}_i}{(2\pi)^2} \frac{d^2\mathbf{q}_i'}{(2\pi)^2} \frac{(2\pi)^2\delta^2(\mathbf{q}_i-\mathbf{q}_i')}{A_\perp} v(0,\mathbf{q}_i) v(0,\mathbf{q}_i') \cdots \\ &= \cdots \frac{\sigma}{A_\perp} \int \frac{d^2\mathbf{q}_i}{(2\pi)^2} |\bar{v}(\mathbf{q}_i)|^2 \int d^2\mathbf{q}_i' \delta^2(\mathbf{q}_i-\mathbf{q}_i') \cdots ,\end{aligned}$$



Equal momenta

$$\begin{aligned}\langle \cdots \rangle_{A_\perp} &= \langle \cdots \int \frac{d^2\mathbf{b}_i}{A_\perp} (-i) \int \frac{d^2\mathbf{q}_i}{(2\pi)^2} v(0,\mathbf{q}_i)\, e^{-i\mathbf{q}_i\cdot\mathbf{b}_i} (-i) \int \frac{d^2\mathbf{q}_i'}{(2\pi)^2} v(0,\mathbf{q}_i')\, e^{-i\mathbf{q}_i'\cdot\mathbf{b}_i} \cdots \rangle \\ &= \cdots (-1) \int \frac{d^2\mathbf{q}_i}{(2\pi)^2} \frac{d^2\mathbf{q}_i'}{(2\pi)^2} \frac{(2\pi)^2\delta^2(\mathbf{q}_i+\mathbf{q}_i')}{A_\perp} v(0,\mathbf{q}_i) v(0,\mathbf{q}_i') \cdots \\ &= \cdots (-1) \frac{\sigma}{A_\perp} \int \frac{d^2\mathbf{q}_i}{(2\pi)^2} |\bar{v}(\mathbf{q}_i)|^2 \int d^2\mathbf{q}_i' \delta^2(\mathbf{q}_i+\mathbf{q}_i') \cdots .\end{aligned}$$



or C.C.

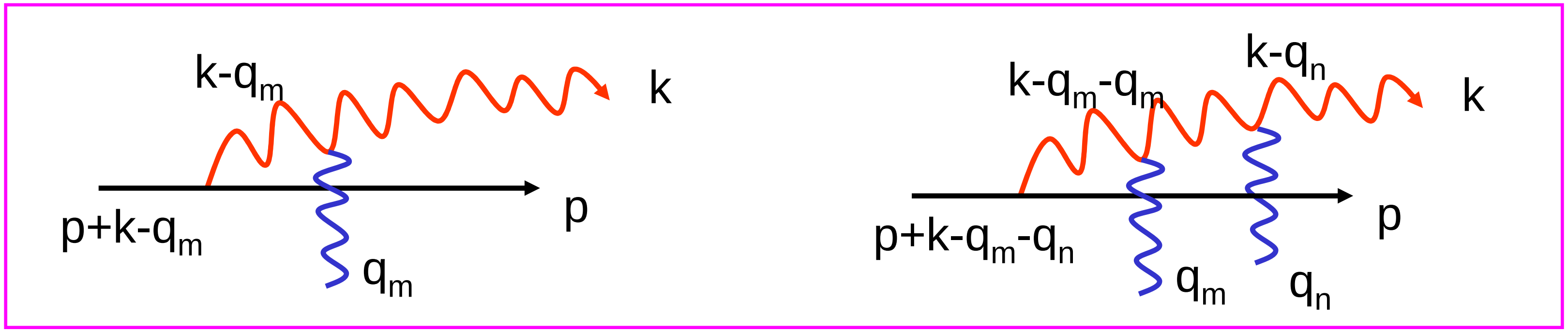
Equal but opposite momenta

The Gluon Scattering Vertices

$$k=[xE^+, k^- \equiv \omega_0, \mathbf{k}] \;, \epsilon(k)=[0, 2\frac{\epsilon\cdot\mathbf{k}}{xE^+}, \epsilon] \;,\; p=[(1-x)E^+, p^-, \mathbf{p}] \qquad E^+ \gg k^+ \gg \omega_{(i\cdots j)} \gg \frac{(\mathbf{p}+\mathbf{k})^2}{E^+}$$

gluon | gluon polarization | parent parton | **Approximations!**

$$\omega_0=\frac{\mathbf{k}^2}{2\omega} \;,\; \omega_i=\frac{(\mathbf{k}-\mathbf{q}_i)^2}{2\omega} \;,\; \omega_{(ij)}=\frac{(\mathbf{k}-\mathbf{q}_i-\mathbf{q}_j)^2}{2\omega} \;,\; \omega_{(i\cdots j)}=\frac{(\mathbf{k}-\sum_{m=i}^{j}\mathbf{q}_m)^2}{2\omega}$$



The difference between QCD and QED is on the self coupling of the gauge field (dynamical color)

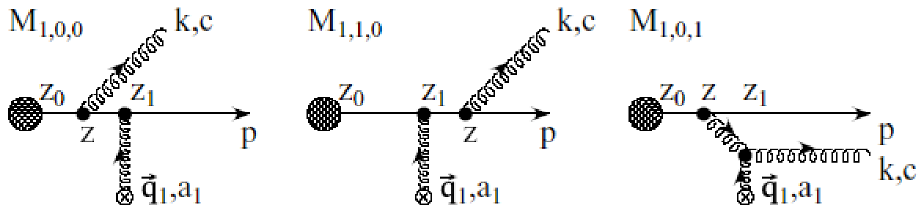
One can also add the color factors

Result:

$$\Gamma_m \equiv (2p+k-q_m)_\alpha \Gamma^\alpha(k;q_m) \approx 2E^+ (\epsilon\cdot(\mathbf{k}-\mathbf{q}_m)),$$

$$\Gamma_{mn} \equiv (2p+k-q_m-q_n)_\alpha \Gamma^\alpha(k;q_n,q_m) \approx 2E^+k^+ (\epsilon\cdot(\mathbf{k}-\mathbf{q}_m-\mathbf{q}_n))$$

Direct Scattering



Jet production amplitude

Average over the momentum transfer

Radiative part with propagators, phases and color factors

- **In the approximations that we outlined we can factorize the answer**

$$M_{1,1,0} = J(p)e^{ipz_0}(-i)\int \frac{d^2\mathbf{q}_1}{(2\pi)^2} v(0,\mathbf{q}_1)\, e^{-i\mathbf{q}_1\cdot\mathbf{b}_1} \times$$
$$\times\,(-2ig_s)\,\frac{\boldsymbol{\epsilon}\cdot\mathbf{k}}{\mathbf{k}^2}\, e^{i\omega_0 z_1}\, c a_1 T_{a_1} \;.$$

Example:

$$M_{1,0,1} \approx J(p+k)e^{i(p+k)z_0}\,[c,a_1]T_{a_1}\,(-i)\int \frac{d^2\mathbf{q}_1}{(2\pi)^2}\, e^{-i\mathbf{q}_1\cdot(\mathbf{x}_1-\mathbf{x}_0)}\, 2g_s\,\boldsymbol{\epsilon}\cdot(\mathbf{k}-\mathbf{q}_1) \times$$
$$\times\, E^+ \int \frac{dq_{1z}}{2\pi} v(q_{1z},\mathbf{q}_1)\Delta(p+k-q_1)\Delta(k-q_1)\, e^{-iq_{1z}(z_1-z_0)} \;.$$

The calculation proceeds via integration in the complex plain: $q_0,\; q_z$

$$I_1(p,k,\mathbf{q}_1,z_1-z_0) \approx i\,\frac{e^{i\omega_0(z_1-z_0)}}{E^+k^+\omega_1}\left(v(-\omega_0,\mathbf{q}_1) - v(\omega_1-\omega_0,\mathbf{q}_1)e^{-i\omega_1(z_1-z_0)}\right)$$
$$\approx iv(0,\mathbf{q}_1)\,\frac{e^{i\omega_0(z_1-z_0)}}{E^+k^+\omega_1}\left(1-e^{-i\omega_1(z_1-z_0)}\right) \;.$$

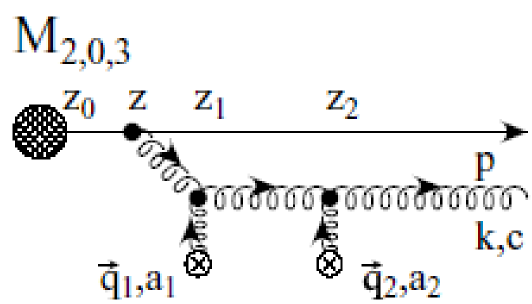
How denominators emerge $k^+\omega_1 = \left(k_\perp - q_{1\perp}\right)^2$

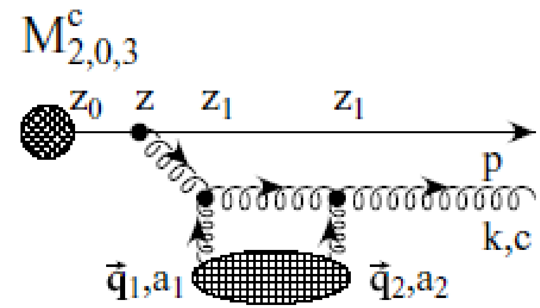
Final result:

$$M_{1,0,1} = J(p)e^{ipz_0}\,(-i)\int \frac{d^2\mathbf{q}_1}{(2\pi)^2} v(0,\mathbf{q}_1)\, e^{-i\mathbf{q}_1\cdot\mathbf{b}_1} \times$$
$$\times\, 2ig_s\,\frac{\boldsymbol{\epsilon}\cdot(\mathbf{k}-\mathbf{q}_1)}{(\mathbf{k}-\mathbf{q}_1)^2}\, e^{i(\omega_0-\omega_1)z_1}\left(e^{i\omega_1 z_1} - e^{i\omega_1 z_0}\right)[c,a_1]T_{a_1}$$

Double Scattering
- The Strictly Unitary Part

- **For double or "virtual" scattering we can perform all the integrals at the amplitude level (We know there will be no scattering in the complementary amplitude at the same position)**





This one will be relevant when we compute larger number of scatterings

$$M^c_{2,0,3} \approx J(p)e^{ipx_0}(-i)\int \frac{d^2\mathbf{q}_1}{(2\pi)^2}\, v(0,\mathbf{q}_1)\, e^{-i\mathbf{q}_1\cdot\mathbf{b}_1}(-i)\int \frac{d^2\mathbf{q}_2}{(2\pi)^2}\, v(0,\mathbf{q}_2)\, e^{-i\mathbf{q}_2\cdot\mathbf{b}_2} \times$$

$$\times \frac{1}{2}(2ig_s)\frac{\epsilon\cdot(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)}{(\mathbf{k}-\mathbf{q}_1-\mathbf{q}_2)^2}\, e^{i\omega_0 z_1}\left(1-e^{-i\omega_{(12)}(z_1-z_0)}\right)\,[[c,a_2],a_1](T_{a_2}T_{a_1})$$

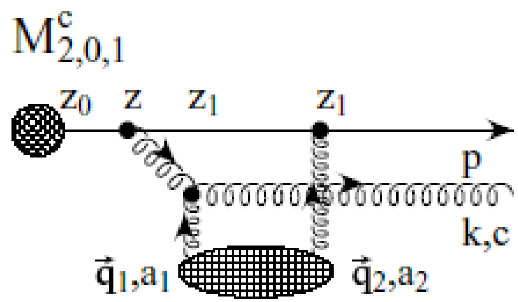
This factor ½ can be considered a symmetry factor in the case of identical momentum exchanges

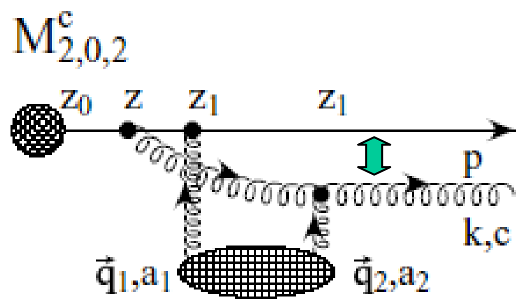
The diagram looks as if there is mo momentum exchange (all the strength in the forward direction $q_1 + q_2 = 0$). The color can also be simplified $[[c,a_1],a_1] = C_A c$

These are unitary corrections to the jet (= parent parton) and gluon elastic scattering

Double Scattering - System Broadening

Looks like the gluon getting a transverse momentum kick



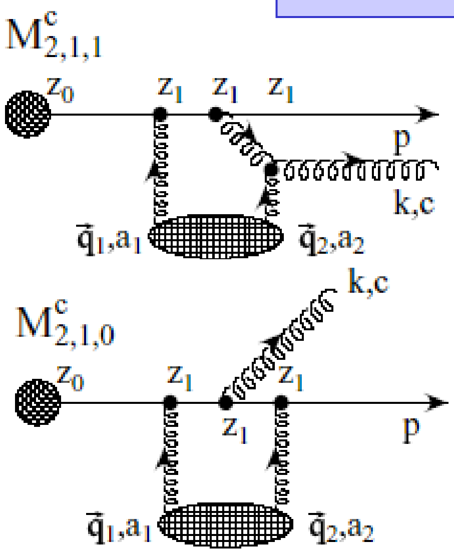


$$M^c_{2,0,1} = J(p)e^{ipx_0}(-i)\int \frac{d^2\mathbf{q}_1}{(2\pi)^2}e^{-i\mathbf{q}_1\cdot\mathbf{b}_1}v(0,\mathbf{q}_1)(-i)\int\frac{d^2\mathbf{q}_2}{(2\pi)^2}e^{-i\mathbf{q}_2\cdot\mathbf{b}_2}v(0,\mathbf{q}_2)\ \times$$

$$\times\ 2ig_s\frac{\epsilon\cdot(\mathbf{k}-\mathbf{q}_1)}{(\mathbf{k}-\mathbf{q}_1)^2}\ e^{i(\omega_0-\omega_1)z_1}(e^{i\omega_1 z_1}-e^{i\omega_1 z_0})\ a_2[c,a_1](T_{a_2}T_{a_1})\ .$$

- The color may or may not be simplified easily (locally)
- These look very similar in structure to the single Born diagrams (one momentum exchange with the gluon)
- There is a "-" sign and two possible attachments that cancel the factor of ½.

Power Suppressed Terms



$$M^c_{2,1,1} \approx J(p)e^{ipx_0}(-i)\int \frac{d^2\mathbf{q}_1}{(2\pi)^2} e^{-i\mathbf{q}_1\cdot\mathbf{b}_1}(-i)\int \frac{d^2\mathbf{q}_2}{(2\pi)^2} e^{-i\mathbf{q}_2\cdot\mathbf{b}_1} \times$$
$$\times\, 2ig_s \frac{\epsilon\cdot\mathbf{k}}{\mathbf{k}^2} e^{i\omega_0 z_0}\, a_2 c a_1 (T_{a_2}T_{a_1})\, (E^+)^2 \int \frac{dq_{1z}}{(2\pi)}\frac{dq_{2z}}{(2\pi)}\, v(\vec{q}_1)v(\vec{q}_2) \times$$
$$\times\, \frac{e^{-i(q_{1z}+q_{2z})(z_1-z_0)}}{(p+k-q_1-q_2)^2+i\epsilon}\left(\frac{1}{(p-q_2)^2+i\epsilon} - \frac{1}{(p+k-q_2)^2+i\epsilon}\right) .$$

- **The integral over q_{z1} is taken first and brings $1/E^+$**

$$I_3 = \int \frac{q_{2z}}{2\pi}\, v(-q_{2z},\mathbf{q}_1)v(q_{2z},\mathbf{q}_2)\left(\frac{1}{(p-q_2)^2+i\epsilon} - \frac{1}{(p+k-q_2)^2+i\epsilon}\right) .$$

$$\mathrm{Res}(i\mu_1) \approx \frac{(4\pi\alpha_s)^2}{E^+(2\mu_1^2)(\mu_2^2-\mu_1^2)}\left(-\frac{k^+}{E^+}\right) ,$$
$$\mathrm{Res}(i\mu_2) \approx \frac{(4\pi\alpha_s)^2}{E^+(2\mu_2^2)(\mu_1^2-\mu_2^2)}\left(-\frac{k^+}{E^+}\right) .$$

- **Note that when summed the potential divergence $\mu_1^2 = \mu_2^2$ goes away. These diagrams are suppressed relative To the others by a large factor k^+/E^+**

- **In terms of time ordered perturbation theory you see that there is no support for the integral (integration range)**

$$M \sim \int_{z_1}^{z_1} f(z)dz = 0$$

"Probability" Result

Propagators:

Hard $\mathbf{H} = \frac{\mathbf{k}}{\mathbf{k}^2}\,,$ $\mathbf{C}_{(i_1 i_2 \cdots i_m)} = \frac{(\mathbf{k} - \mathbf{q}_{i_1} - \mathbf{q}_{i_2} - \cdots - \mathbf{q}_{i_m})}{(\mathbf{k} - \mathbf{q}_{i_1} - \mathbf{q}_{i_2} - \cdots - \mathbf{q}_{i_m})^2}\,,$ Cascade

Bertsch-Gunion $\mathbf{B}_i = \mathbf{H} - \mathbf{C}_i\,,$ $\mathbf{B}_{(i_1 i_2 \cdots i_m)(j_1 j_2 \cdots i_n)} = \mathbf{C}_{(i_1 i_2 \cdots j_m)} - \mathbf{C}_{(j_1 j_2 \cdots j_n)}\,.$

- Up to color factors 2-s, pi-s, average over the momentum transfers the probability of medium induced gluon emission or the medium induced gluon number is

$$P_1 = C_A C_R \left(\mathbf{C}_1^2 - \mathbf{H}^2 + \mathbf{B}_1^2 + 2\mathbf{B}_1 \cdot \mathbf{C}_1 \cos(\omega_1 \Delta z_1) \right) = -2 C_A C_R \mathbf{B}_1 \cdot \mathbf{C}_1 (1 - \cos(\omega_1 \Delta z_1))$$

Explicitly in terms of the momentum transfer:

$$P_1 = C_R C_A \frac{2 k_\perp \cdot q_\perp}{k_\perp^{\,2} (k_\perp - q_\perp)^2} \left(1 - \cos \frac{(k_\perp - q_\perp)^2}{2\omega} \Delta z_1 \right)$$

Dominated by the gluon rescattering

Let us look at a few limits

- In the collinear limit $k_\perp \to q_\perp$ or for very soft gluons $\omega \to 0$ the phases cancel the singularity

- In the collinear limit $k_\perp \to 0$ the angular average over for fixed momentum transfer kills the contribution

$$\frac{1}{k_\perp^{\,2}} \int d\varphi \; k_\perp \cdot q_\perp$$

The answer is well behaved

Radiation Intensity and Formation Time

- Model the medium as gluon dominated and us the approximate elastic gluon -gluon scattering cross section

Approximately $\sigma_{el}^{gg} = \dfrac{9\pi\alpha_s^2}{2\mu^2}$

M.Gyulassy, P.Levai, I.V., Phys.Rev.Lett.85 (2000)

$$\frac{dI^{(1)}}{dx} = \frac{9C_R E}{\pi^2}\int_{z_0}^{\infty} dz\,\rho(z)\int^{|\mathbf{k}|_{\max}} d^2\mathbf{k}\,\alpha_s \int^{|\mathbf{q}|_{\max}} \frac{d^2\mathbf{q}\,\alpha_s^2}{(\mathbf{q}^2+\mu(z)^2)^2} \frac{\mathbf{k}\cdot\mathbf{q}}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}\left[1-\cos\left(\frac{(\mathbf{k}-\mathbf{q})^2}{2xE}(z-z_0)\right)\right] .$$

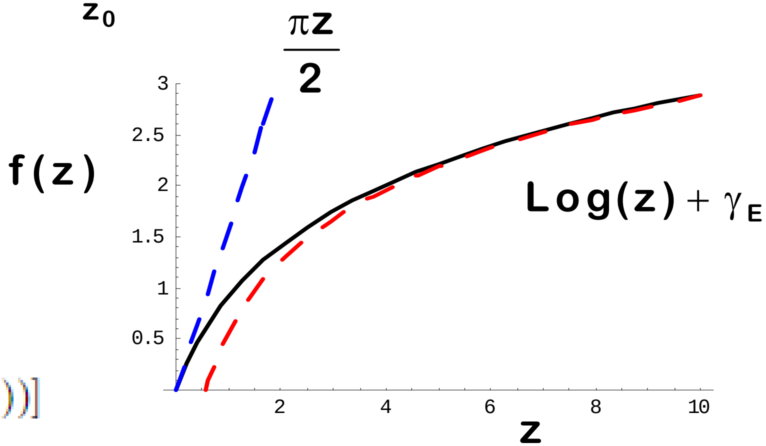
$$\Delta E^{(1)} = \int_0^1 dx\,\frac{dI^{(1)}}{dx} = E_0\,\frac{2C_R\alpha_s}{\pi}\int_0^1 dx\int_{z_0}^{\infty} dz\;\sigma(z)\rho(z,z)\;f(Z(x,z))$$

Formation parameter

$$Z(x,z) = \frac{\mu^2(z)}{2xE}(z-z_0) = \frac{\Delta z}{\tau_{form}}$$

Formation function

$$f(x,z) = \int_0^\infty \frac{du}{u(1+u)}\left[1-\cos\left(uZ(x,z)\right)\right] \approx \frac{\pi Z}{2} + \frac{Z^2}{2}\log(Z) + \mathcal{O}(Z^2) .$$



Analytic limits

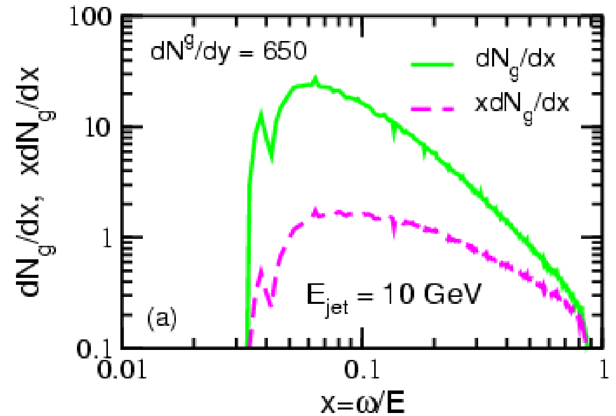
First Order Radiative Energy Loss

$$\Delta E^{(1)} \approx \frac{2C_R\alpha_s}{\pi}\int_{z_0}^{\infty}\frac{dz}{\lambda_g(z)}\left\{\int_{x_c}^{1}\frac{dx}{x}\frac{\pi}{4}\mu^2(z)(z-z_0)+E\int_0^{x_c}dx\,\log\left[\frac{x_c}{x}\right]\right\}$$

$$\approx \frac{C_R\alpha_s}{2}\int_{z_0}^{\infty}\frac{dz}{\lambda_g(z)}\mu^2(z)(z-z_0)\left\{\mathrm{Log}\frac{2E}{\mu^2(z)(z-z_0)}+O(1)\right\}$$

Linear Regime : "Thin Plasma"

$$Z(x,z)\ll 1 \;\Rightarrow\; x_c \equiv \frac{\mu^2(z)}{2E}(z-z_0) \ll x \le 1$$



Dynamically expanding media (Bjorken expansion)

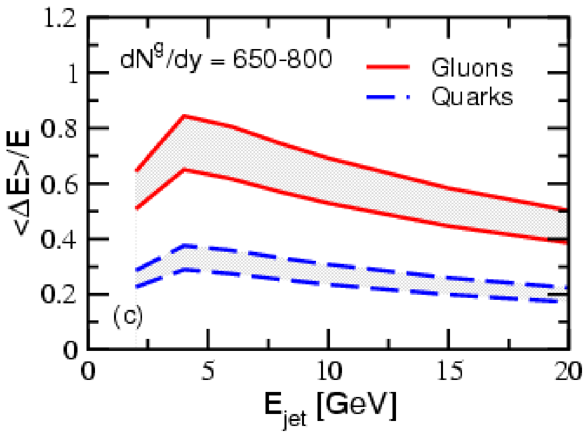
$$\rho(\tau)=\frac{1}{\tau\pi R^2}\frac{dN_g}{dy}$$

$$\Delta E^{(1)} \approx \frac{C_R\alpha_s}{4}\frac{\mu^2 L^2}{\lambda_g}\mathrm{Log}\frac{2E}{\mu^2(L)L}+\dots\,,$$

– Static medium

$$\Delta E^{(1)} \approx \frac{9\pi C_R\alpha_s^3}{4}L\,\frac{1}{A_\perp}\frac{dN^g}{dy}\mathrm{Log}\frac{2E}{\mu^2(L)L}+\dots\,,$$

– 1+1D Bjorken



μ^2/λ - transport coefficient

dN^g/dy - effective gluon rapidity density

Numerically slow $\Delta E/E$ dependence

More Explicit

Just the relevant part of the integrand

Massless

$$P_1 = C_R\, C_A \frac{2k_\perp \cdot q_\perp}{k_\perp{}^2 (k_\perp - q_\perp)^2} \left(1 - \cos \frac{(k_\perp - q_\perp)^2}{2\omega} \Delta z_1 \right)$$

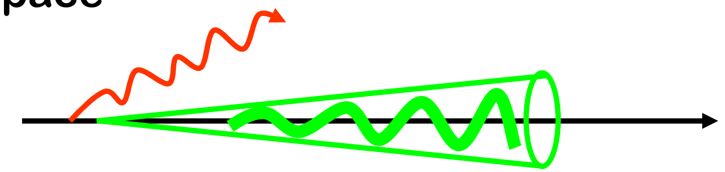
Massive

$$P_1 = C_R\, C_A \frac{2k_\perp \cdot q_\perp}{\left(k_\perp{}^2 + x^2 M^2\right)\left((k_\perp - q_\perp)^2 + x^2 M^2\right)} \left(1 - \cos \frac{(k_\perp - q_\perp)^2 + x^2 M^2}{2\omega} \Delta z_1 \right)$$

Don't keep massive terms here

Mass Effects on E-Loss

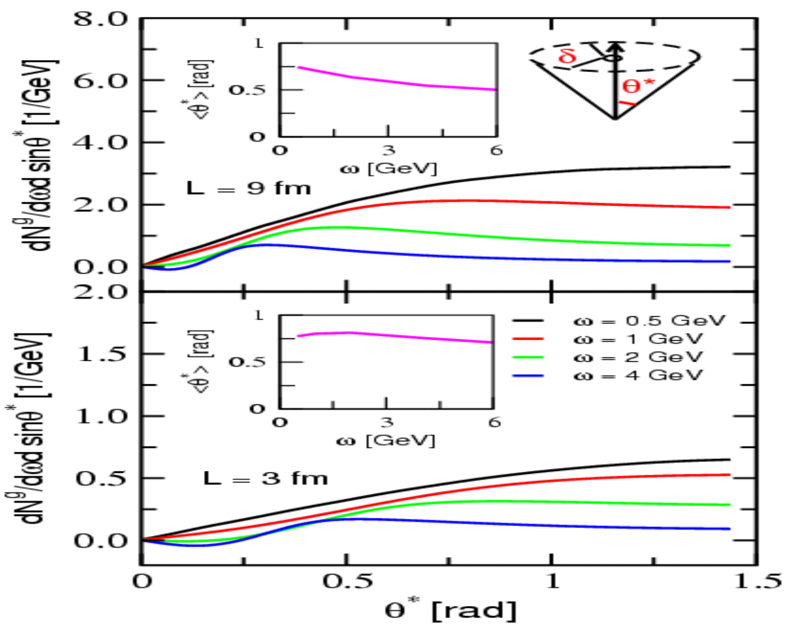
Cutting out part of the available phase space



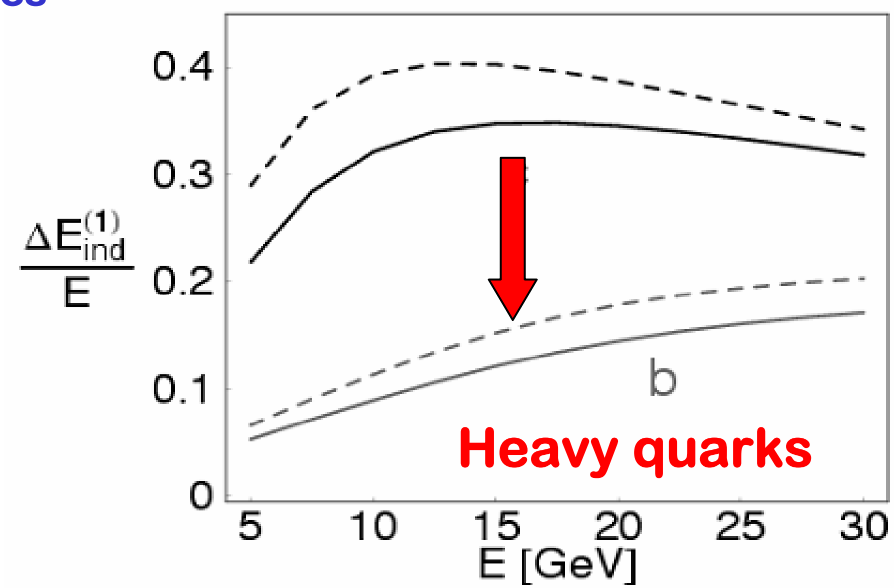
$$\left[\omega_{(1\ldots n)}\right]^{-1} \to \left[\omega_{(1\ldots n)} + \frac{m_g^2 + x^2 M^2}{2xE}\right]^{-1}$$

$$\frac{\vec{k}_\perp}{\vec{k}_\perp^{\,2}} \to \frac{\vec{k}_\perp}{\vec{k}_\perp^{\,2} + m_g^2 + x^2 M^2}, \quad x = \frac{k^+}{p^+} \approx \frac{\omega}{E}$$

Note that the characteristic features of E-loss are related to the interference phases



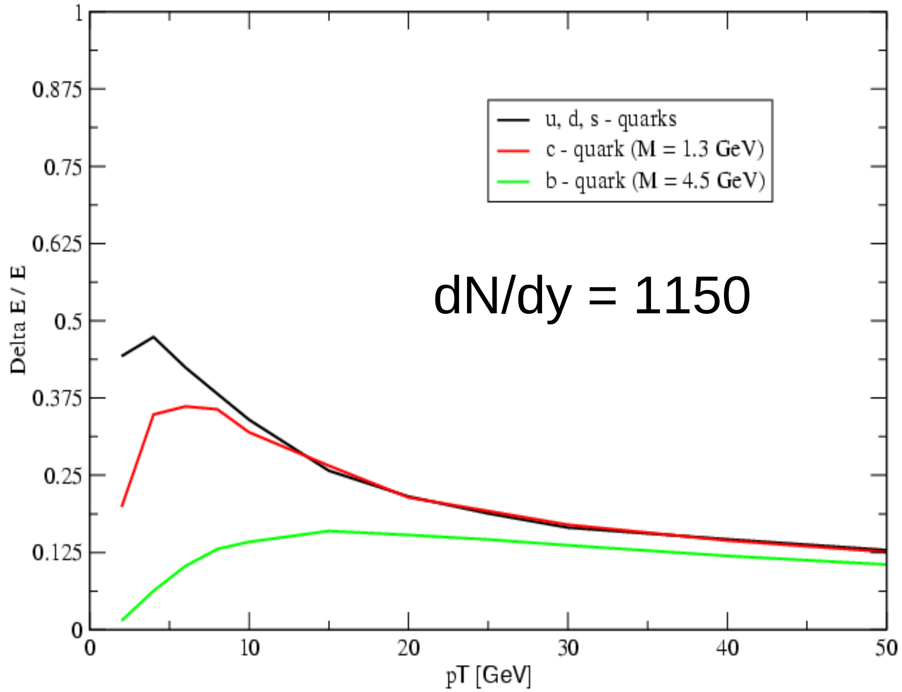
I.V., hep-ph/0501255



Reduction of E-loss

M.Djordjevic, M.Gyulassy, Nucl.Phys.A (2004)

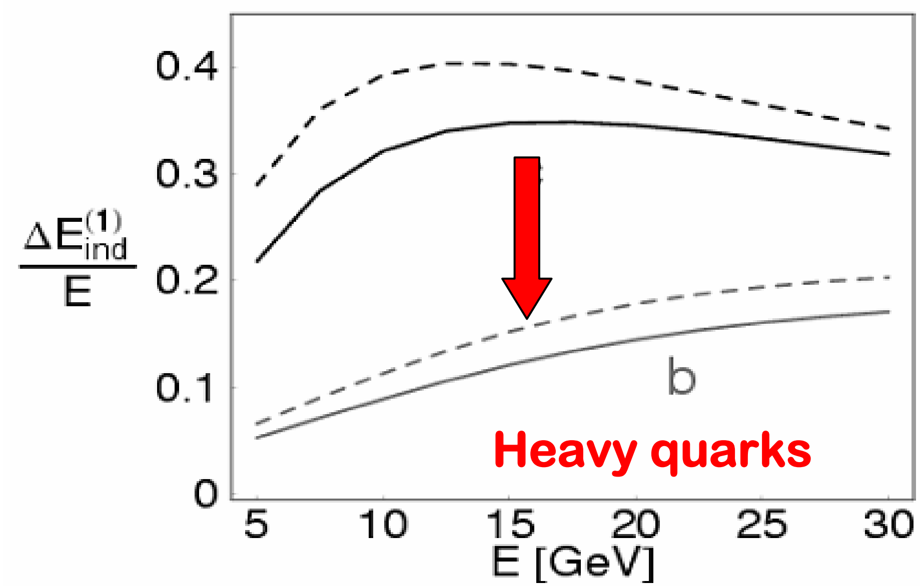
Recent Calculation



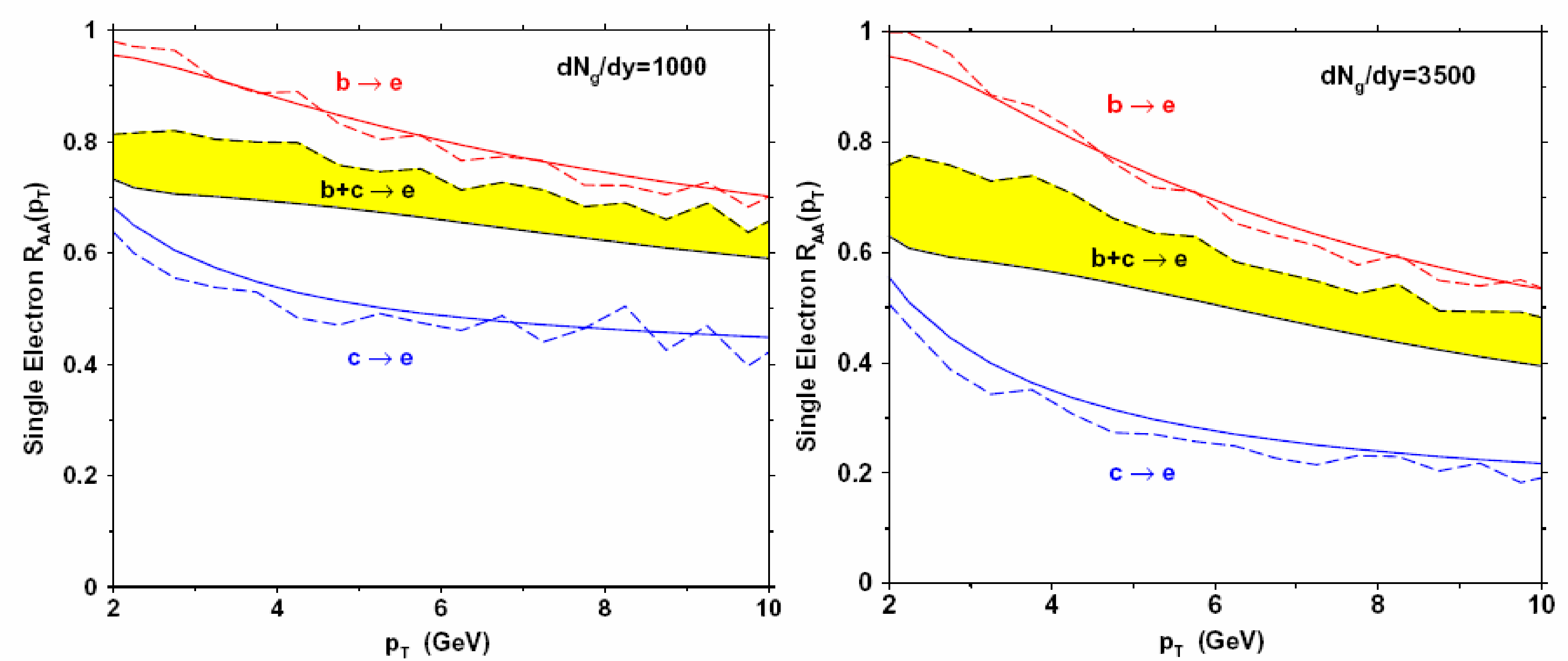
Qualitatively and quantitatively compatible

Small effect relative to light quarks

Next part is the implementation, not yet done



Heavy Quarks R_{AA}



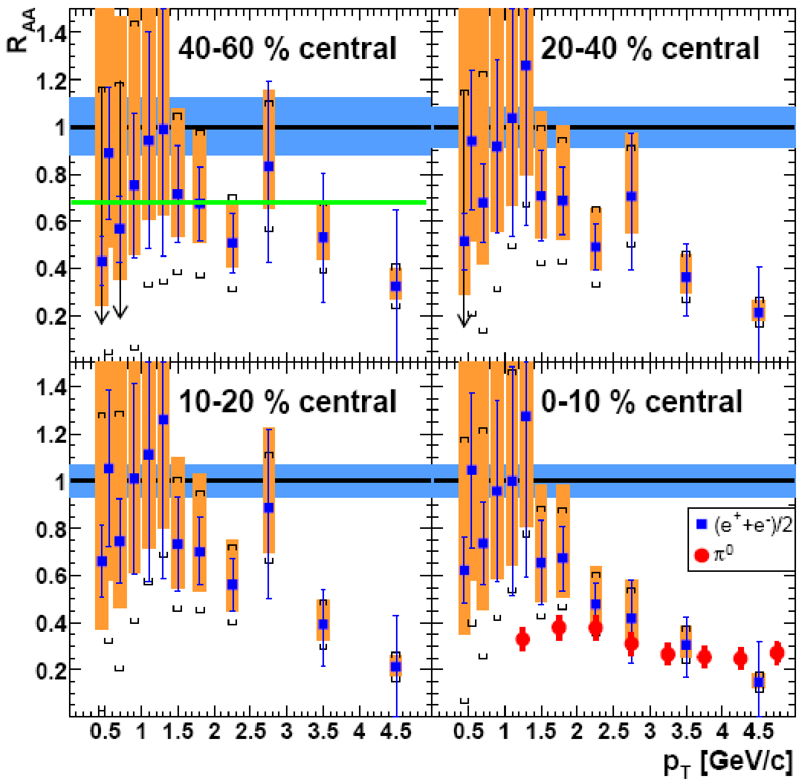
One should be careful about the physical meaning of the parameters!

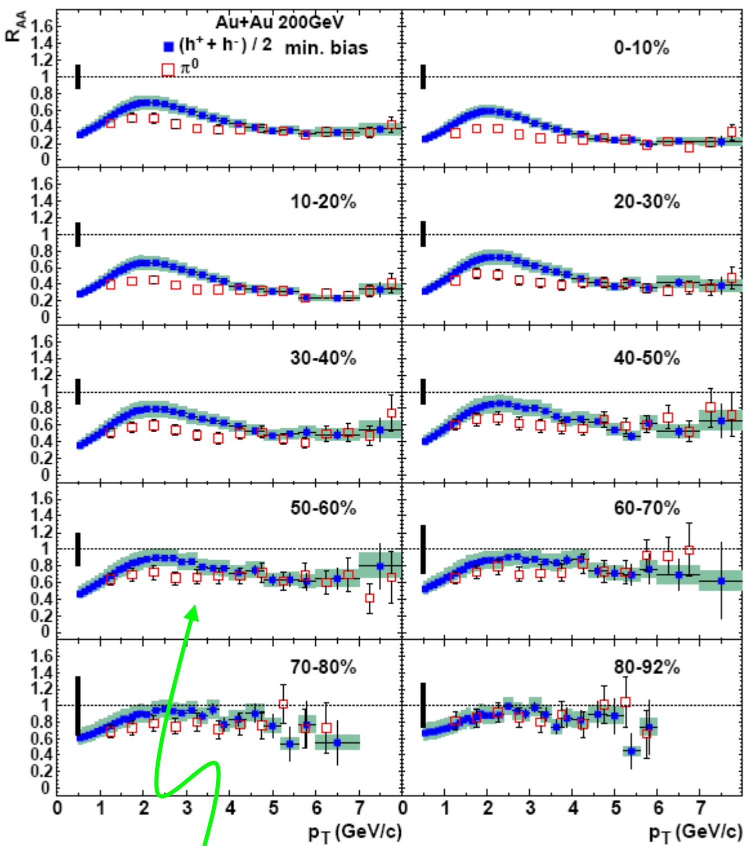
$dN^g / dy = 3500 \qquad \hat{q} = 15\ GeV^2 / fm$ Where does one get such parameters from?

Are these leptons from heavy mesons? (Coctail methods…) FVTX

What are the different attenuation mechanisms for heavy mesons?

Experimental Issues R_{AA}





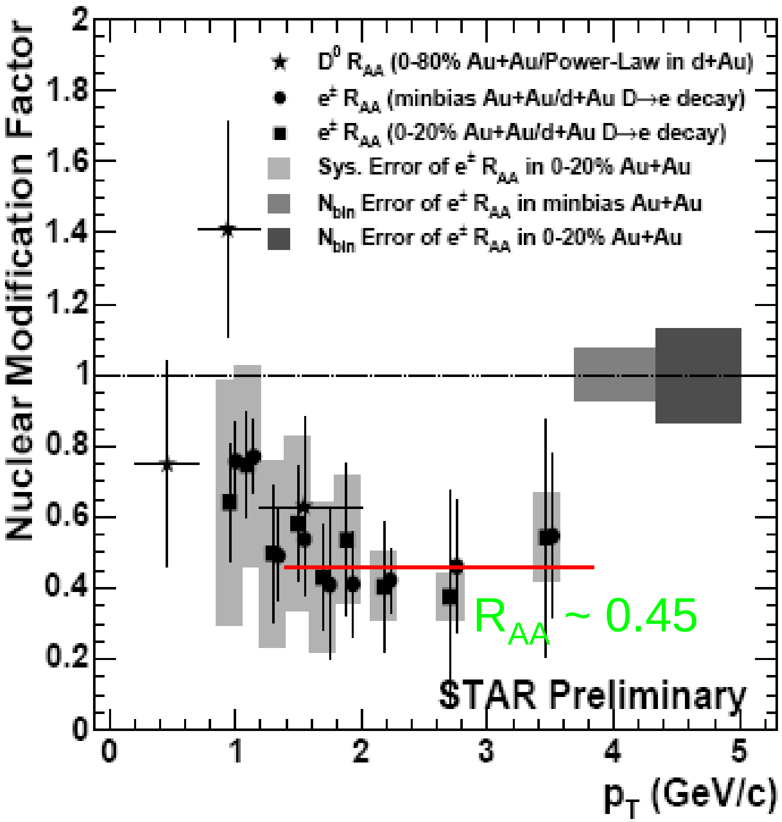
I would be worried about those points

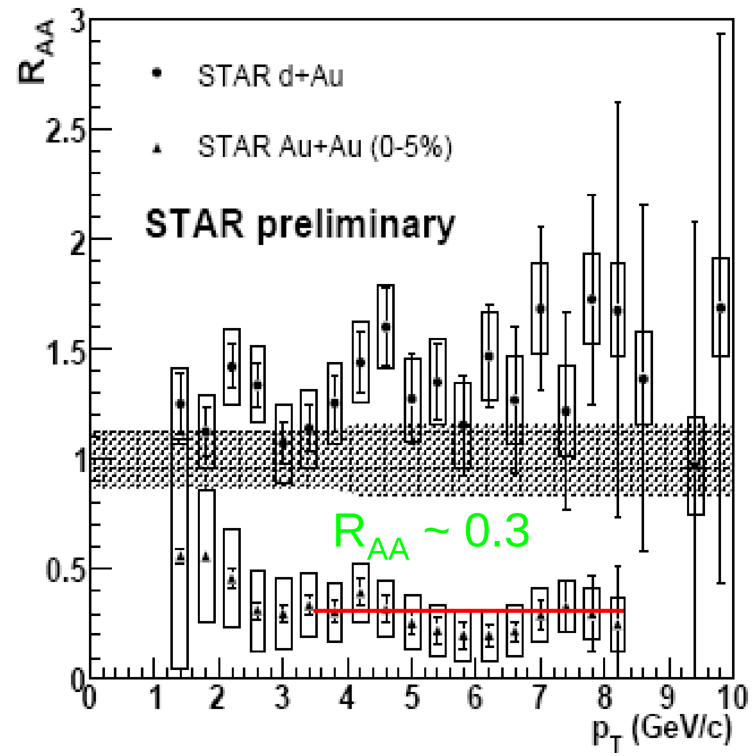
Look here

Experimental Issues R_{AA}

Statement: consistent with very large densities

(itself may be inconsistent)



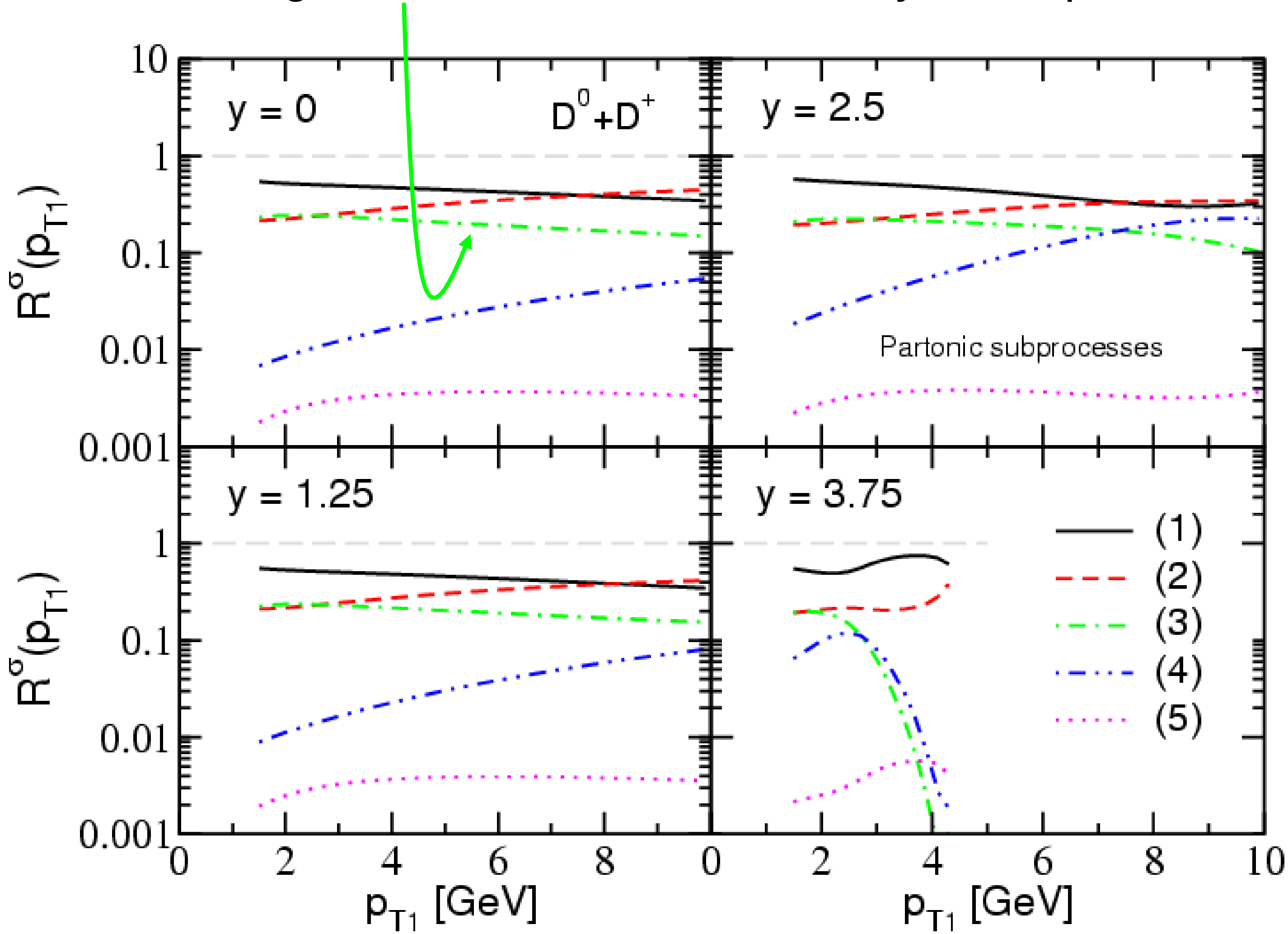


The statement is to be checked

Bottom quark contributions

Certainly the Baseline

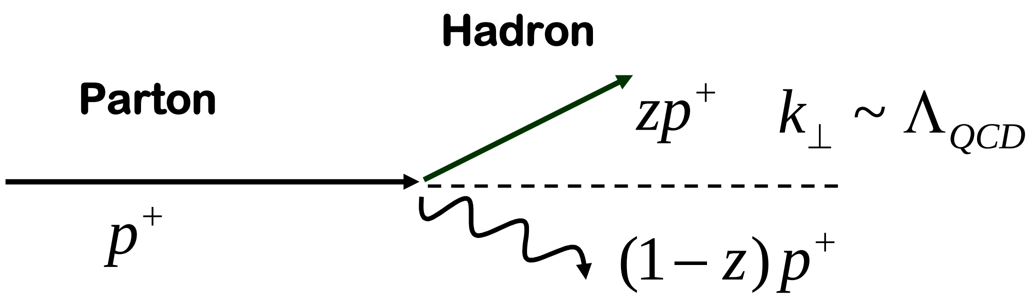
We all know that gluon fusion dominates heavy flavor production, right?



Formation Time

- **From the uncertainty principle:** $\tau_f \simeq 1/\Delta Q$

$$p_q = \left[p^+, \frac{M_q^2}{2p^+}, 0 \right]$$



$$p_h = \left[zp^+, \frac{k_\perp^2 + m_h^2}{2zp^+}, k_\perp \right]$$

$$p_g = \left[(1-z)p^+, \frac{k_\perp^2}{2(1-z)p^+}, -k_\perp \right]$$

$$\Delta y^+ = \frac{1}{\Delta p^-} = \frac{(0.2\ GeV.fm)\ 2z(1-z)p^+}{k_\perp^2 + (1-z)m_h^2 - z(1-z)M_q^2}$$

Formation time (p_T = 5 GeV):

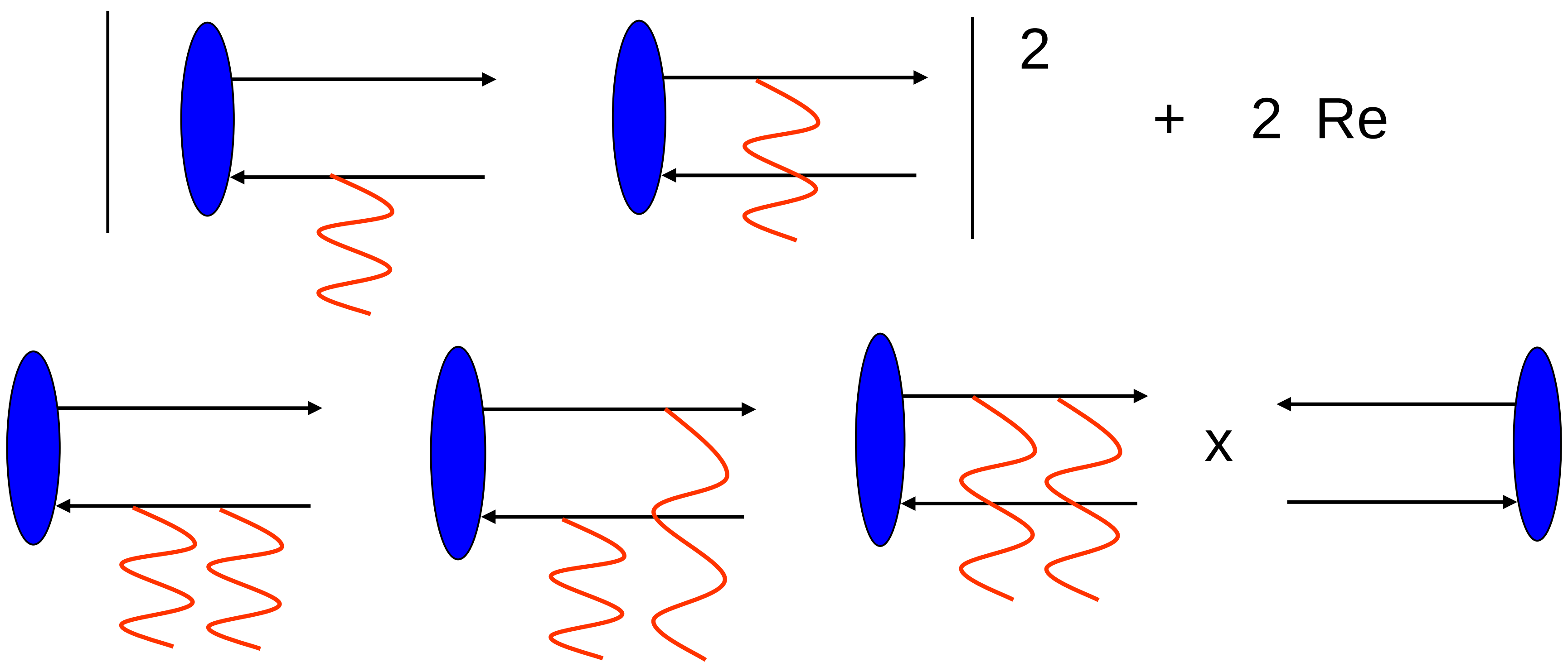
π	D	B
12 fm	**1.5 fm**	**0.25 fm**

The simpler argument: "the gamma boosted size of the hadron" gives qualitatively the same results

$$\tau_f = \gamma_h R_h \approx \gamma_h \times 1\ fm$$

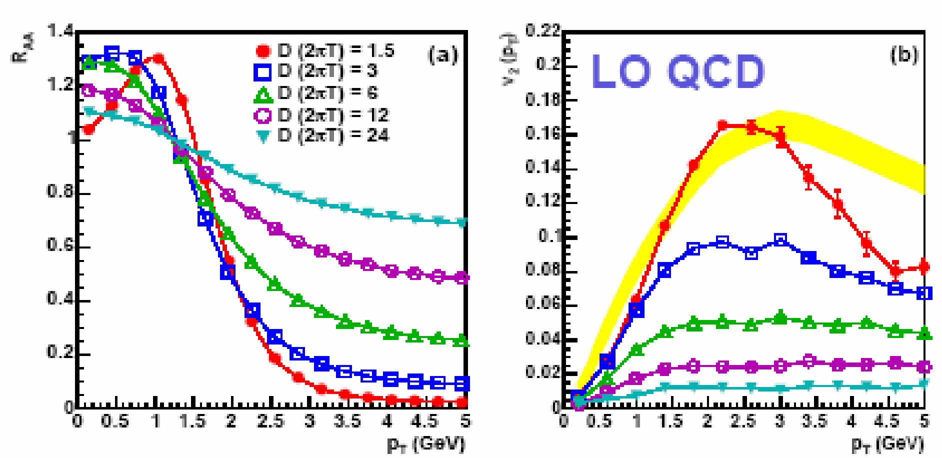
The incorrect argument: $k_T \to p_T \quad \Delta y \sim \dfrac{2z(1-z)}{p_T}$ NOT correct

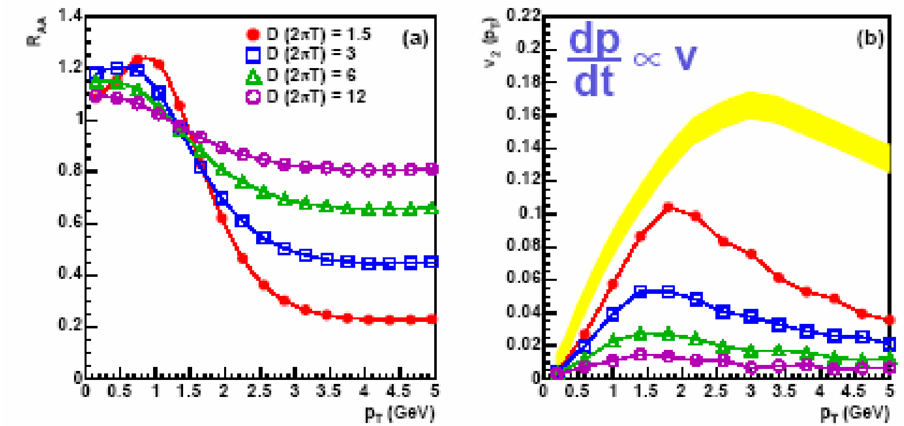
Additional Effects



New probe of the strength of the interactions in the medium

Heavy Quark Diffusion





Relation to viscosity

$$\eta_D(0)^{-1} = \frac{M}{T} D = \frac{2TM}{\kappa_L(0)}$$

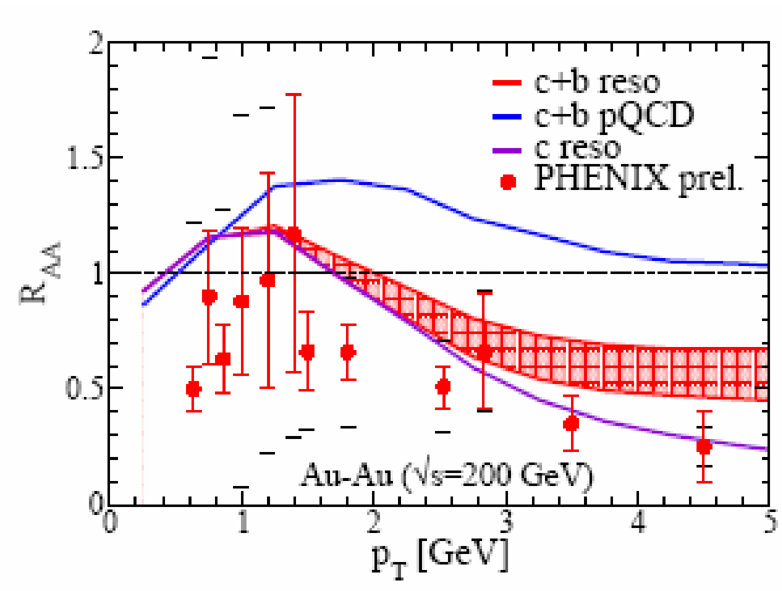
Perturbative

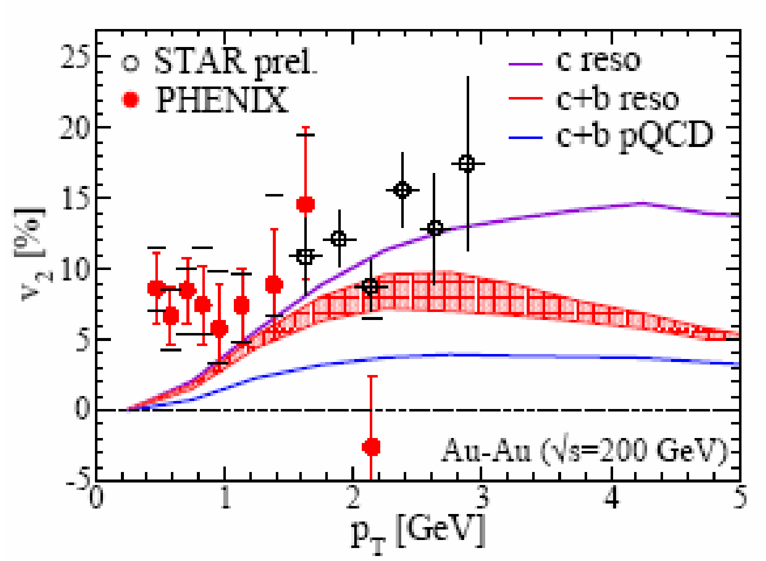
$$D \times (2\pi T) \approx 6\,(0.5/\alpha_s)^2.$$

Does not connect to the energy loss of light quarks

Recovers the known correlation between v_2 and R_{AA}

Charm Resonances





17, 18], cf. also Refs. [19, 20, 21]. Here, we simply assume the existence of the lowest-lying, pseudoscalar D (B) meson as a *resonance* 0.5 GeV above the heavy-light quark threshold [13]. The pertinent effective Lagrangian with chiral and heavy-quark (HQ) symmetry then dictates the degeneracy of the $J^P=0^-$ state with vector, scalar and axial-vector partners. The 2 free model parameters are the resonance masses ($m_{D(B)}$=2(5) GeV, with $m_{c(b)}$=1.5(4.5) GeV) and widths (varied as Γ=0.4-0.75 GeV). For strange quarks we only include pseudoscalar and vector states. The resonant Q-$\bar{q}$ cross sections are supplemented with leading-order pQCD scattering off partons [22] dominated by t-channel gluon exchange and regularized by a Debye mass $m_g=gT$ with $\alpha_s=g^2/(4\pi)=0.4$. When evaluating drag and diffusion coefficients in a Fokker-Planck approach [10], the resonances reduce HQ thermalization times by a factor of $\sim$3 below pQCD scattering [13].

Summary

- **Single inclusive particle quenching at high p_T:**

  Derivation of pQCD factorized formulas
  Derivation of E-loss formulas
  Derivation of Heavy Quark E-loss formulas

- **My perspective of the current data status:**

  Really needs direct measurements

- **Other heavy quark calculations:**

  Elastic energy loss, transport coefficients
  In medium resonances
  Both much closer relation to hydro and transport

- **Possible directions:**

  Dissociation of mesons in the medium via the broadening